Theoretical Study of a Superconducting-Glass Model

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The nature of the metastable magnetic properties of a random Josephson-junction array in the presence of a constant transverse magnetic field is studied extensively by Monte Carlo simulations. The *quenched* magnetization and helicity modulus are calculated as a function of temperature, field, amounts of disorder, dimensionality, and time. The calculated quantities are found to be strongly thermal-history dependent, similar to the magnetic properties found in high-temperature oxide superconductors. We also suggest to test the results of this paper in artificially made arrays of random Josephson junctions.

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Among the many interesting properties found in the high-temperature oxide superconductors are their history-dependent magnetic properties. These were first found by Müller, Takashige, and Bednorz (MTB) in the ceramic superconductor La_2CuO_4:Ba,[1] and have been carefully studied in several of the known high-T_c ceramic superconductors by several groups.[2,3] An understanding of the sources and properties of the metastability found in these materials would be very useful both from the basic as well as the applied physics point of view. There have been essentially two proposals to interpret the metastable properties of these systems: the *superconducting-glass* model and the *flux-creep* model. It was suggested in the MTB paper that the source of the metastability may be similar to that found in spin-glass models. Their conjecture was motivated in part by the behavior of the *zero-field-cooled* (ZFC) and the *field-cooled* (FC) magnetizations which show striking qualitative similarities with those seen experimentally in the canonical spin-glasses like Cu:Mn, including the logarithmic time dependence of the decay of the FC magnetization when the field is switched off, and a de Almeida-Thouless[4] (AT) line. The alternative *flux-creep* interpretation is better known in the literature of conventional type-II superconductors.[5] It was developed by Anderson and was tested in several low-temperature superconductors.[5] Recently, Yeshurun and Malozemoff have found an AT-like nonergodicity line in *single crystals* of the 1:2:3 YBaCuO compound and gave qualitative flux-creep arguments that explain their experimental data.[6]

Our main goals in this paper are to improve our basic theoretical understanding of the glass model as well as to point out how to test and compare the theoretical predictions of the superconducting-glass model in well controlled experimental systems, e.g., Josephson-junction arrays that can be fabricated explicitly by use of modern photolithographic techniques. The model proposed by MTB to explain their data consists of a random array of Josephson junctions (RAJJ) in a transverse magnetic field. There is extensive evidence that the oxide superconductors behave like arrays of Josephson junctions in the low-field regime, and there have been several suggestions as to where the junctions are, from being intragranular to intergranular.[7] Recent experiments by Chaudhari *et al.* seem to indicate that the Josephson connections are mostly formed between the metallurgical grains.[8] The model studied in this paper is defined by the Hamiltonian,

$$\mathcal{H}=\sum_{ij}E_J(i,j)[1-\cos(\phi_i-\phi_j+2\pi f_{ij})]\,, \qquad (1)$$

with $f_{ij}=(1/\Phi_0)\int_i^j \mathbf{A}\cdot d\mathbf{l}$. Here ϕ_i is the phase of the Ginzburg-Landau order parameter of the ith superconducting grain. $E_J(i,j)$ denotes the Josephson coupling between superconducting regions and the frustration parameter $f=\sum_P f_{ij}=\Phi/\Phi_0$, with $\sum_P$ denoting a sum over plaquettes, Φ the magnetic flux through a loop formed by a set of junctions, and Φ_0 the flux quantum. In spite of the simple appearance of this Hamiltonian, its general properties are, to a large extent, unknown. Suffice it to say that even in the periodic array case the model has been studied extensively and only a few selected equilibrium properties of the model are known.[9] Since the grains are located at random, the f_{ij} are themselves random variables. We could also consider $E_J(i,j)$ as a random variable, but this is in some sense included in the randomness of the f_{ij}; thus E_J is taken as a constant.[10] More important are the temperature and magnetic field dependencies of E_J, which, because of the size of the grains can have a significant effect on the results. It is here where a microscopic theory is needed to derive the explicit magnetic field and temperature dependencies of the Josephson coupling. The lattice model given in Eq. (1), with the f_{ij} random, was first studied within the context of granular BCS superconductors by Ebner and Stroud,[11] and a continuous version of the model with random dilution by John and Lubensky.[12] The first explicit study of Eq. (1) aimed at understanding the MTB results has been carried out by Morgenstern, Müller, and Bednorz.[13] They considered a positional disordered model where each site is randomly chosen to be at a maximum radius r from its regular periodic positions. They concentrated their study on the *equilibrium* properties of the model, and, specifically, on deriving the H dependence of $T_c(H)$, which appears to agree with the AT-line form. Here we center our attention on the *non-*

equilibrium, hysteretic, properties of the RAJJ model, with Monte Carlo dynamics, for comparison with the experimental results for the ZFC and FC magnetic properties of the system.

