Effective Theory of the T- and P-Breaking Superconducting State

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We propose an effective theory of superconductivity based on a microscopic theory of the T- and P-breaking spin-liquid state. There are two independent gauge invariances broken by two separate condensates. The theory may be useful for phenomenological calculations. In particular, we find that the H_{c1} are different for magnetic fields with opposite orientations. We also find that the polarization of an electromagnetic wave is rotated after reflection from these T- and P-breaking superconductors.

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Recently it was suggested[1-4] that the ground state of high-T_c materials may break time reversal (T) and parity (P). The T and P breaking may have many experimental consequences both in the normal state and in the superconducting state. We propose here an effective long-distance theory of superconductivity in such a T- and P-broken state. Our theory is not a "pure Landau-Ginsburg" theory in that we incorporate certain features suggested by microscopic considerations.

Firstly, we suppose that the excitations in high-T_c materials may have unusual quantum numbers, as was first proposed by Anderson.[5,6] In particular, we believe that the excitations obey fractional statistics.[7-10] Specifically, Kalmeyer and Laughlin[4] have argued that the excitations are semions (or half-fermions). Their argument has recently received support from general considerations, from field-theoretic models,[2] and from mean-field studies on the lattice.[3]

Secondly, we assume that the ground state of the quantum systems describing high-T_c superconductivity violates time reversal and parity spontaneously. This implies that our effective theory, unlike the standard Landau-Ginsburg theory, can contain T- and P-odd terms. Actually, this second supposition is closely intertwined with the first: In general, particles with fractional statistics violate T and P.

In the microscopic picture described in Ref. 3, we start with the electromagnetic gauge-invariant electron hopping operator $c_i^\dagger c_j \exp(i\int_i^j \mathbf{A}\,d\mathbf{x})$ on the lattice. (Here A_μ denotes the electromagnetic gauge potential and c_i the electron annihilation operator. The indices i and j label lattice sites.) Our convention is such that the covariant derivative of the electron field ψ_e is given by $(\partial_\mu + iA_\mu)\psi_e$. We then introduce an effective gauge potential a_μ in a mean-field analysis by saying that the hopping operator $c_i^\dagger c_j \exp(i\int_i^j \mathbf{A}\,d\mathbf{x})$ can be replaced by $\text{const}\times\exp(-i\int_i^j \mathbf{d}\cdot d\mathbf{x})$. In other words,

$$\langle c_i^\dagger c_j\rangle = \text{const}\times\exp\left[-i\int_i^j (\mathbf{a}+\mathbf{A})d\mathbf{x}\right]. \tag{1}$$

The potentials a_μ and A_μ appear in the combination $-(a_\mu + A_\mu) \equiv \tilde{a}_\mu$. We showed[3] that an effective Chern-Simons term is induced in the Lagrangian:

$$\mathcal{L}_{\text{CS}} = \alpha\epsilon^{\mu\nu\lambda}\tilde{a}_\mu \tilde{f}_{\nu\lambda}. \tag{2}$$

Here $\tilde{f}_{\mu\nu} = \partial_\mu\tilde{a}_\nu - \partial_\nu\tilde{a}_\mu$. At half-filling the quasiparticles are described by an electrically neutral spin-$\frac{1}{2}$ fermionic field ψ_s coupled to the gauge potential $\tilde{a}_\mu$ through the covariant derivative $(\partial_\mu + i\tilde{a}_\mu)\psi_s$. We find it useful to borrow the terminology of the resonating-valence-bond (RVB) discussion and call the quasiparticle the spinon to emphasize its character as a neutral spin-$\frac{1}{2}$ particle. However, our picture differs from the original RVB theory by incorporating fractional statistics and violation of T and P. In particular, the spinon may not obey fermion statistics as proposed in Ref. 6. Because of $\mathcal{L}_{\text{CS}}$, the spinons have a statistics given by $(-)e^{i/8\alpha}$. As mentioned above, theoretical considerations favor $\alpha = 1/4\pi$ so that the spinon is a semion (half-fermion). When two semions are interchanged, their wave function acquires the phase $(-)e^{i\pi/2}$.

