Extreme Nonlinear Damping by the Quasiparticle Gas in Superfluid ^{3}He-B in the Low-Temperature Limit

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The resistance to motion of a wire in superfluid ^{3}He-B has three contributions: that due to the internal friction of the resonator; that due to pair breaking; and that due to the quasiparticle gas. We have been able to separate the quasiparticle contribution and find that it is highly nonlinear with velocity on the scale of the Landau velocity, Δ/p_F, and *falls* with increasing velocity. This indicates that the quasiparticle resistance is also governed by the condensate, almost certainly through Andreev reflection in the flow field around the wire.

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When a macroscopic object (in the present case the wire of a vibrating-wire resonator) is projected through superfluid ^{3}He-B at temperatures much below T_c two things happen. At low velocities the object experiences a drag force from the quasiparticle gas, which varies[1] with temperature as $\exp(-\Delta/k_BT)$. (Despite a large amount of experimental and theoretical effort the actual magnitude of this damping is much higher than any simple theory has been able to predict.) Further, when the velocity reaches a value of around a quarter of the "classical" Landau critical velocity, Δ/p_F, then pair breaking begins and a very much larger drag force appears,[2] which grows extremely rapidly with further increase in velocity. In the present Letter we report that we have been able to separate these two contributions to the damping, and in consequence we can report that the effects of the quasiparticle gas alone are not only highly nonlinear, but show the counterintuitive property that the damping coefficient *falls* as the velocity increases, apparently asymptotically to a value near zero at the Landau velocity.

The experiments are performed on a series of vibrating-wire resonators in superfluid ^{3}He-B, the experimental configuration being similar to that described[3] eariler. The ^{3}He is cooled by contact to copper nuclei, themselves cooled by adiabatic demagnetization to a temperature of around 50 μK. The magnetic excitation field for the resonators is provided by the final demagnetization field, usually 32 mT in the present work.

The resonators used here are of smaller loop diameter (about 3 mm rather than 8 mm) than those used in our previous work (described in some detail in Ref. 3). Otherwise the techniques are similar. The voltage generated by the movement of the wire in the excitation field is detected by a two-phase lock-in amplifier with a high-quality transformer as an input preamplifier. Preliminary characterization of the mechanical resonance of the wire is achieved by stepping the frequency through resonance at a constant drive current. The damping of the resonator as a function of velocity is then observed, in the same configuration, as follows: The frequency is set to a fixed value close to resonance, and the (vector) voltage measured as the drive current is slowly stepped upwards from zero. When small corrections are made for the self-inductance of the wire (negligible in practice) and for any zero error in the lock-in amplifier, the measured voltage simply yields the velocity of the wire as an average along the length perpendicular to the field. The driving current gives a measure of the similar average driving force on the resonator.

This type of measurement is then repeated at a number of temperatures, from base temperature (less than 100 μK at 0 bar pressure, see below) up to around $T_c/3$. The temperature is adjusted by a steady heat input provided by a second wire, driven rather violently, and the ^{3}He temperature is measured (and monitored as constant) from the resonance width of a third gently driven wire. A set of curves of velocity versus drive level taken in this way is shown in Fig. 1. The measurements in the figure all refer to the same vibrating wire, on the same day, and in the same magnetic field.

Similar results have been taken for four different wires. All are single filaments of superconducting NbTi, derived from a multifilamentary magnet wire.[3] The finest wire has a diameter of around 5 μm, and the others are all of diameter around 12 μm. We have made measurements at two pressures, 0 and 6.8 bars, in order to study the influence of the superfluid energy gap and Landau velocity on the damping. The Landau velocity changes by a factor of about $\frac{5}{3}$ between these pressures.

The simplest way to analyze and present the velocity-versus-drive data is in terms of an effective damping function G, which represents the mechanical impedance for the resonator, where the impedance is defined as the ratio of the driving force to the induced velocity, at resonance. Here the force is IBl, where I is the drive current, B is the magnetic field, and l is the effective length of the wire, a length of order the leg spacing, i.e., about 3 mm. The mean velocity of the wire can be expressed as V/Bl, where V is the measured voltage. For a given wire and in a given field (i.e., constant B and l) the impedance or damping function G is thus directly pro-

