Role of the Orbit-Orbit Component of the Fermi-Breit Interaction in a Quark-Model Description of the Baryon Spectra

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(Received 25 January 1989)

The nonstrange and strange baryon resonances have been studied in the framework of the nonrelativistic constituent-quark model involving an SU(6)⊗O(3) oscillator basis spanned by six major shells. It turns out that the inclusion of the orbit-orbit component of the Fermi-Breit interaction obviates to a large extent the necessity of the usual *ad hoc* parametrization of empirical anharmonic effects.

PACS numbers: 14.20.−c, 12.40.Qq

The nonrelativistic constituent-quark model has been extensively employed in the past for correlating a vast body of information on baryonic resonances with impressive overall phenomenological success.[1-13] It is now widely recognized[12,14,15] that judicious choices of such effective parameters as the constituent-quark masses and the strong-interaction coupling constant α_s permit one to reduce the quantitative significance of some of the obvious inherent shortcomings of the model, such as the neglect of relativistic quark dynamics, and the lack of explicit consideration of the gluons as well as the scale dependence.

An important observed feature that characterizes the baryonic excitation spectra in the nonstrange as well as the strange sector is the occurrence of the $L=0$, $N=2$ nodal excitations at energies comparable to those of the odd-parity resonances.[10] This feature has been sought to be rationalized in a number of calculations[2,15] carried out in the past by invoking the deviations from purely harmonic confinement in the framework of the first-order perturbation theory in conjuction with a truncated basis; these calculations have involved the removal of the degeneracy among the $N=2$ states in terms of several parameters characterizing the *diagonal* matrix elements—and these are often as large as the oscillator spacing itself—of the anharmonic terms. The inconsistency inherent in such a phenomenological approach has been brought out by later investigations[11] which indicated that an inclusion of the off-diagonal contributions of the anharmonicities via an exact Faddeev calculation nearly restores the nodal excitations to their original positions. The difficulty of providing a satisfactory description *simultaneously* for the odd- as well as even-parity resonances has also been highlighted by the calculations reported by Carlson, Kogut, and Pandharipande,[9] and by Capstick and Isgur.[12] Whereas the positions (relative to the $N=0$ state) of the odd-parity resonances have turned out *too low* by 50–100 MeV, the positions of the even-parity baryons, on the other hand, appear to be *too high* by roughly the same amount.

It is interesting to note that although the spin-dependent effective two-body potential suggested by the one-gluon-exchange (OGE) graphs has been extensively employed in the past after the initial suggestion by De Rújula, Georgi, and Glashow,[1] none of these nonrelativistic constituent-quark-model treatments reported to date has examined the role of the orbit-orbit interaction which is already inherent in the *same* OGE framework. The purpose of this contribution is to show that the involvement of the heretofore omitted orbit-orbit term results in a significant improvement of the spectra of the positive- and negative-parity baryons; it is seen that this term is particularly efficacious in providing a *parameter-free* description of the empirical anharmonic effects. We elucidate the quantitative significance of the orbit-orbit term by obtaining the positions of the well-established strange as well as nonstrange baryon resonances below $\simeq 2$ GeV/c^2 in a framework that involves this term in conjunction with the usual spin-dependent terms and a scalar confining potential operating in a basis spanned by totally antisymmetric q^3 states covering six oscillator shells. We also demonstrate how one can have qualitative appreciation of the main features of the results obtained with this interaction by recasting it in a form that involves operators with physically transparent roles.