As we dope the system with holes, superconductivity may set in at some finite hole density. With our convention the hole couples to the electromagnetic gauge potential A_μ through the covariant derivative $(\partial_\mu - iA_\mu)\psi_{\bar{e}}$. (The hole here is a charge-e spin-$\frac{1}{2}$ object.) In the T- and P-broken ground state the charged quasiparticles are holons (again to use a sort of RVB terminology), which carry no spin. The holes can be regarded as bound states of the holons and the spinons, $\psi_{\bar{e}} \sim \phi_h\psi_s$. We see the holon field ϕ_h (which is a bosonic field) has a coupling described by $[\partial_\mu - i(\tilde{a}_\mu + A_\mu)]\phi_h$.

There are two gauge symmetries in our model since $\tilde{a}_\mu$ and A_μ can be transformed separately. The Chern-Simons term (2) does not break the gauge symmetry of $\tilde{a}_\mu$. When ϕ_h condenses and obtains a nonzero vacuum expectation value, these two gauge invariances reduce to a single gauge invariance $\tilde{a}_\mu \to \tilde{a}_\mu + \partial_\mu\Lambda$ and $A_\mu \to A_\mu - \partial_\mu\Lambda$. Because of this residual gauge invariance, there can be no Meissner effect and the ϕ_h condensed state is not superconducting. It actually describes a state with quantum Hall effect.[11]

To obtain superconductivity we break both A_μ and $\tilde{a}_\mu$ gauge symmetries. We suggest that the spinons play an important role in breaking the $\tilde{a}_\mu$ gauge symmetry. We suppose that singlet pair condensation of spinons breaks the $\tilde{a}_\mu$ gauge symmetry. Introducing the pairing order parameter $\phi_{ss} \sim \psi_s \psi_s$, we propose the following effective Lagrangian for the T- and P-breaking superconducting state:

$$\mathcal{L} = \phi_h^\dagger i[\partial_t - i(\tilde{a}_0 + A_0)]\phi_h + A_0\rho_0 + \frac{1}{2M}\phi_h^\dagger[\partial_i - i(\tilde{a}_i + A_i)]^2\phi_h - V(\phi_h^\dagger\phi_h)$$
$$+ \phi_{ss}^\dagger i(\partial_t + 2i\tilde{a}_0)\phi_{ss} + \frac{1}{2m}\phi_{ss}^\dagger(\partial_i + 2i\tilde{a}_i)^2\phi_{ss} - V_s(\phi_{ss}^\dagger\phi_{ss}) + \frac{\alpha}{d}\epsilon^{\mu\nu\lambda}\tilde{a}_\mu\tilde{f}_{\nu\lambda} + \frac{1}{16\pi e^2}(\mathbf{E}^2 - \mathbf{B}^2) . \quad (3)$$

Here ρ_0 denotes the background charge density which is equal to the doping concentration. The Lagrangian in (3) describes a three-dimensional system consisting of many weakly coupled two-dimensional layers. The spinons and holons discussed above are confined to the layers. The Chern-Simons term in (2) is defined in each layer. To convert it into a three-dimensional density we need to divide it by the interlayer distance d. In this paper we take the convention that indices i,j take values 1,2 and indices μ,ν take values 0,1,2. The potentials in (3) may be taken to have the standard form $V = -\kappa|\phi_h|^2 + \lambda|\phi_h|^4$ and $V_s = \Delta_{ss}\phi_{ss}|^2 + \lambda_s|\phi_{ss}|^4$. Here Δ_{ss} corresponds to the energy gap of the spinon pair excitations.

While the effective Lagrangian (3) is introduced by a microscopic picture, we would like to suggest that it may be quite robust and largely independent of the fine details of our microscopic theory, as long as one believes that the fundamental quasiparticles obey fractional statistics. The essential physics is the existence of two gauge invariances broken by two separate condensates (or Higgs fields) and the appearance of the T- and P-breaking Chern-Simons term.

In the superconducting state, the ϕ_h and ϕ_{ss} fields obtain some constant vacuum expectation values, v and u, respectively. Varying with respect to $\tilde{a}_\mu$ and A_μ for constant ϕ_h and ϕ_{ss}, we obtain

$$(v^2 - 2u^2) + 2\frac{\alpha}{d}\epsilon_{ij}\tilde{f}_{ij} = 0 , \quad (4a)$$

$$-(v^2 - \rho_0) = \frac{1}{e}J_0 , \quad (4b)$$

$$\frac{2}{M}(\tilde{a}_j + A_j)v^2 + \frac{4}{m}\tilde{a}_j u^2 + 2\frac{\alpha}{d}\epsilon_{jk}\tilde{f}_{0k} = 0 , \quad (4c)$$

$$-\frac{2}{M}(\tilde{a}_j + A_j)v^2 = \frac{1}{e}J_j . \quad (4d)$$

The first two equations are constraints imposed by varying $\tilde{a}_0$ and A_0. We have defined the electromagnetic current

$$J_\mu = -\frac{1}{4\pi e}\sum_{\nu=0}^{\nu=3}\partial^\nu F_{\nu\mu} . \quad (5)$$

