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# Implications of a Half-Millisecond Pulsar

John L. Friedman
*University of Wisconsin, Milwaukee, Wisconsin 53201*

James R. Ipser
*University of Florida, Gainesville, Florida 32611*

Leonard Parker
*University of Wisconsin, Milwaukee, Wisconsin 53201*
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New models of rotating neutron stars show that only a surprisingly narrow range of possible equations of state (EOS's) can simultaneously allow a rotating neutron star with frequency as large as 1968 Hz and a spherical (nonrotating) neutron star with mass as large as $1.44M_\odot$. The mass and baryon mass for the 1968-Hz models exceed $1.5M_\odot$ and $1.7M_\odot$, implying a progenitor mass $> 1.7M_\odot$. Those EOS's that allowed 1968-Hz models have, for spherical stars, a stringent upper mass limit $< 1.7M_\odot$. Each model at 1968 Hz has mass above the spherical upper mass limit for its EOS, implying collapse upon spin down.

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A steady increase in the number of observed neutron stars has been accompanied by a growing interest in their use as what Pines has called cosmic neutron-matter laboratories. Pulsar timing observations and measured upper limits on surface temperatures have led to an improved understanding of the stars's internal structure; and there has long been a hope that observed neutron-star properties could constrain the equation of state (EOS) of matter at high density (nuclear density and somewhat above). Most discussions of such constraints have been restricted to nonrotating models.[1]

Our main purpose in this Letter is to outline how observations of *rapidly rotating* neutron stars can considerably sharpen the constraints on the EOS. But we shall also suggest how such observations could yield important implications for the origin, evolution, and stability of neutron stars. The need for an assessment of rapidly rotating configurations, beyond what is already available,[2] appears especially pressing in view of the recently reported observations[3] of a neutron star in SN 1987A with what appears[4] to be a rotational angular velocity $\Omega = \Omega_{SN} = 1.237 \times 10^4\ \mathrm{s}^{-1}$. Our discussion is based on recent detailed numerical work that extends our earlier study of rapidly rotating neutron stars.[2] As in the earlier work, we have constructed relativistic models of stars in rapid, uniform rotation, using codes that adapt the numerical programs developed by Butterworth and Ipser.[5]

We shall first report results for a particular complex of EOS's that have been proposed in the literature. Most of these EOS's are from the Arnett-Bowers[6] collection and are identified here by the symbols used in that reference. They span the range from very soft EOS's to very stiff ones, although as we shall see, there is a gap precisely in what now appears to be the important part of the range. We have in addition constructed models using the EOS due to Friedman and Pandharipande[7] (denoted by FP), two EOS's from the parametrized set considered by Prakash, Ainsworth, and Lattimer[1] (denoted by PAL1 and PAL2), and a pion-condensed equation of state of Weise and Brown[8] (denoted by $\pi$). We have, in particular, determined upper limits on mass and rotation for EOS's A, B, C, D, F, G, L, FP, PAL1, PAL3, and $\pi$. A more systematic approach would be to examine a parametrized EOS, finding a region in parameter space that can accommodate the properties of observed neutron stars, and we are in the early stages of such a study.

A central issue is the maximum rate of uniform rotation permitted by a given EOS. No uniformly rotating equilibrium can have an angular velocity greater than that of a particle in circular orbit at its equator, and, for a given mass, the configuration with maximum angular velocity rotates at this Keplerian frequency $\Omega_K(M)$. In Fig. 1 we plot, for each EOS considered, the maximum allowed angular velocity $\Omega_K(M)$, for rotating models of neutron stars with gravitational mass $M$. The final point along each curve represents the equilibrium configuration with maximum gravitational mass and baryon mass for a given EOS. Each of these final models rotates with the maximum angular velocity for that EOS, consistent with the constraints of uniform rotation and stability against gravitational collapse (but ignoring, for the moment, the question of stability against nonaxisymmetric perturbations). Although equilibrium configurations with angular velocities greater than this exist, they would quickly collapse: A turning-point stability theorem shows that, along a sequence of rotating neutron stars with fixed angular momentum and increasing central density, the configuration with maximum mass marks the onset of instability to collapse.[9] The instability is secular, leading to gravitational collapse on a time scale associated with redistribution of angular momentum.

Additional properties of these maximum-mass models are exhibited in Table I. We have included, in the table and figure, maximum-mass models for the two softest examples of the parametrized equations of state considered by Prakash, Ainsworth, and Lattimer.[1] These are labeled PAL1 and PAL3 and correspond to the EOS's with their function $F(u)=u$ and $\sqrt{u}$, respectively, and with the compressibility modulus $K=120$ MeV. Finally, $\pi$ is the EOS given by Weise and Brown[8] with the effective axial-vector coupling strength $g_A^*=1.3$.

