Further Evidence of Nonclassical Behavior in Optical Interference

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It is demonstrated in a photon coincidence experiment with two photodetectors, in which signal and idler photons produced by parametric down-conversion are allowed to interfere, that the visibility of the interference pattern is well above 50% and remains unchanged when one of the two light beams is attenuated ninefold compared with the other. These results violate classical probability for light waves.

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Most optical interference effects are describable in completely classical terms. Moreover, it is a feature of all such classical effects that the visibility of the interference pattern depends on the ratios of the interfering light intensities, and in a two-beam interference experiment the visibility becomes small when the two mean intensities are very different.

Recently several optical interference effects have been observed that are only describable in quantum-mechanical terms and violate classical probability.[1-4] Because some of the experiments involve the detection of two photons by two detectors, these effects are usually referred to as fourth-order interference. In one of the experiments signal and idler photons produced simultaneously in the process of parametric down-conversion were allowed to interfere, and the joint probability $P(x_1,x_2)$ of detecting two photons at two positions x_1,x_2 was measured as a function of detector position.[2] The phenomenon depends on the interference of the two two-photon probability amplitudes, and leads to a cosine modulation of $P(x_1,x_2)$ in x_1-x_2 with visibility[2] [cf. Eq. (9) below],

$$\mathcal{V}=\frac{P_{\max}-P_{\min}}{P_{\max}+P_{\min}}=\left[\frac{\sin(\pi\Delta x/L)}{\pi\Delta x/L}\right]^2 . \tag{1}$$

L is the spacing of the classical interference fringes that would be expected for incident waves inclined at the chosen small angle and Δx is the integration interval (slit width) in the experiment. It follows that $\mathcal{V}$ is close to unity for small Δx and independent of the average intensities of the two incident light beams. For interfering classical fields, on the other hand, the visibility $\mathcal{V}$ can be no larger than 50%,[5-8] and it depends on the ratio of the two interfering beam intensities, as we show below. We wish to report on new measurements made in a two-photon interference experiment of this type, that show explicitly that the visibility $\mathcal{V}$ not only exceeds 50% but is independent of the two incident light intensities. We find that $\mathcal{V}$ remains close to 75% even when one light beam is attenuated ninefold compared with the other one.

We consider the fourth-order interference experiment involving two photodetectors that is illustrated in Fig. 1. Two plane, almost monochromatic but randomly phased, light waves with wave vectors $\mathbf{k}_1$ and $\mathbf{k}_2$ and complex amplitudes V_1 and V_2 are incident on a symmetric parallel-sided beam splitter from opposite sides. As a result two new waves emerge from both sides of the beam splitter and fall on two detector apertures located at $\mathbf{r}_a$ and $\mathbf{r}_b$ as shown. If V_a,V_b are the complex amplitudes of the waves at the detector apertures, and we treat the point O as the origin, then we can write

$$\begin{aligned} V_a&=rV_1e^{i\tilde{\mathbf{k}}_1\cdot\mathbf{r}_a}+t'V_2e^{i\mathbf{k}_2\cdot\mathbf{r}_a},\\ V_b&=tV_1e^{i\mathbf{k}_1\cdot\mathbf{r}_b}+r'V_2e^{i\tilde{\mathbf{k}}_2\cdot\mathbf{r}_b}, \end{aligned} \tag{2}$$

where $\tilde{\mathbf{k}}_1,\tilde{\mathbf{k}}_2$ are the wave vectors corresponding to $\mathbf{k}_1,\mathbf{k}_2$ after reflection, and r,t and r',t' are the complex amplitude reflectivity and transmittivity of the beam splitter from one side and from the other side. We shall assume that the angles θ between $\tilde{\mathbf{k}}_1$ and $\mathbf{k}_2$ and between $\tilde{\mathbf{k}}_2$ and $\mathbf{k}_1$ are very small, so that the associated interference pattern has a fringe spacing given very nearly by

$$L\approx\frac{2\pi}{|\tilde{\mathbf{k}}_1-\mathbf{k}_2|}=\frac{2\pi}{|\tilde{\mathbf{k}}_2-\mathbf{k}_1|}\approx\frac{\lambda}{\theta}. \tag{3}$$