(Strictly speaking, while v and u are determined largely by V and V_s, they also have some dependence on $\tilde{a}_j$ and A_j. We ignore such higher-order effects here.) According to (4c), $\tilde{a}_j$ is proportional to A_j, up to higher derivative terms negligible at long distances. As a result of the Meissner effect, $\epsilon_{ij}\tilde{f}_{ij} = 0$ and (4a) implies that $v^2 = 2u^2$. Alternatively, we note that after gauge symmetry breaking, there is an effective mass term for $\tilde{a}_j$ and so finite energy requires $\tilde{f}_{ij} = 0$. Thus the densities of the spinons and the holons are the same. If we also take into account the equation of motions from varying ϕ_h and ϕ_{ss}, we find that $A_0 = 0$ and $\tilde{a}_0 = 0$.

At finite temperatures, the holons are described by the two-fluid model with $|\phi_h|^2$ corresponding to the density of the superfluid of the holons. The density of the normal fluid is given by ρ_h. The total density of holons is $n_h \equiv \rho_h + |\phi_h|^2$. When the holon normal-fluid density is finite, we need to include a term $(\tilde{a}_0 + A_0)\rho_h$ in the effective Lagrangian (3). The constraint (4a) becomes

$$(v^2 + \rho_h - 2u^2) + 2\frac{\alpha}{d}\epsilon^{ij}\tilde{f}_{ij} = 0 . \quad (6)$$

The constraint $v^2 = 2u^2$ in the superconducting state is replaced by $n_h = v^2 + \rho_h = 2u^2$. Equation (4b) is also changed to $-(v^2 + \rho_h - \rho_0) = J_0$. (Note that $\rho_0 = n_h$, so that J_0 remains zero.)

Since we are only interested in physics at long distances, we can solve for the current J_j, dropping terms with two derivatives or higher. After eliminating $\tilde{a}_j$ from the last two equations of (4) and using $\tilde{a}_0 = 0$, we obtain

$$J_j = -\frac{4ev^2u^2}{mv^2 + 2Mu^2}\left[A_j + \frac{\alpha v^2 m^2}{2du^2(mv^2 + 2Mu^2)}\epsilon_{jk}\partial_0 A_k\right] + \text{terms with two or more derivatives} . \quad (7)$$

The first term describes the Meissner effect and thus superconductivity. Note that it vanishes when v or u vanish. Breaking $\tilde{a}_\mu$ gauge symmetry is necessary as well as the condensation of ϕ_h in order to obtain superconductivity. The second term in (7) appears to imply a Hall effect at zero magnetic field. But in the superconducting state the electric field $\mathbf{E} = 0$ and there is no measurable Hall effect. In the normal state $v = 0$ and J_i in (7) vanishes. However, (7) only contains the contribution to the electrical current from the superfluid. When $\mathbf{E} \neq 0$ the normal fluid also contributes to the electrical current so that the normal state is metallic. But we expect no zero-field Hall effect for the normal fluid at least at order $O(e^2/h)$.

We can easily obtain the Hamiltonian (or free energy) of the model (3):

$$H=+\int d^3x_a d^3x_b\, 2e^2\ln|x_a-x_b|[|\phi_h(x_a)|^2+\rho_h-\rho_0][|\phi_h(x_b)|^2+\rho_h-\rho_0]+\int d^3x\frac{1}{8\pi e^2}B^2$$
$$+\int d^3x\left\{-\frac{1}{2m}\phi_{ss}^\dagger(\partial_i+2i\tilde{a}_i)^2\phi_{ss}+V_s(\phi_{ss})\right\}+\int d^3x\left\{-\frac{1}{2M}\phi_h^\dagger[\partial_i-i(\tilde{a}_i+A_i)]^2\phi_h+V(\phi_h)\right\}, \tag{8}$$

where the constraint (6) (at finite temperature) is satisfied. In general the parameters in the potential such as κ and Δ_{ss} depend on doping concentration ρ_0 and the temperatures.

