Instability in a Sand Heap

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When a bead heap is submitted to low-frequency ($\omega/2\pi=10-1000$ Hz) vertical vibrations at a large enough amplitude a, its horizontal free surface becomes unstable and a stationary state is reached. This one is characterized firstly by a free surface which is inclined within an angle θ to the horizontal, secondly by a continuous flow of particles rolling down the free surface, and thirdly by a convective transport inside the material which refills the top of the heap with new beads. We demonstrate that a control parameter is the vibration acceleration $\gamma=a\omega^2$.

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It is well known that among the various processes which govern the physics of granular materials some of them are quite unusual (thixotropy, segregation, fluidization, arching, dilatancy, etc.), so that the properties of such materials are not well understood. Furthermore, the applied field of powders, granular materials, and composite materials exceeds the classical scope of physics since it also concerns many industrial activities such as pharmacology, chemical engineering, food and agriculture, and so on; it also concerns the natural environment since it deals with the formation of dunes, with erosion, wet sand, or land sliding, etc. In order to obtain a deeper understanding of this kind of material, we have undertaken a series of experiments concerning granular media.

For instance, we have recently observed[1] an instability in a cohesionless granular material submitted to vertical vibrations. Beyond a certain threshold, the horizontal free surface becomes unstable and exhibits a slope at an angle θ with the horizontal. It simultaneously appears as a permanent flow of avalanches on the free surface, and there is a convective transport of particles in the bulk from the bottom to the top (Fig. 1).

The experimental setup is the following. A parallelepipedic cell ($10\times5.5\times2$ cm^3) is partially filled up with monodisperse little glass spheres, the diameter of which varies from 0.2 to 2 mm, and the free surface is prepared horizontally. The box is fixed on a loudspeaker so that the bead heap can be vertically shaken, in the range of frequencies 10–1000 Hz. The exact displacement of the cell is precisely evaluated by photoelectric measurement.

At low amplitudes of vibrations, the beads remain motionless in the box reference frame. Beyond a given threshold of amplitude, which appears to depend on the frequency, a relative motion of beads is allowed which leads to internal convection transport together with a new stationary profile.

The final state of the free surface is always inclined within a slope θ. θ is always slightly smaller than the maximum angle of repose of the static heap; however, it lies near the angle. The choice of the direction of the slope corresponds to a spontaneous breaking of symmetry, but is reproducible (i.e., repeating the same experiment under the same conditions leads to the same result).

For much larger amplitude of vibrations, another permanent state appears, this convective instability is destroyed, and a chaotic behavior is observed, with temporal and spatial intermittencies of jets of beads at the free surface. The mean profile of the time average of the surface becomes horizontal again, if one can still define a surface.

de Gennes[2] has proposed a theoretical approach to this instability based on a series of alternate passive and active regimes.

(a) Passive regime.— When the cell is raised up, the beads are compacted so that no possible intergranular motion is allowed; the bead heap can be considered as a solid.

(b) Active regime.— When the cell is carried down, with an acceleration γ of the cell larger than the acceleration g of gravity, the system of beads is fluidized.

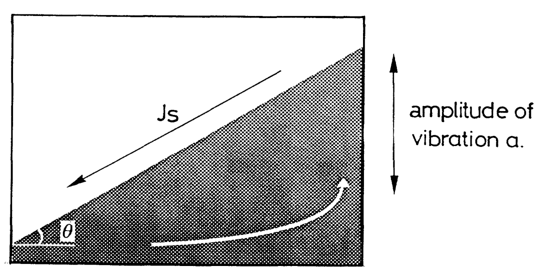

FIG. 1. Beyond a certain threshold of amplitude of vibrations, the free surface of a bead packing becomes inclined at θ to the horizontal. Simultaneously, there is a convective transport of matter from the bottom to the top and a permanent current J_s rolling down the surface face.

During these active fractions f of the period, first relative interparticles motion are allowed, and second, beads are submitted to an apparent gravity $g_a \sim \gamma - g$ reversed upward. The surface of the fluidized bead therefore becomes unstable and exhibits a gravitational instability. In this case, a sinusoidal perturbation of the surface of wave-number k grows exponentially with time as $\exp[(g_a|\mathbf{k}|)^{1/2} ft]$. In addition, de Gennes[2] introduces a mechanism which limits the increase of bumps at the interface. During the passive fraction of the period, the granular material behaves like a solid. So, the slope of any bump cannot pass the maximum value for the angle of repose of the bead heap at equilibrium. The excess of matter which has been raised up on bumps during the active lapse of time will then flow as avalanches downward during the passive regime. So, de Gennes' model[2] predicts the existence of a permanent surface flow of particles, as is experimentally observed.[1]

Our purpose has been to determine the efficient parameters of the instability threshold. For instance, plausible relevant parameters are the magnitude a and the frequency $\omega/2\pi$ of the alternative displacement of the cell, the size of the particles, and the dimensions of the container. In the scheme of a gravitational instability of the free surface, it is obvious that an important parameter is the amplitude $\gamma = a\omega^2$ of the acceleration of the cell compared to the acceleration g of gravity. A parameter which would likely take place for the fluidization process of the granular medium is the ratio of the size of the beads to the magnitude a of vibrations of the container. The dimension of the cell has to be considered when spatial extension of boundary effects reaches the dimensions of the box.