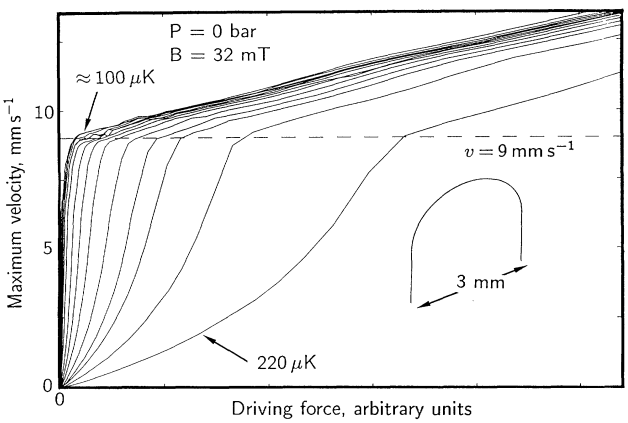

FIG. 1. The response in ^{3}He-B of an approximately 5-μm-diam wire resonator, expressed as maximum velocity (see text), as a function of drive level for a number of temperatures from below 100 to 220 μK. The rapid change in slope at $v_{max} \simeq 9$ mm s^{-1} represents the onset of pair breaking. The nonlinear behavior at lower velocities is clearly seen in the higher-temperature curves. The pair-breaking edge is also accompanied by a great deal of fine, and highly reproducible, structure. Inset: The geometry of the resonator.

portional to I/V.

At this point we need to add two words of caution. First, since in practice the data are not taken precisely at resonance, we use for the effective force the in-phase component of the current $I\cos\theta$ instead of I, where θ is the phase angle from resonance. This is a minor adjustment only in all the data presented here. Second, if conversion to *absolute* values of velocity or drive is needed, then we have to take into account the geometry of the loop. The measurements are made in terms of rms values of current and voltage. Our high-Q resonators oscillate (virtually) harmonically, so that conversion of the average over time into a time variation is straightforward. However, since all parts of the wire are not moving with the same velocity, there is also spatial averaging, and this is not so straightforward to handle. We estimate[2] that the maximum velocity of the extreme top of the wire is related to the measured average rms velocity by a factor of about 1.75. It is unfortunate that we cannot make use of a geometry of uniformly and transversely moving cylinders, rather than the contorted arrangement we are restricted to. Nevertheless the conversion can be relied upon to about 10%.

The analysis of the data follows straightforwardly if we assume that the damping function G for a vibrating wire in superfluid ^{3}He-B as a function of wire velocity v is made up of three principal contributions. We write

$$G(v) = G_0 + G_1 + G_2 , \quad (1)$$

where the terms have the following origins.

G_0 is the *vacuum-damping* term, the damping which would be present in the absence of the ^{3}He. We believe that this term is a constant, i.e., independent of v, in the velocity range covered in these experiments. Nonlinearities in the vacuum response do occur, but typically at 10 times the maximum velocity used here. The maximum amplitude employed is of order 10 μm, comparable to the wire diameter. At the lowest temperatures this vacuum-damping term dominates the low-velocity damping.

The second term G_1 arises from the *scattering of existing thermal quasiparticles* by the wire. We shall find that this term, when separated, turns out to be strongly nonlinear in v.

The third term G_2 relates to *quasiparticle pair production.* To a good approximation, for any one wire, this term is a function only of the reduced velocity, $v^* = v/v_L$, and arises from the breaking of superfluid Cooper pairs to form quasiparticle excitations. As previously noted,[2] the main onset of dissipation occurs at about $v^* = 0.25$ to 0.30 for all wires. The present work shows that G_2 has no significant dependence on temperature up to about $0.25T_c$, the highest temperature at which these fine-wire resonators can be readily studied. This is to be expected since we assume that this term depends directly on the superfluid density ρ_s, which is essentially constant throughout our temperature range.

There are two additional small effects. The detailed form of G_2 is wire specific, to be expected since the wires are not perfectly smooth cylinders, and neither are the loops perfectly flat. Physical irregularities of the wire surface must exist, and these will distort the superfluid backflow. Furthermore, we see pressure-dependent steps and other structure at the pair-breaking "edge" which will be discussed in a later publication.

Using the ideas discussed above, we can analyze the experimental results to bring out the properties of the new quasiparticle damping term, G_1 of Eq. (1).

First, we take the experimental data for one particular wire and experimental run (as shown in Fig. 1), calculate the impedance G from V and I as discussed above, and plot G against velocity v. The family of curves so derived is then most usefully displayed with G normalized to the low-velocity value $G(0)$. [The magnitude of $G(0)$ corresponds to the low-level frequency width Δf_2 which we use for thermometry, and can itself be used as a secondary thermomenter for the ^{3}He.] The data of Fig. 1 are plotted in this way, i.e., as $G/G(0)$ against v, in Fig. 2.