Although the effect of T and P breaking does not appear in the uniform state, it does appear in nonuniform states. For example, a vortex with positive magnetic flux has different energy from that of a vortex with negative flux. Assume $(\phi_h, \phi_{ss}, A_\mu, \tilde{a}_\mu)$ is the solution for a vortex with positive flux. Then if $\alpha=0$, from the Hamiltonian (8) and the constraint (6), we see that $(\phi_h^\dagger, \phi_{ss}^\dagger, -A_\mu, -\tilde{a}_\mu)$ is also a solution corresponding to a vortex with negative flux. Both vortices have the same energy. But if $\alpha\neq 0$, the second vortex configuration does not satisfy the constraint (6) if the first one does. Therefore vortices with opposite flux in general have different energies. This implies that the lower critical field H_{c1} has different values for magnetic fields with opposite orientations.

Noticing that ϕ_h carries unit charge, one may wonder whether the magnetic flux is quantized in units of hc/e rather than $hc/2e$ in the superconducting state. Actually, there are two kinds of fundamental vortices.[12] (A) We can have a vortex in which the phase of ϕ_h twists around 2π at infinity [thus implying, according to (7), $\oint d\mathbf{x}\cdot(\tilde{\mathbf{a}}+\mathbf{A})=2\pi$], while ϕ_{ss} has an asymptotically constant phase (thus implying $\oint d\mathbf{x}\cdot\tilde{\mathbf{a}}=0$). The electromagnetic flux and the statistical "magnetic" flux are then $(\Phi,\tilde{\Phi})=(2\pi,0)$. (B) We can also have a vortex in which the phases of both ϕ_h and ϕ_{ss} twist around 2π. According to (7) this implies $\oint d\mathbf{x}\cdot(\tilde{\mathbf{a}}+\mathbf{A})=\oint d\mathbf{x}\cdot 2\tilde{\mathbf{a}}=2\pi$ and thus $(\Phi,\tilde{\Phi})=(\pi,\pi)$. In other words, the smallest unit of magnetic flux is π (or $hc/2e$ in traditional units) as carried by a type-B vortex. A type-A vortex carries two units of magnetic flux. All other vortices can be thought of as combinations of these two vortices. The same argument applies to the magnetic flux through a superconducting ring.

In their analysis of flux quantization, Byers and Yang[13] noted that time-reversal invariance implies that the free energy is a symmetric function of the flux. Thus, it has been suggested[14] that T and P breaking may result in a shift in the value of the magnetic flux through a vortex so that it becomes $hc/2e+\text{const}$. Although our effective theory breaks T and P, such a shift does not appear here. This is because the T- and P-breaking term in our theory appears as the Chern-Simons term which has one more derivative than the gauge-symmetry-breaking terms $\tilde{a}_j^2u^2$ and $A_j^2v^2$. The T- and P-breaking effects only appear at short distances and high frequencies.

The change in the polarization of an electromagnetic wave reflected off a superconducting sample provides another signal of T and P violation. The propagation of an electromagnetic wave is described by the phenomenological equation

$$\sum_{\nu=0}^{\nu=3}\partial^\nu F_{\nu 0}=0, \tag{9}$$

$$\sum_{\nu=0}^{\nu=3}\partial^\nu F_{\nu i}+\mu^2A_i+\beta\epsilon_{ij}\partial_0A_j=0, \tag{10}$$

where the parameters μ^2 and β may be read off from (4b) and (7). Outside the superconductor, μ^2 and β vanish, of course. We consider an idealized situation in which $z<0$ is empty space while $z>0$ is filled with the superconducting material. We study a solution of normal incidence (and also with the direction of propagation perpendicular to the planar layers) with $A_0=0$ and $A_j\propto e^{-i\omega t}$ independent of x and y. The equation of motion

$$(\partial_z^2+\omega^2-\mu^2)A_i+i\beta\omega\epsilon_{ij}A_j=0 \tag{11}$$

implies the boundary condition that A_i and ∂_zA_i must be continuous. We have, outside the superconductor,

$$A_j=b_je^{ikz}+c_je^{-ikz}, \tag{12}$$

and inside,

$$A_j=a_{j+}e^{-\kappa_+z}+a_{j-}e^{-\kappa_-z}. \tag{13}$$

Here c_j represent the reflected wave.