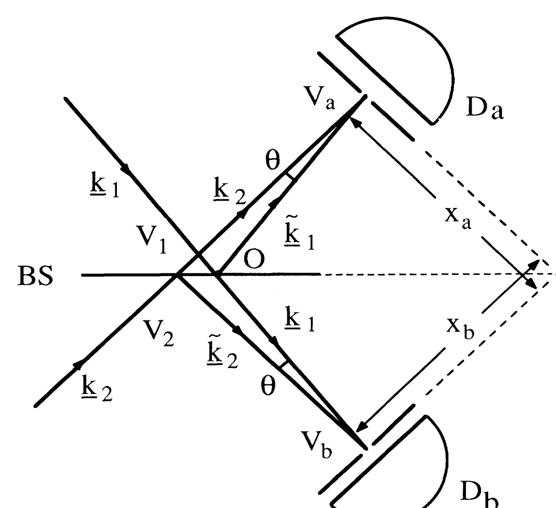

FIG. 1. Outline of the geometry for the experiment.

If the beam splitter is 50%:50%, then $|r|=|t|=1/\sqrt{2}$ and we can simplify Eqs. (2) in the form

$$V_a=\frac{e^{i\theta_t}}{\sqrt{2}}[iV_1e^{i\tilde{\mathbf{k}}_1\cdot\mathbf{r}_a}+V_2e^{i\tilde{\mathbf{k}}_2\cdot\mathbf{r}_a}]\,,$$
$$V_b=\frac{e^{i\theta_t}}{\sqrt{2}}[V_1e^{i\mathbf{k}_1\cdot\mathbf{r}_b}+iV_2e^{i\tilde{\mathbf{k}}_2\cdot\mathbf{r}_b}]\,. \qquad (4)$$

We have put $r'=r=|r|e^{i\theta_r}$, $t'=t=|t|e^{i\theta_t}$, and have made use of the well-known general relation $\theta_r-\theta_t=\pm\pi/2$ for a symmetric beam splitter.[9-13] From Eqs. (4) we immediately obtain for the corresponding light intensities

$$I_a=|V_a|^2 =\tfrac{1}{2}[I_1+I_2]+\tfrac{1}{2}[iV_1V_2^*e^{i(\tilde{\mathbf{k}}_1-\mathbf{k}_2)\cdot\mathbf{r}_a}+\text{c.c.}]\,,$$
$$I_b=|V_b|^2 =\tfrac{1}{2}[I_1+I_2]+\tfrac{1}{2}[-iV_1V_2^*e^{i(\mathbf{k}_1-\tilde{\mathbf{k}}_2)\cdot\mathbf{r}_b}+\text{c.c.}]\,. \qquad (5)$$

Now it is well known that the joint probability P for registering photodetections at both detectors simultaneously is proportional to the classical ensemble average $\langle I_aI_b\rangle$.[14] With the assumption that V_1,V_2 have random phases we then obtain from Eqs. (5)

$$P=K\langle I_aI_b\rangle=\tfrac{1}{4}K\{\langle(I_1+I_2)^2\rangle-2\langle I_1I_2\rangle\cos[(\mathbf{k}_2-\tilde{\mathbf{k}}_1)\cdot\mathbf{r}_a-(\tilde{\mathbf{k}}_2-\mathbf{k}_1)\cdot\mathbf{r}_b]\}$$
$$=\tfrac{1}{4}K\{\langle(I_1+I_2)^2\rangle-2\langle I_1I_2\rangle\cos[2\pi(x_a-x_b)/L]\}\,, \qquad (6)$$

