

Clean Boundary between Anisotropic Superconductors as a Weak Link

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If two anisotropic superconductors are in perfect contact, the persistent current normal to a sufficiently long interface must be accompanied by spontaneous creation of a vortex chain at the boundary (in zero external field). The phenomenon is due to peculiar boundary conditions which should be imposed upon currents at the interface. The vortices are subject to the Lorentz force along the interface, thus providing the possibility of an extra dissipation and reduction of the critical current across the grain boundary.

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In a series of remarkable experiments,¹⁻³ Chaudhari and co-workers have recently demonstrated that clean boundaries of misaligned crystallites of YBa₂Cu₃O₇ suppress the *boundary* critical current j_c ; the suppression rapidly increases with the misalignment. A qualitative argument is given in this Letter to show that the supercurrent through the boundary between two misaligned anisotropic superconductors must be accompanied by a nonzero persistent current *along* the boundary, which flows in opposite directions on the two sides of the interface. This is due to peculiar conditions at the interface separating anisotropic superconductors. The extra current results in a magnetic field parallel to the boundary, which must be quantized. In other words, a current through the boundary initiates spontaneous creation of a chain of vortices bound to the interface, causing the contact of two misaligned banks to acquire properties of a weak link. The model provides a basis for the qualitative understanding of the IBM experiments¹⁻³ and sheds light on the "bottleneck" problem of critical currents in polycrystalline high- T_c materials.

Let us consider a plane boundary, $x=0$, between two anisotropic grains. It is assumed—for simplicity—that the z axis is a common principal direction, say \hat{c} , for both crystallites, while the remaining two principal directions on two sides of the boundary are misaligned. The misalignment may be characterized by two angles, θ^R at the right and θ^L at the left [Fig. 1(a)], between one of the "in-plane" principal directions, call it \hat{a} , and the normal \hat{x} to the interface on the right of the boundary (where $x > 0$).

Let us also assume that a small macroscopically uniform current j_x flows through the system in the direction normal to the boundary. This, of course, implies that the grains can sustain the current by having a proper distribution of pinning sites. Although vortices can be present in the grain's interior, we assume that a layer of thickness λ (average penetration depth) around the interface is free of pins and vortices; this is to single out the properties of the boundary itself. In materials of interest, the London theory can be applied provided distances on the order ξ (the coherence length) are of no importance.

The anisotropic version of the London equations reads, in standard notation (see, e.g., Ref. 4),

$$h_i = e_{ikl} \frac{\partial}{\partial x_l} \left[\frac{4\pi\lambda^2}{c} m_{kn} j_n \right], \tag{1}$$

where the components of the dimensionless mass tensor m_{ik} in the x - y frame are

$$\begin{aligned} m_{xx} &= m_a \cos^2\theta + m_b \sin^2\theta, \\ m_{yy} &= m_a \sin^2\theta + m_b \cos^2\theta, \\ m_{xy} &= (m_a - m_b) \sin\theta \cos\theta. \end{aligned} \tag{2}$$

Here $m_{a,b}$ are the eigenvalues of m_{ik} in the x - y plane; the crystals are not assumed uniaxial, i.e., $m_a \neq m_b$.⁵ The angle θ acquires the superscript R for the right grain and

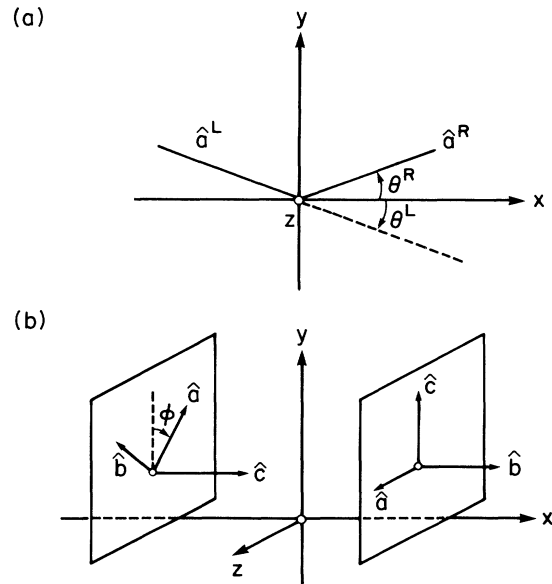


FIG. 1. (a) Example of symmetric grain boundary at $x=0$: $\theta^L = -\theta^R$. The \hat{c} axes coincide with \hat{z} on both sides of the boundary. (b) A situation in which the grain boundary at $x=0$ does not have the weak-link property: $\hat{c}^L = \hat{b}^R = \hat{x}$.