Inside, we find that

$$\kappa_\pm=+\left(\mu^2-\omega^2\pm\beta\omega\right)^{1/2}. \tag{14}$$

We will consider the low-frequency region so that $\kappa_\pm$ are real. Notice that there is a frequency regime in which κ_+ remains real while κ_- becomes imaginary. At high frequencies, our effective Lagrangian treatment breaks down. Also we have

$$a_{1\pm}=\mp ia_{2\pm}. \tag{15}$$

Outside, we have $k=\omega$, of course. Matching the boundary condition, we can readily determine c_j and $a_{j\pm}$ in terms of the incoming wave parameters.

Let us parametrize the reflected wave by

$$c_1=c\cos\varphi,\quad c_2=c\sin\varphi e^{i\eta}. \tag{16}$$

(Note that, in general, c_i, b_i, and a_i can be complex.)

Then we find

$$\frac{1+i\tan\varphi e^{i\eta}}{1-i\tan\varphi e^{i\eta}}=\frac{\kappa_-+ik}{\kappa_++ik}\frac{\kappa_+-ik}{\kappa_--ik}\frac{b_1+ib_2}{b_1-ib_2}. \tag{17}$$

For the case of a linearly polarized incoming wave $b_2=0$, we find that in the low-frequency regime, since both κ_+ and κ_- are real, $\eta=0$ and the outgoing wave remains linearly polarized but with the polarization axis rotated by an angle φ. For $\mu^2-\omega^2 \gg \beta\omega$, we find

$$\varphi=\frac{\beta\omega^2}{(\mu^2-\omega^2)^{3/2}}. \tag{18}$$

At zero temperature we have

$$\beta=8\pi\frac{\alpha e^2}{d}\frac{m^2}{(m+M)^2}. \tag{19}$$

In the intermediate- and high-frequency regime, $\kappa_\pm$ may become imaginary so that η is no longer zero, leading to an elliptically polarized outgoing wave.

Our theory admits three possible normal states (nonsuperconducting states) depending on the values of the parameters in the theory: (I) $\phi_{ss}=\phi_h=0$, (II) $\phi_h\neq 0$, $\phi_{ss}=0$, and (III) $\phi_h=0$, $\phi_{ss}\neq 0$. In all these states, it is energetically favorable to have uniform charge density $|\phi_h|^2=\rho_0-\rho_h$. In the states (I) and (II) we have $\tilde{f}_{12}=-\rho_0 d/4\alpha$ due to the constraint (6). In (II) we also have $B=F_{12}=-\tilde{f}_{12}$ due to the effective mass term $|\phi_h|^2(\tilde{a}_i+A_i)^2$. A magnetic field is generated. In (III), however, the effective mass term $|\phi_{ss}|^2\tilde{a}_i^2$ enforces $\tilde{f}_{ij}=0$. Thus we must have $|\phi_{ss}|^2=\rho_0/2$ due to the constraint (6). The phase transition from the normal state (I) or (II) to the superconducting state is a first-order transition because $|\phi_{ss}|^2$ jumps from zero to $\rho_0/2$ after the transition, while the transition between (III) and the superconducting state is a second-order transition as observed in high-T_c materials. Therefore it appears that only the normal state (III) is consistent with experimental observations. Our consideration here is limited to uniform states. It may be possible to have domain structures with different phases in different domains.

We would like to point out that our effective Lagrangian (3) is not the most general Lagrangian for the T- and P-breaking superconducting state. We did not include a possible "Maxwell" term $\tilde{f}_{\mu\nu}^2$ and possible cross terms like $|\phi_h|^2|\phi_{ss}|^2$ in the potential. The inclusion of these terms would not affect our previous results about the T- and P-breaking properties in an essential way. However, the energies of the three possible normal states do depend on these terms. Which of the three normal states is actually realized depends on those energies. When the coefficient in front of $\tilde{f}_{\mu\nu}^2$ is large we see that (III) is favored. The equal density of spinons and holons suggests the binding of spinon and holon. In phase (III), the excitations may include the vortex in the spinon pair field ϕ_{ss} with statistics dual to that of the spinon pair field and hence semion statistics.[15]

In the above discussion we have used a fermionic field to describe the spinon. We may alternatively use a bosonic field to describe the spinon and change α from $1/4\pi$ to $-1/4\pi$ to keep the statistics of the spinon unchanged. In this case the holon is described by a fermionic field so that it has the same statistics as before. Now the superconducting state is realized as a result of spinon condensation and holon pair condensation. Although the effective theory has a very different appearance, the qualitative properties are the same as before. We expect the properties discussed here to be generic for the T- and P-breaking state.

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