where K is characteristic of the detectors and x_a,x_b are the positions of the detector apertures, as illustrated in Fig. 1. L given by Eq. (3) is the spacing of the interference fringes corresponding to the given inclination θ between wave vectors $\mathbf{k}_1$ and $\tilde{\mathbf{k}}_2$ and between $\tilde{\mathbf{k}}_1$ and $\mathbf{k}_2$ $(L\approx\lambda/\theta)$. The second line in Eq. (6) follows immediately from the first when we note that $\mathbf{k}_2-\tilde{\mathbf{k}}_1$ is a vector of magnitude $k\theta$ that points along x_a, whereas the projection of $\mathbf{r}_a$ onto this vector is x_a-const. In practice it is difficult to make observations at "points" x_a and x_b, and each measurement represents an average over the width Δx of the entrance slit at the detector. As is well known,[15] this reduces the observable modulation in Eq. (6) by the factor $[\sin(\pi\Delta x/L)/(\pi\Delta x/L)]^2$.

Equation (6) describes a fourth-order interference effect. If we include the effect of the finite aperture Δx, then, according to classical optics, the visibility $\mathcal{V}$ of the interference is given by

$$\mathcal{V}=\frac{2\langle I_1I_2\rangle}{\langle I_1^2\rangle+\langle I_2^2\rangle+2\langle I_1I_2\rangle}\left[\frac{\sin(\pi\Delta x/L)}{\pi\Delta x/L}\right]^2, \qquad (7)$$

and as $\langle I_1^2\rangle+\langle I_2^2\rangle\geq 2\langle I_1I_2\rangle$, it follows that $\mathcal{V}\leq\frac{1}{2}$, which is a classical limit that has been derived several times before.[5-8]

Of particular interest to us here is the dependence of the visibility on the ratio of the two mean light intensities, $R\equiv\langle I_2\rangle/\langle I_1\rangle$. For simplicity, let us suppose that the normalized autocorrelation and crosscorrelation of any light intensity fluctuations are all equal, i.e.,

$$\langle\Delta I_1\Delta I_2\rangle/\langle I_1\rangle\langle I_2\rangle=\langle(\Delta I_1)^2\rangle/\langle I_1\rangle^2=\langle(\Delta I_2)^2\rangle/\langle I_2\rangle^2\,.$$

Then Eq. (7) yields

$$\mathcal{V}=\frac{2\langle I_1\rangle\langle I_2\rangle}{(\langle I_1\rangle+\langle I_2\rangle)^2}\left[\frac{\sin(\pi\Delta x/L)}{\pi\Delta x/L}\right]^2 =\frac{2R}{(1+R)^2}\left[\frac{\sin(\pi\Delta x/L)}{\pi\Delta x/L}\right]^2, \qquad (8)$$

and an identical result is obtained if I_1,I_2 do not fluctuate at all. The visibility has its largest value when $R=1$, and falls by a factor of about 3 when $R=9.4$ or 1/9.4. It is interesting to note, however, that $\mathcal{V}$ given by Eq. (7) would be unity and independent of R if one had $\langle I_1^2\rangle=0=\langle I_2^2\rangle$, which is of course impossible for a classical field of nonzero mean. From the standpoint of quantum mechanics, on the other hand, $\langle I^2\rangle$ can vanish for single photons entering from each side of the beam splitter in place of the classical waves V_1,V_2 in Fig. 1, if one identifies I^2 with the intensity squared in normal order. Such photon pairs can be generated in the parametric down-conversion process. Indeed quantum mechanically one finds that under these conditions the probability of two-photon detection is given by[2,5-8]

$$P=KN_P\left\{1-\left[\frac{\sin(\pi\Delta x/L)}{\pi\Delta x/L}\right]^2\cos\left[2\pi\frac{x_a-x_b}{L}\right]\right\}, \qquad (9)$$

where N_P is the number of incident photon pairs, and K is characteristic of the detectors as before. We note that this time the visibility can exceed 50% and is independent of the photon beam intensity. Attenuation of one of the incident photon beams relative to the other one merely reduces the number of photon pairs N_P, and therefore the coincidence rate, but not the visibility of the interference pattern. Below we report on an experiment of this kind, in which the fringe visibility was observed to be about 75% and to remain unchanged when one light intensity was attenuated by a factor 9.4 compared with the other.