There is a significant amount of experimental evidence that shows that the physical properties of the oxide superconductors are highly anisotropic, with the conduction mechanism taking place mainly along the CuO_2 planes. Thus, we begin by considering a two-dimensional model and later on discuss the effects of adding interactions between the layers. The RAJJ model considered here consists of arrays of junctions equidistant along the y axis but with random separations along the x axis. The lattice sites along the x axis are located at points $x_i = ia + \delta a$, with i an integer and a the fundamental spacing in the periodic lattice, that we shall set equal to 1; δ is an independent random variable defined in the interval $[a-\delta, a+\delta]$ and determined by a probability distribution $P(\delta)$. We find that even when we consider randomness in the y axis as well as the x axis, the results are qualitatively analogous. Similarly, calculations for uniform and Gaussian probability distributions *do not lead* to significant qualitative differences in the results. Thus, here we present results only for the uniform probability distribution. The simulations were carried out in two-dimensional and quasi-three-dimensional lattices of fixed size with magnitudes $L_x \times L_y$ and $L_x \times L_y \times L_z$, respectively. In the quasi-three-dimensional calculations, each x column is chosen random and independent from the others. The model emphasizes the trapping of particular amounts of flux along the y axis; thus it has correlated disorder along the y axis. This RAJJ model is of interest for several reasons: First, it can be easily made in two dimensions with any of the modern photolithographic techniques employed to fabricate arrays.[14,15] Second, in the strong-field limit, a similar model has been shown analytically to correspond to a Sherrington-Kirkpatrick long-range mean-field-theory model,[16] while for weak fields it leads to an AT line.[17] Here we are interested in the weak-field intermediate regime (the analog of the Abrikosov regime $H_{c1} \leq H \leq H_{c2}$ for Josephson-junction arrays.[18]

Given the Hamiltonian and the $P(\delta)$, we calculate the experimentally measurable magnetization $M(T,\delta,H)$ and the helicity modulus $\Upsilon(T,\delta,H)$, following the standard Metropolis algorithm. If we choose the Landau gauge, the magnetization is given as

$$M(T) = -\frac{1}{N}\left\langle \sum_{i,j} (\pi E_J/\Phi_0)\sin(\phi_i - \phi_j + 2\pi f_{ij})(x_i + x_j)\right\rangle_c ,$$

where x_i denotes the ith lattice point in the x direction, N is the total number of lattice points, and $\langle\ \rangle_c$ stands for configurational average. Υ represents the increase in the free energy of the system when there is a uniform twist of the phases at one end of the sample, and is directly related to the superfluid density.[19] The corresponding expression for Υ along the x axis is

$$\Upsilon_x = \frac{1}{N}\left\{\left\langle \sum_{ij} x_{ij}^2 \cos(\phi_i - \phi_j + 2\pi f_{ij})\right\rangle_c - \beta\left\langle \left[\sum_{ij} x_{ij}\sin(\phi_i + \phi_j + 2\pi f_{ij})\right]^2\right\rangle_c + \beta\left[\left\langle \sum_{ij} x_{ij}\sin(\phi_i - \phi_j + 2\pi f_{ij})\right\rangle_c\right]^2\right\},$$

with $x_{ij} = x_i - x_j$. An equivalent expression is obtained when the twist is along the y axis.

In doing our calculations we follow essentially the same procedures as in the experiments.[1] We start by generating a random lattice from the $P(\delta)$ distribution. At low temperatures, the system is equilibrated in the absence of an external magnetic field. A field is then turned on and the system is warmed up slowly, allowing it to reach *local equilibrium* at each temperature. This gives us the ZFC properties of the system. In the periodic case, without a field, the model has a Berezinshkii-Kosterlitz-Thouless critical temperature $T_{BKT} \sim 0.9$; thus the system is warmed up past this temperature, always with the constant field on. Typically, at about $T \sim 1.5$, the process is reversed and the system is slowly cooled down to its initial low-temperature value. Once again the system is slowly warmed up to the high-temperature region: this process defines the FC magnetic properties of the system. This procedure is repeated for each member of the configurational ensemble. We found that the configuration average *was necessary* to get statistically reliable results, although the number of members in the ensemble did not need to be large (typically for $5 \leq n \leq 10$ the results were statistically stable). Nonetheless, the calculations were computationally intensive.