The complete Hamiltonian employed in the present work can be expressed in the form

$$H=\sum_{i=1}^{3}\left(m_i+\frac{p_i^2}{2m_i}\right)+\sum_{i<j}[V_c^G(i,j)+V_c^{\mathrm{conf}}(i,j)+V_\sigma^G(i,j)+V_t^G(i,j)+V_D^G(i,j)+V_{\text{o-o}}^G(i,j)]\,. \quad (1)$$

Here, V_c^G, V_σ^G, V_t^G, V_D^G, and $V_{\text{o-o}}^G$ are the usual spin-independent central, spin-dependent central, tensor, Darwin, and orbit-orbit terms, respectively, that result from the Fermi-Breit reduction of the OGE amplitude:

$$V_c^G(i,j) = -2\alpha_s/3r_{ij}\,, \quad (2a)$$

$$V_\sigma^G(i,j) = (3m_i m_j/2)^{-1}\mathbf{S}_i\cdot\mathbf{S}_j[\Delta_{(r_{ij})}V_c^G]\,, \quad (2b)$$

$$V_t^G(i,j) = -(3m_i m_j)^{-1}(3\mathbf{S}_i\cdot\hat{\mathbf{r}}_{ij}\mathbf{S}_j\cdot\hat{\mathbf{r}}_{ij} - \mathbf{S}_i\cdot\mathbf{S}_j) \times [D_{(r_{ij})}V_c^G]\,, \quad (2c)$$

$$V_D^G(i,j) = [(m_i^{-2}+m_j^{-2})/8][\Delta_{(r_{ij})}V_c^G]\,, \quad (2d)$$

and

$$V_{\text{o-o}}^G(i,j) = -(2m_i m_j)^{-1}V_c^G[\mathbf{p}_i\cdot\mathbf{p}_j + \hat{\mathbf{r}}_{ij}\cdot(\hat{\mathbf{r}}_{ij}\cdot\mathbf{p}_i)\mathbf{p}_j]\,, \quad (2e)$$

where

$$D_\rho \equiv d^2/d\rho^2 - \rho^{-1}d/d\rho\,.$$

The confinement term appearing in Eq. (1) is given by

$$V_c^{\text{conf}}(i,j) = a + kr_{ij}[1-\exp(-\gamma_0^2 r_{ij}^2)]\,. \quad (3)$$

The term containing the parameter γ_0 ensures proper analytic behavior of the confinement potential at the origin.

In the present work we have regularized the Coulomb term by approximating it by a sum of five Gaussians over the interval $[r_{\min},r_{\max}]$; the term $V_c^G(i,j)$ is replaced by

$$\tilde{V}_c^G(i,j) = -(2\alpha_s/3)\sum_{i=1}^{5}c_i\exp(-\gamma_i^2 r_{ij}^2)\,, \quad (4)$$

where the parameters c_i and γ_i are uniquely related to the interval $[r_{\min},r_{\max}]$.

The use of the interaction $\tilde{V}_c^G(i,j)$ in Eqs. 2(b)-2(e) circumvents the problems that would normally arise due to the presence of singular δ functions in the context of a variational calculation. In the present work we have employed throughout the range $[r_{\min},r_{\max}] \equiv [0.25\text{ fm},1.00\text{ fm}]$.

The energy eigenvalues of the Hamiltonian H given by Eq. (1) in the basis of harmonic-oscillator wave functions were calculated by setting up and diagonalizing the Hamiltonian matrices in the space of totally antisymmetric q^3 states characterized by total angular momentum, isospin, strangeness, and parity, with the spatial part of the wave functions involving six oscillator shells. In view of the dependence of the eigenvalues on the size parameter b—an artifact of the present calculational framework with a truncated basis—we have determined its *optimum* value for *each* eigenvalue via the variational constraint:

$$\delta\langle\Psi(b)\,|\,H\,|\,\Psi(b)\rangle\,|_{b=b_{\text{opt}}} = 0\,. \quad (5)$$

In order to provide an estimate of the efficacy of the orbit-orbit interaction *vis-à-vis* the description of seventeen nonstrange as well as nineteen strange baryon resonances of three-star and four-star status (up to and including the positive-parity excited states), we have calculated the resonance positions separately for the following cases: (a) calculations involving the Hamiltonian $H - V_{\text{o-o}}^G$ of Eq. (1) and (b) calculations involving the Hamiltonian H of Eq. (1).

