Comment on "Topological Quantum Effects for Neutral Particles"

Recently Aharonov and Casher[1] (AC) conceived a striking variant of the classic Aharonov-Bohm (AB) effect,[2] which itself is already long confirmed by experiment[3] and by consistency requirements of quantum mechanics.[4,5] In the AB effect, electrons diffracted around either side of a long impenetrable flux tube aligned perpendicularly to the plane of electron motion exhibit an interference pattern which shifts as the flux changes, repeating when the change is one flux unit hc/e.

In the new AC effect, neutral particles with magnetic moment aligned perpendicularly to their plane of motion are diffracted by an impenetrable line charge, also aligned perpendicularly to the plane. As in the AB effect, there is a negligible velocity-dependent force, but still a phase shift occurs, this time proportional to the line-charge density.

The purpose of this Comment is to emphasize several aspects which make the AC effect remarkable in its own right, illuminating matters which do not arise for the AB effect. First, the very fact that a magnet aligned in the specified way feels no force in the line-charge electric field requires demonstration. It is neither sufficient nor correct to rely on the standard formula for the force on a magnet in a slowly varying magnetic field,

$$\mathbf{F} = \nabla \mu \cdot \mathbf{B} . \tag{1}$$

Secondly, if indeed Eq. (1) has indicated the correct force law, the fringe shift predicted by AC would have been quenched, so that experimental verification of the new effect would do more than reconfirm the AB effect; it would also give a subtle quantum check of the principle of momentum conservation. Finally, the AC system is not only different, but also intrinsically richer in possibilities than that of AB.

The elements of the needed force analysis were presented by Thomson[6] in 1904, and by Costa de Beauregard,[7] Shockley and James,[7] and Coleman and Van Vleck[7] in 1967 and 1968. As realized also by AC, the key point one needs to grasp is that for total momentum conservation the force on a current-loop magnet must include a term equal to $d\mathbf{P}_{\rm em}/dt$, where $\mathbf{P}_{\rm em}$ is the electromagnetic momentum localized on the magnet,

$$\mathbf{P}_{\rm em} = \mathbf{E} \times \mu / c , \tag{2}$$

and $\mathbf{E}$ is the electric field at the magnet. Because of this extra term, the total force (evaluated in the magnet's rest frame) due to faraway sources is the same whether the magnet is made of a current lop or a pole-antipole pair,

$$\mathbf{F} = \mu \cdot \nabla \mathbf{B} + \mathbf{E} \times d\mu / cdt . \tag{3}$$

For the case of AC this indeed leads to a force-free, velocity-dependent interaction, and hence a new form of the AB effect.

Boyer[8] has found the amusing result that if the "naive" force expression $\nabla \mu \cdot \mathbf{B}$ is used then there will be a spatial retardation of the one beam with respect to the other by exactly the fraction of a wavelength yielding the phase shift of the AC effect. However, since this incorrect force is nonconservative, one should also take into account phase shifts due to time dependence of the energy, with the result that a complete calculation with this wrong force law actually gives a much smaller net phase shift than the AC result. Thus experimental verification of the AC effect would confirm total momentum conservation in the interactions of magnets and charges, as well as illustrate the principles of the AB effect in a new way.

Among the important differences between the two effects are the following:

(1) In AB the flux tube must be endless but may be curved arbitrarily, even into a finite toroid, for which the effect has been verified experimentally,[9] but in AC the line charge must be straight and parallel to the magnetic moment.

(2) In AB the electromagnetic field strength vanishes in the region traversed by the scattered particles, while in AC the force vanishes for correct alignment, but the electric field does not.

(3) In AC there is an extra degree of freedom which deserves further attention, the spin orientation of the particle. When that spin is a quantum operator with noncommuting components, the interaction of a nonrelativistic magnetic particle with a Maxwell field becomes isomorphic to the interaction with a Yang-Mills potential of a particle carrying isospin but not spin, making such potentials observable at least in principle.[10]

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