Our results for the maximum rotation are roughly consistent with the prediction of Shapiro, Teukolsky, and Wasserman that the maximum frequency $\Omega_K$ for a given equation of state is proportional to $(M_s/R_s^3)^{1/2}$, where $M_s$ and $R_s$ are the maximum mass and radius of the corresponding spherical models.[10] That is, to better than 10% accuracy we find

$$\Omega_K/(10^4\ \mathrm{s}^{-1})=0.72(M_s/M_\odot)^{1/2}[R_s/(10\ \mathrm{km})]^{-3/2}.$$

It is clear from Fig. 1 that the existence of submillisecond rotation periods places severe restrictions on the EOS. Indeed the existence of neutron stars with angular velocity $\Omega \geq \Omega_{SN}$ would immediately rule out the EOS's stiffer than A, namely C, D, L. M, N, and O. In addition, of those EOS's in our complex that survive (that allow $\Omega \geq \Omega_{SN}$), all except B, G, and $\pi$ require a neutron star with $\Omega = \Omega_{SN}$ to lie extremely close to the model with maximum frequency $\Omega_K$. One expects that an object so near the termination point will be unstable to nonaxisymmetric modes[11] that radiate its excess angular momentum in gravitational waves and will spin down on a time scale $\ll 10^{11}$ s, the lower limit reported[3] for SN 1987A. In this connection, recent numerically exact solutions,[12] for the normal modes of pulsation of rapidly rotating Newtonian configurations, lead to the prediction that the critical angular velocity, $\Omega_c$, at which nonaxisymmetric instability sets in is given by $\Omega_c \approx 0.9\,\Omega_K$ for a neutron star of the age of that in SN 1987A. This result follows from using the value $T \sim 10^9$ K for neutron-star temperature[13] and using neutron-neutron nonsuperfluid viscosity[14] to estimate the stabilizing effects of viscous dissipation. (It is, however, possible that exotic

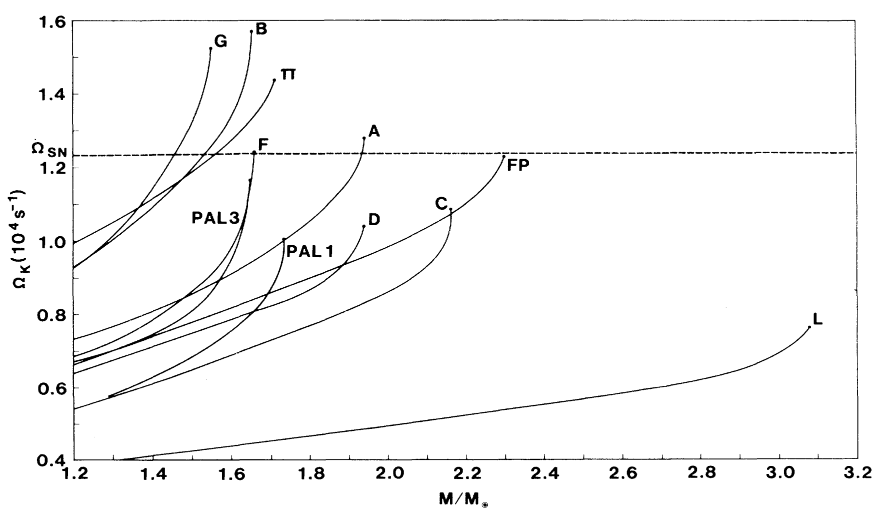


FIG. 1. Maximum angular velocity $\Omega_K$ vs $M$ for eleven equations of state (EOS's). Single letters labeling curves follow the notation of Arnett and Bowers. FP refers to the Friedman-Pandharipande equation of state, $\pi$ to an EOS with pion condensation due to Weise and Brown, and PAL1 and PAL3 are equations of state in the parametrized family considered by Prakash, Ainsworth, and Lattimer (see text). For each equation of state, the final dot represents the configuration with maximum mass and with the maximum angular velocity consistent with stability against collapse. The dotted line marks the frequency, $\Omega_{SN}=1.237\times 10^4\ \mathrm{s}^{-1}$, observed in SN 1987A.