An outline of the experiment, which is closely related to several previously reported experiments,[2-4] is shown in Fig. 2. Degenerate signal and idler photons are produced in the process of parametric down-conversion from an incident argon-ion laser beam at 351.1 nm that interacts with a nonlinear crystal of $LiIO_3$. The down-converted photons provide the two inputs to the beam

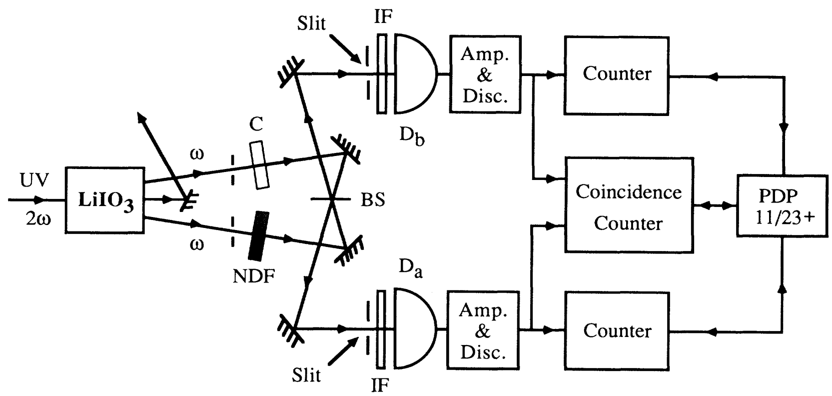

FIG. 2. Outline of the experiment.

splitter. The beam splitter outputs are received and measured by two photomultipliers D_a and D_b mounted on micrometer stages, whose positions x_a,x_b can be varied transversely to the incident light. The photomultiplier pulses, after amplification and pulse shaping are fed to the two inputs of a coincidence counter with 7.7 nsec resolving time T_c. The number N_c of coincident pulses recorded in some measurement interval of order 25 min provides a measure of the joint detection probability $P(x_a,x_b)$, after accidental coincidences are subtracted out. The expected number of accidentals in a measurement interval of length T is given by $r_a r_b T_c T$, where r_a,r_b are the counting rates in the two detector channels. In performing the experiment we actually vary x_a by moving one of the detectors, but hold x_b constant.

The two interfering light beams are inclined to each other at an angle of about 1 mrad, leading to interference fringes with a spacing of about 0.8 mm at a wavelength of 700 nm. With slits of width $\Delta x \approx 0.1$ mm in front of the detectors, the factor $[\sin(\pi\Delta x/L)/(\pi\Delta x/L)]^2$ comes to about 0.95. In practice the observed visibility fell consistently about 20% below this value, possibly because of imperfect alignment between the slits and the interference pattern, which is not of course directly visible. Moreover, because of the angular distribution of the down-converted photons, the illumination in the interference plane was not uniform, but fell by nearly 40% within ± 1 mm from the center of the interference pattern, as shown by measurements of the light intensity. In order to incorporate these effects we need to modify Eq. (9) somewhat and we write for the coincidence counting rate

$$\mathcal{R}_c = C N_P f(x_a) \left\{ 1 - \eta \left[\frac{\sin(\pi\Delta x/L)}{\pi\Delta x/L} \right]^2 \cos\left[2\pi \frac{x_a - x_b}{L} \right] \right\}, \tag{10}$$

where C is another constant. Here $\eta \sim 0.8$ and from measurements of the beam intensity the function $f(x_a)$ is found to be reasonably well approximated over the range $x_a = 5.7 \pm 1$ mm by the Gaussian function