Typical results for the ZFC and FC runs for M and Υ are shown in Fig. 1. Note that the results for M are strikingly similar to the experimental results found by MTB and other authors.[2,3] After the first warming up, we repeated the cooling and warming procedure several times, essentially retracing the same FC curve, indicating that the FC branch corresponds to the equilibrium value for the magnetization. In the inset we show the temperature dependence of the amount of metastability by the difference $\Delta M(T,H,\delta) = M_{FC} - M_{ZFC}$. We notice that close to $T_c(H)$, ΔM decays to zero approximately linearly with $1 - T/T_c(H)$. In Fig. 1(b) we show the helicity modulus for the same parameter values as in Fig. 1. Again we see that there is a clear hysteretic behavior as a function of temperature with a transition temperature $T_c(H)$ about the same as in the $M(T)$ results. The temperature dependence of $\Delta\Upsilon = \Upsilon_{FC} - \Upsilon_{ZFC}$ can be read off from the plot, while in the inset we show the magnetic field dependence of the width $\Delta\Upsilon$. Note

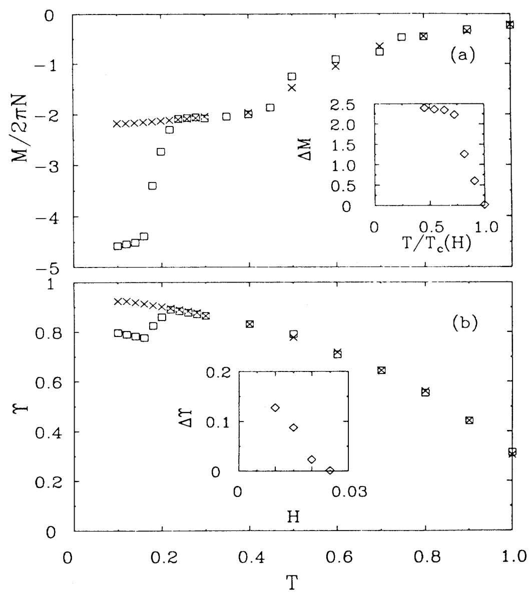

FIG. 1. Zero-field-cooled (□) and field-cooled (×) runs for (a) M and (b) ϒ, in a lattice of 16×16 sites, with $H=0.01$ and $\Delta=0.1$. The results for each configuration corresponds to runs of 6×10^4 MCS/angle discarding 10^4 MCS/angle for equilibration. N denotes the total number of angles, with ten members for the configurational quenched averages. M and T are normalized by E_J, and M and H also by $\Phi_0/\langle S\rangle$, with $\langle S\rangle$ the average area of a plaquette. The size of the error bars are about the size of the symbols. The insets are discussed in the text.

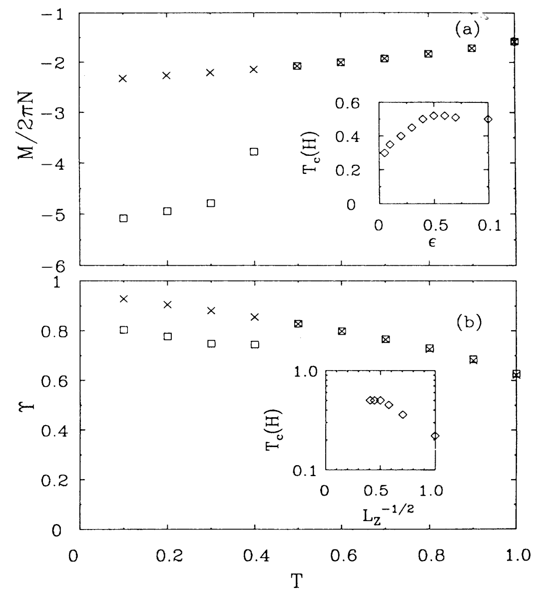

FIG. 2. M and ϒ as in Fig. 1, with the same H and δ but lattice size $16\times 16\times 5$ for one configuration and the same number of MCS/angle as in Fig. 1. The insets are discussed in the text.

that the superfluid density is finite above $T_c(H)$, indicating that the zero-resistance temperature is higher than the onset of nonergodic behavior, as seen in the experiments. For fixed H and temperature, the magnetization is essentially constant as function of δ (for $0.1\le\delta\le 0.25$), whereas the critical temperature $T_c(\delta)$ decreases monotonically with δ. In Fig. 1(a) it would appear that there is a smaller jump about $T=0.5$. We did calculations for a large number of members in the ensemble ($n=10$) and the apparent discontinuity became smooth, proving that doing configurational averages can be important. We also analyzed the size dependence of the results, for $n=5$, taking $L_x\times L_y=8\times 8$ 16×16, and 20×20, with the same number of Monte Carlo steps per angle (MCS/angle). As expected, since for smaller systems the nonergodic line decays to the equilibrium line in a shorter time, we found that the results were qualitatively analogous in smaller lattices but with ΔM smaller, given that the system needs an equilibration time which is of exponential order in N.