TABLE I. Models with maximum mass and rotation for the equations of state mentioned in the text. The properties listed are $\Omega$, angular velocity in $10^4\ \mathrm{s}^{-1}$; $\epsilon_c$, central density in $10^{15}\ \mathrm{g\,cm}^{-3}$; $M/M_\odot$ and (%), gravitational mass and its percentage increase over the maximum mass of the spherical model; $M_0/M_\odot$, baryon mass; $R$, equatorial radius [(proper circumference)/$2\pi$] in km; $T/W$, ratio of rotational energy to gravitational energy; $I$, moment of inertia in $10^{45}\ \mathrm{g\,cm}^2$; $cJ/GM^2$, dimensionless ratio of angular momentum $J$ to $M^2$; $e$, eccentricity; and $\beta$, injection energy.

| Equation of state | $\Omega$ | $\epsilon_c$ | $M/M_\odot$ | (%) | $M_0/M_\odot$ | $R$ | $T/W$ | $I$ | $cJ/GM^2$ | $e$ | $\beta$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| L | 0.76 | 1.11 | 3.18 | (20) | 3.72 | 17.3 | 0.122 | 7.87 | 0.68 | 0.69 | 0.34 |
| PAL1 | 1.00 | 3.11 | 1.73 | (15) | 1.93 | 13.0 | 0.090 | 1.54 | 0.58 | 0.70 | 0.45 |
| D | 1.04 | 2.78 | 1.94 | (17) | 2.21 | 12.7 | 0.11 | 2.00 | 0.63 | 0.69 | 0.40 |
| C | 1.11 | 2.71 | 2.16 | (17) | 2.47 | 12.9 | 0.11 | 2.42 | 0.64 | 0.68 | 0.35 |
| PAL3 | 1.16 | 3.98 | 1.65 | (15) | 1.85 | 11.3 | 0.094 | 1.23 | 0.59 | 0.69 | 0.47 |
| FP | 1.23 | 2.5 | 2.30 | (17) | 2.71 | 12 | 0.13 | 2.41 | 0.70 | 0.67 | 0.28 |
| F | 1.24 | 4.1 | 1.66 | (13) | 1.87 | 11 | 0.094 | 1.16 | 0.60 | 0.67 | 0.39 |
| A | 1.28 | 3.29 | 1.94 | (17) | 2.25 | 10.8 | 0.117 | 1.71 | 0.66 | 0.67 | 0.33 |
| $\pi$ | 1.54 | 4.47 | 1.74 | (15) | 2.02 | 9.18 | 0.121 | 1.15 | 0.66 | 0.66 | 0.30 |
| B | 1.57 | 5.16 | 1.65 | (17) | 1.91 | 9.2 | 0.107 | 0.98 | 0.64 | 0.66 | 0.31 |
| G | 1.52 | 5.5 | 1.55 | (14) | 1.73 | 8.6 | 0.101 | 0.86 | 0.62 | 0.62 | 0.34 |

particles—pion condensates or axions, for example—could lead to substantially faster cooling, and if $T \lesssim 10^7$ K, viscosity would probably be high enough to preclude any nonaxisymmetric instability.)

In summary, for all EOS's that we have considered, except B, G, and $\pi$, instability is likely in recently born neutron stars with $\Omega \gtrsim \Omega_{\rm SN}$. If the cooling is as slow as the standard model[13] indicates, confirmation of a pulsar in the SN 1987A remnant rotating with frequency 1968 Hz would itself probably rule out every EOS in the Arnett-Bowers collection except B and G.

EOS's B and G have their own problems. Assuming general relativity, the measured mass of the binary pulsar is $(1.442 \pm 0.003)M_\odot$.[15] This exceeds $1.414M_\odot$ and $1.358M_\odot$, the maximum nonrotating values allowed by EOS's B and G, respectively (the rotation of the $1.44M_\odot$ member of the binary pulsar has a negligible effect on its mass). Hence, because they do not allow large enough spherical masses, B and G also fail. Thus, within the standard model for temperature and viscosity, it appears that the existence of young neutron stars with stable angular velocities $\Omega \gtrsim \Omega_{\rm SN}$ would rule out every EOS in the Arnett-Bowers collection as well as EOS FP, but not EOS $\pi$.

This rather strong conclusion must be tempered by the uncertainty in the cooling rate and in neutron-star viscosity; and by the fact that our estimates of relativistic instability points are extrapolations from the Newtonian theory, because nonaxisymmetric instability points along relativistic sequences have not yet been computed. However, even if one ignores the question of nonaxisymmetric instability, only EOS's $\pi$, F, FP, and A simultaneously allow a rotating neutron star with frequency as large as 1968 Hz and a spherical neutron star with mass as large as $1.44M_\odot$. We have, in addition, looked at models based on two of the parametrized EOS's considered by Prakash, Ainsworth, and Lattimer[1] with the smallest compression modulus ($K = 120$ MeV) that they considered. To our surprise, neither EOS allowed a model with angular velocity as large as $\Omega_{\rm SN}$, suggesting that the range of EOS's considered in their Letter [i.e., with their choice of functions $F(u)$ and with $K \geq 120$ MeV] is ruled out. The range of EOS's considered by Glendenning[1] similarly seems too narrow to allow a model with $\Omega \geq \Omega_{\rm SN}$: We find for the softest EOS he considered ($K = 210$ MeV) a maximum frequency below $0.95 \times 10^4\ \mathrm{s}^{-1}$.