The calculations presented here involve the *single* set of six parameters listed in Table I. The optimized values of the oscillator parameter b_{opt} determined in calculation (a) are also used for calculation (b).

The results presented in Fig. 1 bring out the significance of the orbit-orbit term. Comparing the results of calculations (a) and (b) for the nonstrange baryon resonances one observes a significant selective lowering of the positions of the excited positive-parity states with respect to the negative-parity states as well as an increase of the intraband splitting of both groups of resonances.

Also in the case of the strange resonances one observes significant improvements in the calculated positions of some *specific* excitation; for example, the results shown in Fig. 1 display a preferential lowering—by about 100 MeV—of the resonances $\Lambda(\frac{1}{2}^+,1600)$, $\Lambda(\frac{1}{2}^+,1810)$, and $\Sigma(\frac{1}{2}^+,1660)$ relative to the positions of the other members of the $2\hbar\omega$ multiplet. In general, the orbit-orbit interaction is seen to *raise* the centroid of the odd-parity states, while *lowering selectively* the members of the $N=2$ band with quantum numbers $J^\pi=\frac{1}{2}^+$ (and, in the case of the Δ resonances, also the $J^\pi=\frac{3}{2}^+$)—a feature that is in consonance with the trends displayed by the available experimental information. The resulting rms deviations

$$\delta_{\text{rms}} = \left[\sum(M_{\text{expt}} \pm \Delta M - M_{\text{calc}})^2/N\right]^{1/2}, \quad (6)$$

where the experimental values are allowed to float within a range determined by the experimental uncertainties, are $\delta_{\text{rms}}=135$ and 144 MeV for the nonstrange and strange resonances, respectively.

We hasten to add that the large (residual) rms deviation in the calculation (b) even after the inclusion of the orbit-orbit interaction is simply a reminder of the fact that we have not considered several important state-of-the-art ingredients (along with the associated set of nonfundamental additional parameters)—such as the three-body potential[9] motivated by the constraint of the minimization of the energy contained in the color-electric flux tubes associated with the gluonic fields, relativsitic modifications of the interquark interactions,[12] as

TABLE I. Interaction parameters [see Eqs. (2)-(4)] employed for obtaining the results presented in Fig. 1.

$\alpha_s=0.54$
$a=-436.0$ MeV
$k=532.0$ MeV/fm
$\gamma_0=395.0$ MeV
$m_u=m_d=396.0$ MeV
$m_s=645.0$ MeV