L for the left. The z component of the current as well as $\partial/\partial z$ can be set equal to zero, so that the m_{zi} 's (of which the only nonzero one is $m_{zz}=m_c$) are not needed. It is assumed, again for simplicity, that $\theta^R = -\theta^L > 0$, a "symmetric misalignment" shown in Fig. 1(a). Then, the diagonal elements of m_{ik} on both sides are the same, while $m_{xy}^R = -m_{xy}^L$; we imply hereafter that m_{xy} (without a superscript) belongs to the right.

The boundary conditions to Eq. (1) consist of the continuity requirements for the field \mathbf{h} , for the normal component of \mathbf{j} (i.e., j_x), as well as for the tangential component of the vector potential \mathbf{A} . Comparing the definition $h_i = e_{ikl} \partial A_l / \partial x_k$ with Eq. (1), one concludes that A_i can be chosen as $-(4\pi\lambda^2/c)m_{lk}j_k$. This relation between \mathbf{A} and \mathbf{j} implies a certain gauge: $m_{ik} \partial A_k / \partial x_i = \partial j_i / \partial x_i = 0$, the situation analogous to that of the isotropic London theory, where $\text{div} \mathbf{A} = 0$. The continuity of A_y yields

$$(m_{yk}j_k)^L = (m_{yk}j_k)^R. \quad (3)$$

This boundary condition was introduced first by Grishin who considered the interaction of vortices with the interface between anisotropic superconductors.⁶

Owing to the central role of Eq. (3), it is worth indicating yet another way to obtain the boundary condition for the London equations: While performing the minimization of the London free energy (to derive the anisotropic London equations, see, e.g., Ref. 4) for the system of two misaligned crystals, one integrates the "kinetic energy" term, $\int m_{ik} \text{curl}_i \mathbf{h} \text{curl}_k \mathbf{h} dV$, by parts. This results in the surface contribution to the free energy which should be minimized as well, thus yielding the condition (3).

Note that for the symmetric misalignment of Fig. 1(a), the interface $x=0$ is *not* a symmetry plane in the presence of a persistent current j_x (under $x \rightarrow -x$, j_x changes sign). However, the combined operation $x \rightarrow -x$ and the time inversion leaves the current j_x unchanged. Therefore, the component j_y should have the property that $j_y(x) = -j_y(-x)$; in particular, $j_y(+0) = -j_y(-0)$. Condition (3) now yields the following for currents at the interface:

$$j_x(m_{yx}^R - m_{yx}^L) = [j_y(+0) - j_y(-0)]m_{yy},$$

or using the symmetry,

$$j_y(+0) = -j_y(-0) = -(m_{xy}/m_{yy})j_x. \quad (4)$$

Thus, the current component along the boundary is discontinuous, the current is "refracted," while no fundamental law is violated (in a better theory the discontinuity should be smeared over a distance $\sim 2\xi$). This unusual, at first sight, result is rooted in a peculiar feature of charged anisotropic superfluids: The directions of the mass flow (that of the momentum \mathbf{Q}) and of the charge flow \mathbf{j} are not the same (unless both \mathbf{Q} and \mathbf{j} point in one of the principal directions). Indeed, the current is given

by $j_i = 2en_s m_{ik}^{-1} Q_k$, where n_s is the density of the "superconducting carriers" and $\mathbf{Q} = \hbar \nabla \eta - 2e\mathbf{A}/c$ is the gauge-invariant "supermomentum" with η being the order-parameter phase. Equation (3), then, represents the continuity of the tangential component of the momentum, Q_y , which must be fulfilled at the plane interface.

The current $j_y(x)$ should be attenuated away from the boundary. To estimate this effect, take the curl of Eq. (1) and solve for $j_y(x)$, assuming j_y is y independent: $j_y(x) = j_y(+0) \exp(-x/\lambda \sqrt{m_{yy}})$ for $x > 0$. The same current flows along y at the left, but in the opposite direction. The current j_y produces the magnetic field h_z with the total flux $\Phi = 8\pi j_x \lambda^2 m_{xy}/c$ per 1 cm of the y axis. Clearly, this flux should be quantized; i.e., it should break up into vortices centered at the interface and each containing the flux quantum ϕ_0 (if the boundary is sufficiently long). In other words, the actual situation is not as simple as in the "laminar solution" just outlined. In particular, because of the repulsive interaction, the vortices should form a chain with a period

