Comment on "Reflected Phase-Conjugate Wave in a Plasma"

Nebenzahl, Ron, and Rostoker[1] (NRR) consider the mixing of two strong counterpropagating transverse electromagnetic "pump" waves (labeled 1 and 3, and having amplitude E_0, frequency ω_0, and wave vectors $\pm\mathbf{k}_0$) with an imposed weak wave (labeled 2, and having frequency ω_s and wave vector $\mathbf{k}_s$) in a homogeneous plasma to generate a fourth wave (labeled 4 and having frequency $2\omega_0-\omega_s$). This is the standard "phase-conjugation" geometry of "four-wave mixing." Resonant enhancement is obtained by arranging that $\omega_0-\omega_s$ equal the ion-acoustic wave frequency $\Omega=v_a q$, where $q\equiv|\mathbf{k}_0-\mathbf{k}_s|$ and v_a is the acoustic velocity. NRR claim that this geometry provides a special enhancement that allows strong wave mixing to be observed at intensities several orders of magnitude below those required to produce, for example, significant stimulated Brillouin (acoustic-wave) scattering. Here we correct their derivation to show that no such enhancement exists, just as it is known not to exist when the Langmuir resonance is used instead[2] or when the acoustic resonance in nonconducting media is used.

The main result of NRR is that the wave 4 has an amplitude whose magnitude is $\tan|\kappa|L$ times that of the incident wave after interacting for a distance L. In their Eq. (12) for the peak κ, the acoustic linewidth Γ_a appears, having been introduced in their (unnumbered) expression following Eq. (6) for the linear dielectric function $\epsilon_a(\mathbf{q},\nu)$ of the plasma near the ion-acoustic resonance ($\nu\sim\Omega$). In fact the Γ_a appearing there should be multiplied by $(k_D/q)^2$, where k_D is the Debye wave number.[3] This factor moves unchanged through the remaining analysis with its approximations for q and k_s so that, in the final result (12) of NRR, one must replace the κ by the modified

$$\kappa_{\text{mod}}=\kappa(q/k_D)^2. \tag{1}$$

If we call the scattering angle between the beams labeled 1 and 2 by θ_{12}, and assume, as NRR did, that the acoustic frequency Ω is small compared to the optical pump frequency ω_0 (where the refractive index is near unity), then $q=(2\omega_0/c)\sin(\theta_{12}/2)$ and $k_D=\omega_p(mc^2/T_e)^{1/2}$, so that we may write

$$\kappa_{\text{mod}}=\kappa(T_e/mc^2)(\omega_0/\omega_p)^2 4\sin^2(\theta_{12}/2). \tag{2}$$

Using the numerical example of the authors, one finds that the corrected κ_{mod} is 4 orders of magnitude smaller than their value for κ. We next show that κ_{mod} shows no enhancement over the usual stimulated Brillouin growth coefficient, and hence the whole analysis must be modified to include nonlinear Brillouin currents.

If, in the geometry of NRR, wave 3 is turned off, then the weak wave 2 simply experiences exponential growth, i.e., its intensity is proportional to $\exp(\beta_s l)$, where l is the distance traveled along its wave vector. In a plasma in which there is no bound-state nonlinear response[2]

$$\beta_s=\pi E_0^2 e^4\,\mathrm{Im}R_e(\mathbf{q},\nu)/\omega_s\omega_0^2\hbar m^2 c\ \mathrm{cm}^{-1}, \tag{3}$$

where $R_e(\mathbf{q},\nu)$ is the electron density response function defined, for quantum (or classical) plasmas, by

$$\int_0^\infty dt\int d^3x\, e^{i\nu t-i\mathbf{q}\cdot\mathbf{x}}\, i\langle n(\mathbf{x},t),n(0,0)\rangle. \tag{4}$$

Here n is the usual electron number density operator, and the $\langle\,\rangle$ signifies the appropriate average. We may define similarly the ion density response function $R_a(\mathbf{q},\nu)$ by replacing n in (4) by the ion number density operator N. For ν near the acoustic frequency Ω, the electrons follow the ions nearly perfectly and we may use $R_e\approx Z^2R_a$ in (3). Here Ze is the ionic charge. If we know the spectral shape of $R_a(\mathbf{q},\nu)$ we may normalize it by the Thomas-Kuhn-Reich identity $\int_{-\infty}^{\infty}\nu\,d\nu\,R_a(\mathbf{q},\nu)\equiv\pi\hbar q^2\bar{N}/M$, where M is the ion mass and $\bar{N}$ is the average ion density. If there is a Lorentzian acoustic resonance of full width at half maximum γ (rms) then the peak value of $\mathrm{Im}R_a(\mathbf{q},\Omega)$ is $\hbar q^2\bar{N}/\Omega\gamma M$ and (3) gives for the maximum Brillouin gain coefficient

$$\beta_{sm}=\pi E_0^2 e^4 Zq^2 n_0/\omega_s\omega_0^2 m^2 c\,\Omega\,\gamma M\ \mathrm{cm}^{-1}, \tag{5}$$

where $n_0=Z\bar{N}$ is the average electron density. With the approximation of NRR that $\Omega=q(T_e/M)^{1/2}$, and $\Gamma_a=\gamma/\Omega$, we calculate the ratio of (5) to (3), assuming the same total pump intensity in each case, and find

$$\beta_{sm}/\kappa_{\text{mod}}=4Z\omega_0^3/\omega_s c^2 q^2. \tag{6}$$

Since $cq\lesssim 2\omega_0$ this ratio is always greater than, or nearly equal to, unity. Realizing that the Brillouin growth coefficient β_{sm} is always larger than the pure four-wave mixing coefficient κ_{mod}, one must redo the analysis of NRR with the (Stokes and anti-Stokes) Brillouin currents included, and interfering coherently, with the four-wave mixing current. The solution is no longer of the form $\tan|\kappa|L$. Such complete solutions have been studied for the Langmuir resonance, with the result "that for waves near the matched condition ($|\mathbf{k}_1-\mathbf{k}_2+\mathbf{k}_3-\mathbf{k}_4|L\lesssim 1$) the growth rate is always lower than that for unmatched waves."[2] This is also commonly the case for nonconducting media. We would not be surprised if it were the case here also.

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[3] *Radiation Processes in Plasmas*, edited by G. Bekefi (Wiley, New York, 1966).