Can the EOS governing neutron-star matter satisfy the stronger constraint of nonaxisymmetric stability and still allow a $1.44M_\odot$ spherical star? We think so. It, like EOS $\pi$, may lie in a rather narrow range of stiffness between EOS's A and B. In the density range $3 \times 10^{14}\ \mathrm{g/cm}^3 < \epsilon < 10^{15}\ \mathrm{g/cm}^3$, EOS A is slightly too stiff to allow a stable neutron star with $\Omega = \Omega_{\rm SN}$, while B is slightly too soft to allow a spherical neutron star with $M = 1.44M_\odot$. Note, however, that of all of the equations of state we examined, *none* simultaneously allows a spherical model with mass greater than $1.7M_\odot$ and a rotating model with angular velocity 1968 Hz (stable or unstable to nonaxisymmetric perturbations). Any observation of a spherical neutron star with mass much above $1.7M_\odot$ would be very difficult to reconcile with the existence of a half-millisecond pulsar. There are a few approximately measured masses that would be too high—for example, $(1.85 \pm 0.3)M_\odot$ for 4U0900-40[15]—if the error bars were smaller.

For an EOS in the allowed gap, a neutron star could evolve to $\Omega \sim \Omega_{\rm SN}$ in the following way (see also Kluzniak *et al.*[16]). Since viscosity $\propto T^{-2}$ and since $\Omega_c$ decreases as viscosity decreases,[14] one envisages an initial hot ($T \gtrsim 10^{10}$ K), rapidly rotating state with $\Omega \gtrsim 0.9\,\Omega_K > \Omega_c$. In this state normal modes of pulsation with spherical harmonic indices $l=m\gtrsim 3$ are unstable and the neutron star spins down on a time scale

$$\tau \approx 10^6 (M/1.4M_\odot)^{-4} \times [R/(10\,\mathrm{km})]^5 \left[\frac{\Omega-\Omega_c}{0.1\Omega_c}\right]^{-(2m+1)} \mathrm{s},$$

where $R$ is the stellar radius. As the neutron star cools to $T\sim 10^9$ K on the time scale of years,[13] viscosity increases and $\Omega_c$ rises to $\sim 0.9\Omega_K$.[12] $\Omega_c$ and $\Omega$ thus approach each other. In this phase, which would be the current one for SN 1987A in this picture, the spin-down time scale becomes[17]

$$\tau \sim 10^9 (\Omega-\Omega_c)^{-1} \left[\frac{M}{1.4M_\odot}\right]^{1/2} \left[\frac{R}{10\,\mathrm{km}}\right]^{1/2} \times \left[\frac{\epsilon}{10^{15}\,\mathrm{gm}^{-3}}\right]^{-5/4} \left[\frac{T}{10^9\,\mathrm{K}}\right]^2,$$

where $\epsilon$ is the mean density of the neutron star. It is also possible that rapid spin down in the initial hot state left the neutron star rotating with frequency $\Omega$ less than the current value of $\Omega_c$.

Our results also have implications for the possibility of gravitational collapse of neutron stars and for the masses of neutron-star progenitors. For every EOS in our complex, we find that the mass of a neutron star rotating with $\Omega \geq \Omega_{\mathrm{SN}}$ exceeds the maximum nonrotating mass. (The excess mass ranges from 3% for EOS $\pi$ to 17% for EOS A.) If spun down sufficiently far, say by a pulsar mechanism, such an object will become unstable to gravitational collapse.[9] Hence one is led to speculate on the existence of a class of black holes formed in this way with masses $\sim 1.5M_\odot$ to $2M_\odot$. A magnetic field of $10^9$ G (corresponding to the observed luminosity of the remnant SN 1987A) could be expected to spin such a pulsar down in $10^7$–$10^8$ yr, while a field of $4\times 10^7$ G would be sufficient for spin down within the age of the Universe ($10^{10}$ yr). If a similarly rapid pulsar is the typical outcome of the supernova of a similar progenitor star (initially a main sequence star of mass $<20M_\odot$), one can conservatively estimate $10^6$ black holes to have formed in this way in the lifetime of the galaxy.[18] Finally, in this connection, all our neutron-star models have (gravitational) masses $M\gtrsim 1.5M_\odot$ and baryon, or rest, masses $M_0\gtrsim 1.7M_\odot$ when $\Omega\gtrsim\Omega_{\mathrm{SN}}$. This bound on $M_0$ places a lower limit on the total mass of an immediate progenitor (assuming negligible gravitational binding energy for the progenitor). That is, a stellar core that collapses to form a neutron star with $\Omega\gtrsim\Omega_{\mathrm{SN}}$ must have $M\approx M_0 \gtrsim 1.7M_\odot$.

After completion of this paper we received reports by Sato and Suzuki[19] and Goldman[20] which give some of the same conclusions reached here.

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