In Fig. 2 we show the magnitude of the displacement of the cell at threshold as a function of the frequency. We deduce from it that the threshold magnitude can accurately be fitted with the following law:

$$A = \gamma_t / \omega^{2.0 \pm 0.1} .$$

This confirms that the acceleration $\gamma = a\omega^2$ is a key parameter which in turn strengthens the theoretical approach of de Gennes,[2] since $a\omega^2$ is the maximum acceleration undergone by the cell during a cycle. This experiment has been repeated on glass spheres of different diameters (0.2, 0.4, 1, and 2 mm).

As shown in Fig. 2, the parameter γ_t seems to be independent of the size of the beads (at least for the 0.2-, 0.4-, and 1-mm cases). This is inconsistent with a range of boundary effects proportional to the size of the beads. On the contrary, no instability has been evidenced with the 2-mm-diam bead for this geometrical shape of container. Such a phenomenon might be related to the existence of arch effects which would inhibit internal convective motion.

The experimental value for the parameter γ_t is 12.5 ± 1 m/s^2 and is undoubtedly larger than g but still of the same order of magnitude. Furthermore, the ratio γ_t/g seems to be independent of the diameter of the beads. A tentative explanation is that γ must fairly exceed g to produce an efficient mechanism of fluidization of the granular medium.

We have also repeated these measurements in vacuum. No difference for the value of the instability threshold has been noted. This means that the collective behavior of the glass beads is independent of the surrounding fluid.

A hypothesis of de Gennes[2] is that the surface flux of particles may obey a power law as a function of the

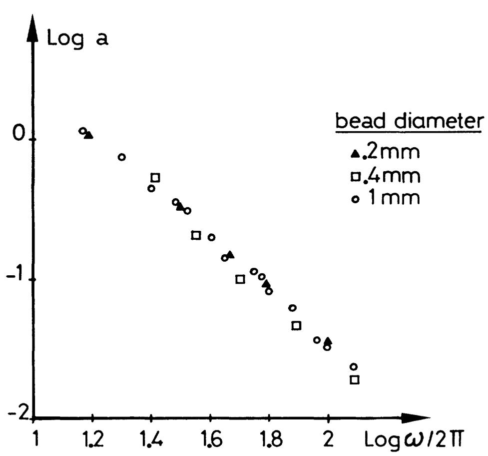

FIG. 2. Threshold amplitude of vibrations required to generate the instability of the free surface (see Fig. 1) plotted as a function of the frequency $\omega/2\pi$ (log-log plot). This experiment has been repeated for different sizes of beads (▲ = 0.2 mm, □ = 0.4 mm, ○ = 1 mm).

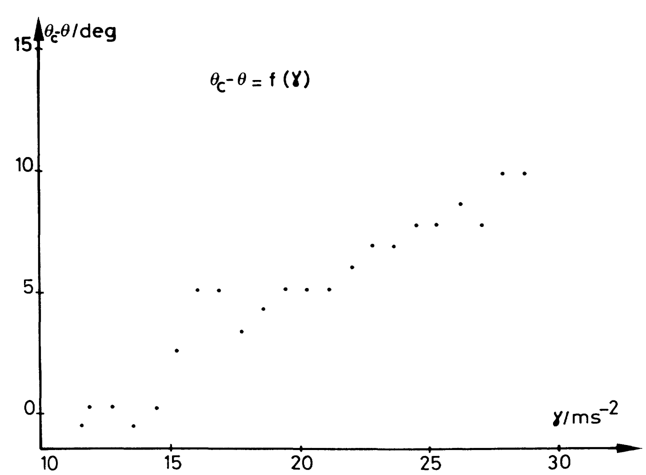

FIG. 3. Difference $\theta_c - \theta$ between the static maximum angle of repose θ_c and the dynamic angle of the free surface in the stationary regime θ as a function of the control parameter $a\omega^2$.

difference $(\theta-\theta_c)$ between the nonequilibrium slope θ of the heap in the permanent convective regime and the static slope θ_c:

$$J_s \sim (\theta-\theta_c)^m .$$

This behavior for J_s vs $\theta-\theta_c$ has been obtained in simulations by Tang and Bak[3] who modeled an avalanche process.

However, we have noticed that in the permanent convective regime the slope θ of the heap is always smaller than the static equilibrium one, θ_c. A plausible explanation might be that at the end of the active phase the surface beads have a nonzero speed, so that avalanches may exist for angles smaller than θ_c.

In Fig. 3 we show the difference $\theta_c-\theta$ as a function of the experimental control parameter $a\omega^2$. It does not exhibit any discontinuity or reentrance. We conclude that the reported instability of the free surface is a supercritical transition.

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[1]J. Rajchenbach and P. Evesque, C. R. Acad. Sci. Ser. 2 **307**, 1 (1988); P. Evesque and J. Rajchenbach, C. R. Acad. Sci. Ser. 2 **307**, 223 (1988).

[2]P. G. de Gennes, to be published.

[3]C. Tang and P. Bak, Phys. Rev. Lett. **60**, 2347 (1988).