Several points stand out. Most significant is the nonlinearity at low velocities (i.e., below the onset of pair breaking) in all the curves except the coldest. For the curves taken at base temperature, the quasiparticle damping term G_1 is everywhere negligible, and therefore from Eq. (1), $G = G_0 + G_2$. The absence of nonlinearity in G for these curves at $v < 0.15v_L$ reflects the expected linear behavior of the vacuum-damping term, with the pair-production term being the first deviation from linearity. Had our previous measurements not been

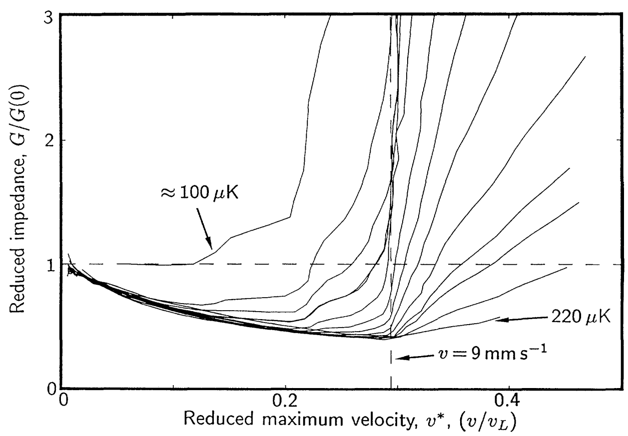

FIG. 2. The data of Fig. 1 presented in the form of the impedance G (see text) as a function of reduced maximum velocity $v^* = v/v_L$, where the Landau velocity v_L is 30.6 mm s^{-1}. The curves have all been normalized to unity at low velocities by dividing by the low-velocity value of G, $G(0)$. The sudden increase in impedance at $v_{max} \simeq 9$ mm s^{-1} is clear, as is the nonlinear behavior at lower velocities.

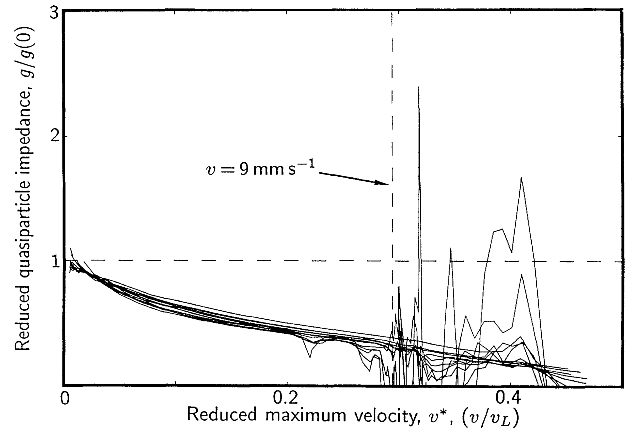

FIG. 3. The normalized quasiparticle impedance $g/g(0)$ (see text) for the data of Fig. 1, plotted on the same scale as Fig. 2. The form of $g/g(0)$ is clearly universal and extends well above the pair-breaking region, apparently set to fall to near zero at the Landau velocity. The noise on the lower-temperature curves is caused by the subtraction process where g is only a small fraction of the original impedance G.

made invariably at base temperature, we would have seen sooner the nonlinear damping reported here.

As soon as the temperature is increased to the point where thermal quasiparticles are dominant, we see from Fig. 2 that the curves of $G/G(0)$ become universal at low reduced velocity v^*. We know from earlier work[1] that at low velocities the quasiparticle damping is found to be proportional to $\exp(-\Delta/k_BT)$. We can add the effect of velocity to an excellent approximation by factorizing G_1 as

$$G_1 = A\exp(-\Delta/k_BT)f(v^*)\,, \tag{2}$$

where A is a constant, the exponential factor being proportional to the product of the mean quasiparticle group velocity and the normal fluid density (see below) at temperature T. The final highly nonlinear factor $f(v^*)$ is of universal importance since it appears to be a function only of $v^* = v/v_L$, and is independent of temperature T, of wire diameter, and of pressure (i.e., of v_L and Δ). One thing we can say at this stage is that from the absence of nonlinearities at base temperature we can put an upper limit on G_1, which in turn puts an upper limit on the base temperature of ~ 100 μK at 0 bar, or that $T_c/T \sim 10$.

One significant point about the use of Eq. (2) should be stressed. The data analysis in this paper nowhere requires an independent knowledge of temperature. The temperatures quoted are themselves derived from the data. The thermometric quantity chosen is simply the low-velocity limit $G_1(0)$ of G_1, i.e., $G(0) - G_0$, an experimentally accessible quantity. Indeed, Eq. (2) could (for the purpose of data analysis) be rewritten as $G_1 = G_1(0) \times f(v^*)$.