In Fig. 2 we present typical results for M and ϒ for quasi-three-dimensional model calculations. Note that ΔM is larger while Δϒ is about the same magnitude as in their corresponding two-dimensional cases. In the inset of Fig. 2(a) we show results for $T_c(H)$ as a function of the interplane interaction strength $\epsilon=E_J(\perp)/E_J(\parallel)$. We find that the critical temperature $T_c(H)$ increases monotonically as a function of ϵ, and then *saturates* after a value of $\epsilon\sim 1$, and a fixed number of planes. The increase in the critical temperature was seen in the calculations of Ref. 13 for the value of $\epsilon=1$, but not for the systematics shown in the inset of Fig. 2(a). This trend is analogous to that seen in the increase of T_c for the oxide superconductors under pressure.[20] In the inset of Fig. 2(b), the logarithm of the critical temperature is plotted as a function of $1/\sqrt{L_z}$. For values of $L_z\le 5$ the curve is essentially linear while for $L_z\ge 5$ it flattens out. The $1/\sqrt{L_z}$ behavior is analogous to the one proposed to fit the experimental change of $T_c(H=0)$ as a function of the number of CuO_2 layers.[21] Thus, it appears that there is *an upper bound* in $T_c(H)$ as a function of the

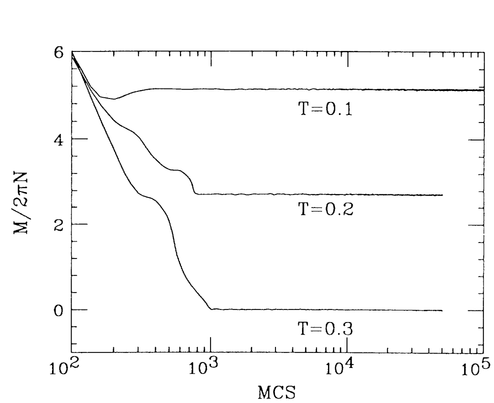

FIG. 3. Decay of remanent magnetization M for a lattice of size 16×16, and with $H=0.02$, $\delta=0.2$. The critical temperature $T_c(H)\sim 0.25$.

number of layers, and the magnitude of the interlayer interaction.

As in the MTB experiments, to further check the dynamics of the trapped flux in the model, we take an equilibrium point in the FC branch and then turn the field off. The Monte Carlo time decay of M is shown in Fig. 3. Note that in Fig. 3 the magnetization has a slow decay, as in the experiments, as a function of $\ln t$. Initially it is almost linear in $\ln t$ and then it becomes essentially constant for the runs shown here. At low temperatures (~ 0.1) the constant value persisted even for runs up to 10^6 MCS/angle, while for higher temperatures for times larger than 10^5/angle, the magnetization decays to a lower value of M. For clarity in the figure we only show three values of T; two below $T_c(H)$ (0.1 and 0.2) and one above $T_c(H)$ (~ 0.25). The point here is just to stress that the decay in this model is nonexponential in contrast to the exponential decay found in the model studied in Ref. 13.

The results contained in the figures clearly indicate that there are energy barriers between different metastable flux states that have to be jumped before the system reaches thermodynamic equilibrium. These thermally activated flux transitions are analogous to those in the flux-creep model. Thence the two models are, in a sense, qualitatively similar and their predictions can be considered as complementary to each other. In fact, logarithmic or nonexponential time decays are found in many systems that show metastability. The pervasiveness of this type of behavior can be traced to some very general properties of the thermal activated decay of metastable states. Furthermore, the model given in Eq. (1) can be transformed to a vortex representation, with standard duality transformation techniques,[10] and the discussion about vortex penetration or exclusion can, in principle, be made as in the flux-creep treatment. Theoretically, however, the study of the dynamics of the problem in terms of its vortex representation appears much harder to do.

The conclusion we arrive at from the results presented above is that the RAJJ model leads to a number of results that are in striking similarity with those seen experimentally. Furthermore, we suggest testing the predictions of our studies by precise measurements of Υ using techniques similar to or those employed by Martinoli and his group.[22,23] Apart from the fact that such studies would be of interest in themselves, they may serve as a bridge in the understanding of the underlying physics of metastability in the oxide superconductors. A preliminary report of related results can be found in Ref. 24 while a detailed discussion of these and other results will appear elsewhere.[25]

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