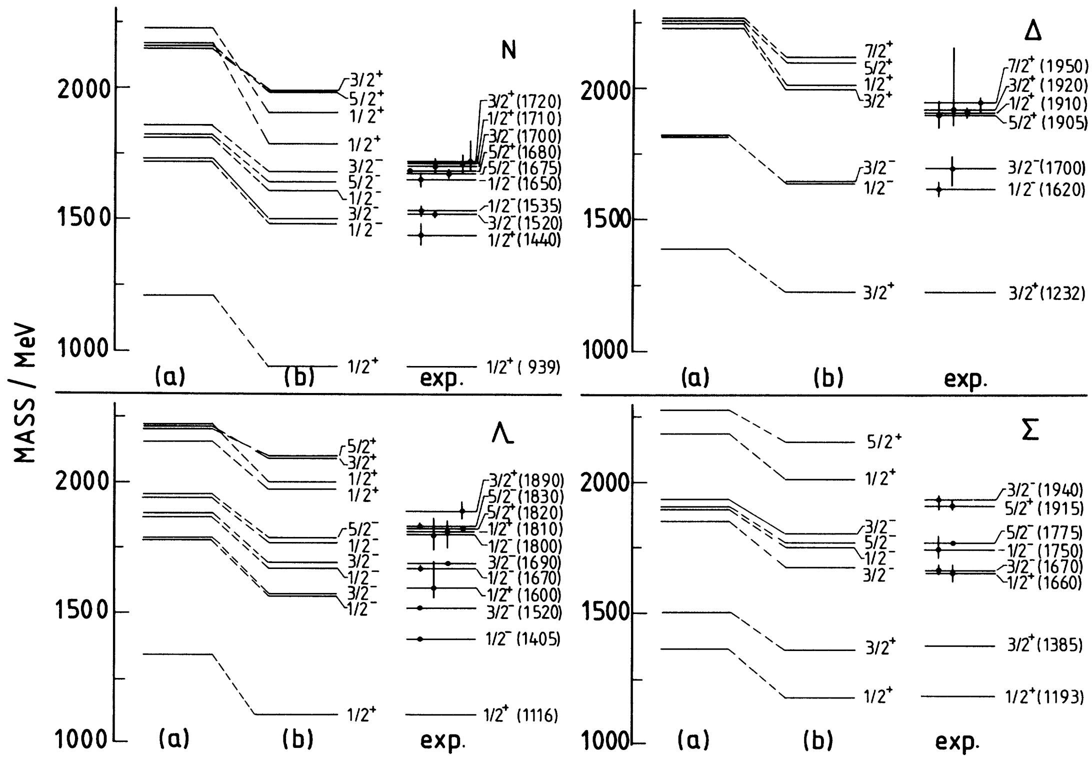

FIG. 1. Experimental (Ref. 16) and calculated (see text) spectra of the nonstrange and the strange (with $S=-1$) baryon resonances. The error bars on the observed levels denote the relevant uncertainties associated with the baryon masses.

well as the coupling to meson-decay channels[13]—of the quark-model spectroscopic calculations.

In what follows we show how one can understand qualitatively the role of the orbit-orbit component of the Fermi-Breit interaction *vis-à-vis* both the spacing between *radial* excitations as well as that between members of the $N=1$ and 2 bands. Introducing the Jacobi coordinates

$$\boldsymbol{\rho}=\mathbf{r}_1-\mathbf{r}_2,\quad \boldsymbol{\lambda}=(\mathbf{r}_1+\mathbf{r}_2)/2-\mathbf{r}_3\,, \tag{7}$$

and corresponding momenta, one can rewrite the orbit-orbit component [Eq. (2e)] as

$$V^G_{\text{o-o}}(1,2)=-(2\alpha_s/3)(2m_1m_2)^{-1}\rho^{-1} \times\{p_\rho^2+\hat{\boldsymbol{\rho}}\cdot(\hat{\boldsymbol{\rho}}\cdot\mathbf{p}_\rho)\mathbf{p}_\rho-\tfrac{1}{4}[p_\lambda^2+(\hat{\boldsymbol{\rho}}\cdot\mathbf{p}_\lambda)^2]\}\,. \tag{8}$$

The use of the identities

$$\hat{\boldsymbol{\rho}}\cdot(\hat{\boldsymbol{\rho}}\cdot\mathbf{p}_\rho)\mathbf{p}_\rho=\rho^{-2}L_\rho^2+2i\rho^{-1}\hat{\boldsymbol{\rho}}\cdot\mathbf{p}_\rho+p_\rho^2\,,$$
$$[p_\rho^2,\rho^{-1}]=2i\rho^{-2}\hat{\boldsymbol{\rho}}\cdot\mathbf{p}_\rho-[\Delta_{(\rho)}(\rho^{-1})]\,, \tag{9}$$

next permits one to write Eq. (8) in the suggestive form (where we have replaced V_c^G by its regularized version $\tilde{V}_c^G$)

$$V^G_{\text{o-o}}(1,2)=-(2m_1m_2)^{-1}\{\tilde{V}_c^G\rho^{-2}L_\rho^2-(\tilde{V}_c^G p_\rho^2+p_\rho^2\tilde{V}_c^G) +\tfrac{1}{4}\tilde{V}_c^G[p_\lambda^2+(\hat{\boldsymbol{\rho}}\cdot\mathbf{p}_\lambda)^2]\}\,. \tag{10}$$