We can make one more significant step. If the ideas above are correct, the lowest-temperature curves for G simply give $G_0 + G_2$. However, since G_0 and G_2 should both be independent of temperature we may hence estimate G_1 by simply taking the data at temperature T and subtracting the data at base temperature. The resulting damping function, say $g = G - G(\text{base})$, can then be analyzed as before by dividing by the low-velocity limit $g(0)$. The data of Figs. 1 and 2 plotted as $g/g(0)$ against v^* are given in Fig. 3. According to the model, all the data should lie on a single universal curve, that of $f(v^*)$ against v^*. Below the pair-breaking edge the data clearly lie on a universal curve. However, the fact that the curve is continuous through the pair-breaking region and is clearly followed to considerably higher velocities is a very strong confirmation indeed, especially when we recall that the damping above the critical velocity is very high, and thus $f(v^*)$ is derived from the subtraction of two large and similar numbers.

We have used one set of data here for all three figures so that the reader can see how the process follows through. This is in no way our best "typical" data. We have many families of curves from different wires all showing the same features with the same clarity. Comparison of data from different wires shows that $f(v^*)$ is independent of the details of wire properties, unlike the detailed fine structure in G_2. Comparisons at 0 and 6.8 bars show that the dependence is on v^* rather than on v alone. This is a good test, since v_L changes by a factor of $\frac{5}{3}$ between the two pressures.

What can we say about the nonlinearity in the damping with velocity? As can be seen in Fig. 3 the nonlinearity is extremely strong, f falling to about 0.3–0.4 of its low-velocity value at the critical velocity.

There have been two puzzles in the way wire resonators behave in superfluid ^{3}He-B. First, as mentioned above, the low-velocity damping, although having a reasonable temperature dependence, has a much higher magnitude than expected. The basic problem has been stated in an earlier paper.[3] We can use a simple kinetic argument that the damping is caused by the collision of quasiparticles with the wire. We assume that the quasiparticle density is proportional at $T \ll T_c$ to $(1/\sqrt{T}) \times \exp(-\Delta/k_B T)$, and that the quasiparticles travel with group velocity proportional to $\sqrt{T}$ and exchange momentum of order p_F with the wire on each collision. This approach yields the correct temperature dependence but seriously underestimates the magnitude of the damping, by a factor of 100 to 1000. It has been pointed out by Hall[4] that the correct order of magnitude is obtained from the kinetic argument if ρ_n/m^* is used instead of the simple quasiparticle density, introducing a factor of order $E_F/k_B T$. However, there is no firm theoretical basis for this way around the problem (see Sec. 3.2.1 of Ref. 3).

The second puzzle is why pair breaking is observed to set in at a relative velocity between wire and liquid of only 0.5 to 0.6 of the Landau velocity, apparently independent of wire diameter and surface condition. Any simple argument would suggest pair breaking should start at a wire velocity $v = v_L/2$, i.e., a relative velocity of v_L, unless recourse is taken to convoluted arguments about the surface roughness (excrescences increase the local relative velocity between wire and liquid).

With reference to the form of the impedance in Fig. 3, we note that the scale on which the quasiparticle impedance is changing is indeed that of v_L. Indeed the data look very much as if $g/g(0)$ is asymptotically approaching zero as $v \to v_L$. We can thus say that the superfluid component definitely plays some role. This strongly suggest that the mechanism of the nonlinearity lies in the interaction of the quasiparticles with the condensate in the form of the flow field of superfluid around the wire. The relative velocity of the backflow implies local modifications to the quasiparticle dispersion relation of magnitude $E = \mathbf{p}_F \cdot \mathbf{v}$, which become dominant in the superfluid when $v \sim v_L$. This modification of the quasiparticle dispersion near the wire means that quasiparticles can transfer momentum to the wire from Andreev[5] processes taking place out in the flow field, thus greatly increasing the effective cross section. Unfortunately Andreev processes only transfer relatively small momenta compared with normal processes ($\Delta p_F/E_F$ as against p_F), and the direct effect of the increased cross section is largely lost.

The precise shape of the $g/g(0)$ curve is very odd. One would expect any mechanism, whether involving the condensate or not, to give deviations from linearity (i.e., from g constant) at low amplitude which are even powers of v. However, Fig. 3 shows deviations down to the lowest velocities measured. (In fact, $1/g$ can be fitted by a virtually straight line in v.) The continuity of the term through the pair-breaking region is quite amazing given the small size of g compared with the total G from which it is extracted. Nevertheless, why should the impedance fall with increasing velocity, apparently to near zero at the Landau velocity? There are many unsolved questions here.

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