$$f(x_a) = e^{-0.5(x_a - 5.7)^2} \quad (x_a \text{ in mm}) . \tag{11}$$

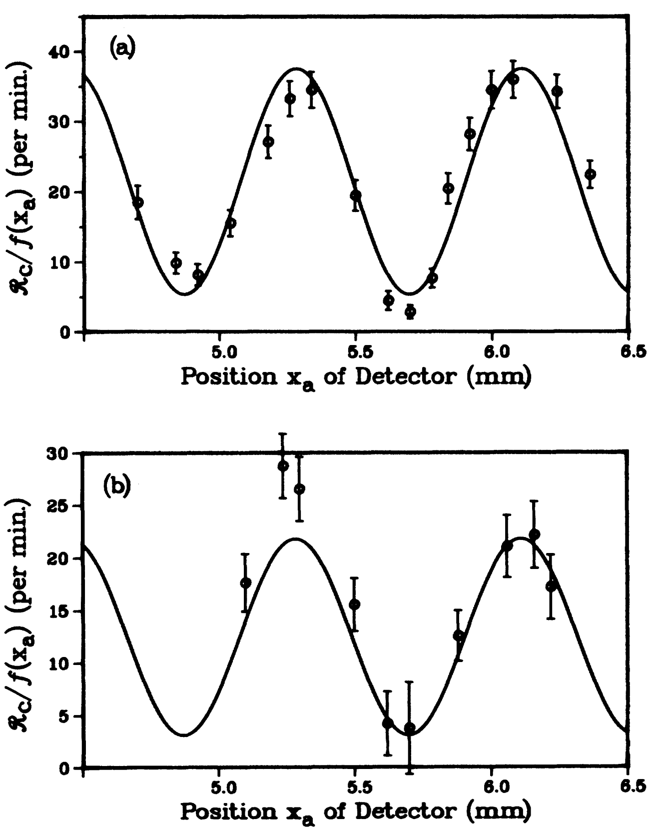

FIG. 3. Measured photon coincidence rates $\mathcal{R}_c$ divided by $f(x_a)$ as a function of detector position x_a for (a) unattenuated signal and idler beams and (b) signal beam attenuated elevenfold and idler-beam strength reduced by a factor of 0.84. The full curves are theoretical and are based on Eq. (10). The visibility $\mathcal{V}$ is just over 75% in both cases.

Figure 3(a) shows several measured values of the coincidence rate, together with their standard deviations, as a function of detector position x_a, with the other detector held fixed at x_b. The full curve is based on Eq. (10) with $x_b=5.7$ mm, $L\approx0.83$ mm, $\eta=0.79$, and $CN_P=21.4$/min. The precise values of x_b, L, η, and CN_P were chosen by a least-squares-fitting procedure, because they could not be determined very accurately by direct measurement. It will be seen that the interference pattern has a visibility $\mathcal{V}$ of about 75%, in clear violation of the canonical upper bound of 50% required by classical wave optics.

We then repeated the experiment with an 11:1 neutral-density filter (NDF) inserted in the signal photon beam, with a compensating glass plate C (with 0.84 transmittance) to produce an equal time delay inserted in the idler beam, as shown in Fig. 2. In order to partially compensate for the reduction in the resulting photon coincidence rate, the power of the pump laser beam was increased about 7.7 times. The net effect was a reduction in the number of photon pairs N_P by a factor $7.7\times\frac{1}{11}\times0.84=0.58$ compared with the first experiment. The results of the second series of measurements are shown in Fig. 3(b). The full curve is again based on Eq. (10) with CN_P reduced by the factor 0.58, but with all other parameters unchanged. Evidently there is no change of visibility of the interference pattern, as predicted by quantum mechanics, in violation of Eq. (8) for classical light waves, which predicts an almost threefold reduction of the visibility.

We have therefore demonstrated another nonclassical feature of light in an interference experiment for which a classical wave picture is often considered to be adequate.

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