$$L = \phi_0/\Phi = c\phi_0/8\pi j_x \lambda^2 m_{xy}. \quad (5)$$

Thus, in the absence of an external field, the transport current through the interface is accompanied by a vortex chain at the boundary (provided the boundary length exceeds L). The intervortex spacing L in the chain diverges for perfectly aligned grains ($\theta^R = \theta^L = 0$ and $m_{xy} = 0$) and in the absence of the current j_x through the interface; i.e., in these cases the chain does not exist. The Lorentz force $j_x \phi_0/c$ acting on each vortex will move the whole chain along the boundary, unless the interface contains pinning sites. Therefore, *the ideal, pinning-free, plane boundary cannot support any current across itself without dissipation* (except in some special situations discussed below). A real imperfect interface can be characterized by a maximum Lorentz force that the chain can withstand; i.e., the pinning force per unit length of the boundary, P , can be introduced by $P = j_x \times \phi_0/cL = (8\pi\lambda^2/c^2)m_{xy}j_x^2$. Therefore, the critical current $j_c \propto m_{xy}^{-1/2} \propto (\sin 2\theta)^{-1/2}$, i.e., the boundary critical current should fall off with increasing misalignment. For the perfect alignment this does not impose any restriction upon j_c ; in this case j_c is governed by the bulk pinning.

Applying a small field H_z , one can reduce the line density of vortices in the chain, provided the direction of the applied field is opposite to that of the quantized field associated with the chain. Therefore, a small applied field H_z can increase or suppress the j_c , depending on the sign of H_z . In other words, the critical current as a function of the applied field should peak at a field, H_p , which compensates the flux of the chain. The field H_p is estimated by equating the flux, $H_p \lambda \sqrt{m_{yy}}$, through 1 cm of the chain to Φ :

$$H_p = (8\pi/c)j_x \lambda (m_{xy}/\sqrt{m_{yy}}). \quad (6)$$

Unfortunately, a comparison with the asymmetry in $j_c(H)$ reported in Ref. 3 cannot be done because the observed asymmetry might be related to other than the boundary sources of irreversibility. Still, the simple model outlined here describes—at least qualitatively—major experimental facts.¹⁻³

The twin boundary is a special case of the symmetric misalignment; e.g., for $\text{YBa}_2\text{Cu}_3\text{O}_7$, $\theta \approx \pi/4$, $m_{xy} \approx (m_a - m_b)/2$, and $m_{xx} \approx m_{yy} \approx (m_a + m_b)/2$. Therefore, twinned grains of orthorhombic materials (such as those of the $\text{YBa}_2\text{Cu}_3\text{O}_7$ family) should have “built-in” weak-link features (with respect to currents normal to twin boundaries).⁷ These may well be associated with the “intrinsic” weak-link properties of single crystals discussed in the literature.⁸⁻¹²

Only special symmetric misalignments have been treated above. The general case, of course, is more complicated and rich in possibilities. To mention only one: Contacts between isotropic and anisotropic superconductors should be dissipative for a general orientation of the anisotropic bank. Generally speaking, the “weak-link” property of the interface should be present for any misalignment, except some special cases which are clearly of a practical interest. One of these situations comes about when the normal \hat{x} to the interface coincides with a principal direction of both the right- and left-hand sides; an example is shown in Fig. 1(b). Choosing the y and z axes along the c and a of the right-hand side, one has

$$m_{ik}^R = \begin{pmatrix} m_b & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_a \end{pmatrix}, \quad m_{ik}^L = \begin{pmatrix} m_c & 0 & 0 \\ 0 & m_{yy} & m_{yz} \\ 0 & m_{zy} & m_{zz} \end{pmatrix},$$

where the yz components of m_{ik}^L are readily expressed in terms of m_a , m_b , and the angle ϕ , shown in Fig. 1(b). The boundary conditions $(m_{ik}j_k)^L = (m_{ik}j_k)^R$ with $t=y,z$ then yield $m_{yy}^L j_y^L + m_{yz}^L j_z^L = m_c j_y^R$ and $m_{zy}^L j_y^L + m_{zz}^L j_z^L = m_a j_z^R$. The tangential currents, $j_{y,z}$, of this case are *decoupled* from the transport current j_x through the boundary. There always exists a zero solution for tangential currents; in other words, the interface supports the transport current without dissipation.

It should be noted that though the term “weak link” is used throughout this text to describe the dissipative property of a current-carrying boundary between anisotropic superconductors, one should not confuse this boundary with Josephson-type junctions. To underline the difference, it is enough to mention again that a pinning-free grain boundary cannot support any finite persistent current across itself without dissipation. It is interesting to note that current-voltage characteristics of the boundary reported in Ref. 1 are more reminiscent of a system governed by pinning than of a Josephson-type junction, although the data are not sufficient for a definite conclusion.

Effects of the anisotropic penetration depth upon properties of interfaces between misaligned anisotropic superconductors, which have been considered in this Letter, do not exhaust all possible anisotropy effects. If the gap Δ is also anisotropic, the properties of the boundary in question should differ from those described above. However, given the little experimental information now available, the role of the gap anisotropy is hard to estimate.

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