This expression facilitates an understanding of the overall role of the orbit-orbit component largely in terms of the effect of the first two (λ-independent) terms in the curly brackets of Eq. (10). The term $\tilde{V}_c^G\rho^{-2}L_\rho^2$ raises the excitation energy of the $N=1$, $L^P=1^-$ 70-plet of $SU(6)_{SF}$ and removes the degeneracy among the $2\hbar\omega$ multiplets; one obtains quasirotational splittings between the masses of the multiplets with $L^\pi=0^+$, 1^+, and 2^+, a scenario that has earlier been (phenomenologically) achieved via the first-order perturbative effect of the (postulated) nonharmonic central potentials. The second term provides a *downward* shift that is roughly proportional to the quantum number N_ρ ($\equiv 2n_\rho+l_\rho$). It, therefore, lowers the centroid of the $2\hbar\omega$ multiplets relative to that of the 70-plet, favoring thereby, as a net effect, the nodal excitations which are unaffected by the L_ρ^2 term.

Summarizing, we have presented and discussed calculations that have highlighted (we believe for the first time) the significance of the orbit-orbit component of the Fermi-Breit interaction *vis-à-vis* a consistent simultaneous description of the positive- and negative-parity resonances. Apart from enhancing the quantitative efficacy of the calculations involving the nonrelativistic-quark-potential model, the orbit-orbit interaction also provides, to a good extent, an understanding of some conspicuous empirical features from a dynamical perspective.

One of us (S.K.S.) wishes to thank the Alexander von Humboldt Stiftung, Bonn, for financial support.

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[1]A. De Rújula, H. Georgi, and S. Glashow, Phys. Rev. D **12**, 147 (1975).

[2]N. Isgur and G. Karl, Phys. Lett. **72B**, 109 (1977); **74B**, 353 (1978); Phys. Rev. D **18**, 4187 (1978); **19**, 2653 (1979); **21**, 3175 (1980); N. Isgur, G. Karl, and D. W. L. Sprung, *ibid.* **23**, 163 (1981); N. Isgur, G. Karl, and R. Koniuk, Phys. Rev. Lett. **41**, 1269 (1978); Phys. Rev. D **25**, 2394 (1982).

[3]D. Gromes and I. O. Stamatescu, Nucl. Phys. **B112**, 213 (1976).

[4]W. Celmaster, Phys. Rev. D **15**, 1391 (1977).

[5]D. Gromes, Nucl. Phys. **B130**, 18 (1977).

[6]L. J. Reinders, J. Phys. G **4**, 1241 (1978).

[7]U. Ellwanger, Nucl. Phys. **B139**, 422 (1978).

[8]C. S. Kalman, Phys. Rev. D **26**, 2326 (1982).

[9]J. Carlson, J. Kogut, and V. R. Pandharipande, Phys. Rev. D **27**, 233 (1983); **28**, 2807 (1983).

[10]M. V. N. Murthy, M. Brack, R. K. Bhaduri, and B. K. Jennings, Z. Phys. C **29**, 385 (1985).

[11]B. Silvestre-Brac and C. Gignoux, Phys. Rev. D **32**, 743 (1985).

[12]S. Capstick and N. Isgur, Phys. Rev. D **34**, 2809 (1986), and the references given therein.

[13]W. Blask, M. G. Huber, and B. C. Metsch, Z. Phys. A **326**, 413 (1987).

[14]S. Capstick, S. Godfrey, N. Isgur, and J. Paton, Phys. Lett. B **175**, 457 (1986).

[15]H. J. Weber and H. T. Williams, Phys. Lett. B **205**, 118 (1988).

[16]Particle Data Group, G. P. Yost *et al.*, Phys. Lett. B **204**, 1 (1988).