Degenerate Quantum-Beat Laser: Lasing without Inversion and Inversion without Lasing

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A single lasing mode driven by a three-level "quantum-beat" atomic configuration can show gain without population inversion or optical absorption into an excited state without spontaneous or stimulated emission.

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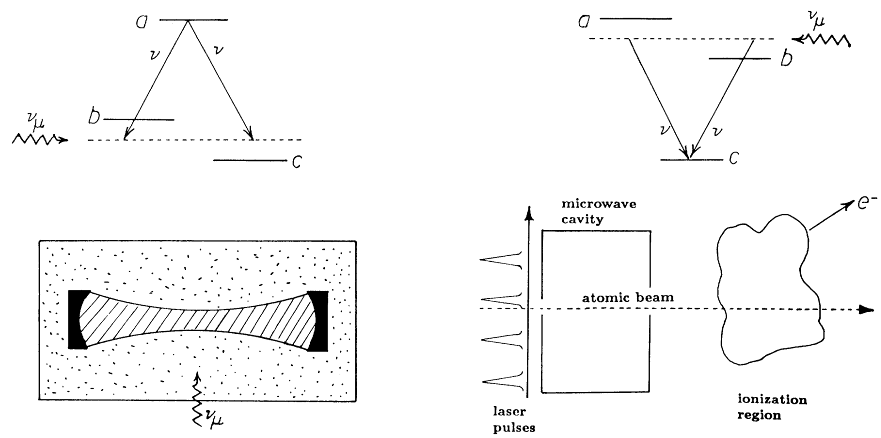
The quantum-beat laser[1-5] concept was originally advanced as a means of quenching spontaneous emission noise. The lasing medium in such a device consists of three-level atoms with the upper two (closely spaced) levels driven by a coherent microwave field as shown. In the present Letter we investigate the degenerate quantum-beat laser wherein the two optical fields with different frequencies ν_1 and ν_2 are replaced by a single frequency field as shown in Figs. 1(a) and 1(b). In the "V quantum-beat laser" of Fig. 1(b), we find the surprising possibility that the atoms in an appropriate superposition of the two upper levels will not decay to the lower one via spontaneous or stimulated emission into the lasing mode. On the other hand, we find that the "Λ quantum-beat laser" [Fig. 1(a)] allows gain without population inversion, i.e., can display gain even when only a small fraction of the atoms are in the upper level $|a\rangle$.

Lasing without population inversion was suggested some time ago by utilizing the splitting of emission and absorption spectra caused by atomic recoil.[6,7] The recoil splitting will be large enough to have practical usage only for very high-frequency light, e.g., x rays. A most interesting possibility of obtaining noninversion lasing or amplification has recently been proposed by Harris.[8] He analyzes the difference between the emission and absorption spectra due to Fano interferences[9] between two lifetime-broadened discrete levels which decay to the same continuum.[8-10]

We find that the degenerate Λ quantum-beat laser can, in principle, be realized in several ways (e.g., using microwaves or coherent picosecond excitation to estab-



LASING WITHOUT INVERSION (a) — INVERSION WITHOUT LASING (b)

FIG. 1. Degenerate quantum-beat lasers and corresponding atomic-level schemes: (a) Λ-type laser showing lasing without inversion. Laser action takes place between the two mirrors, and laser radiation exists only in the cross hatched region. Atoms pass through lasing region with an average transit time τ. (b) V-type system showing inversion without lasing. Atoms excited into the $|a\rangle$ state, by a picosecond pulse, decay to the ground state if μ waves are turned off. But if μ waves couple $|a\rangle$ and $|b\rangle$, we predict no decay to the $|c\rangle$ state. After exciting the cavity, $|a\rangle$ and $|b\rangle$ state atoms are ionized but $|c\rangle$ atoms are not.

lish coherence between lower levels) and in different regimes (transit-time broadened, radiation broadened, or collision dominated). However, we will concentrate here on the experimental arrangement of Fig. 1(a), which illustrates the physics in its simplest form. In this figure we consider a microwave cavity containing three-level atoms, the lower two levels of which are driven by an intense microwave field. There is also a weak, incoherent, pumping mechanism which populates level a. The lasing field exists only between the mirrors, as indicated in Fig. 1(a). We consider the gas to be dilute so that we are in the collisionless regime, and the levels to be "long lived" compared to the average transit time τ for an atom to cross[11] the lasing region.

The main result of this paper is the linear gain equation for the average photon number, $\bar{n}$, of the laser field. For the Λ laser case of Fig. 1(a), we find

$$\frac{d}{dt}\bar{n}=\alpha\rho_{aa}^{0}(\bar{n}+1)-\alpha\rho_{cc}^{0}\bar{n}(1-\cos\phi)\,, \tag{1}$$

where α is the linear gain, ρ_{ll}^{0} denotes the initial population in the $|l\rangle$ state, and ϕ is the phase of the microwave field. For $\phi=0$, we see that the absorption of light by the lower levels vanishes; however, the linear gain (ρ_{aa}^{0} term) is not affected. Hence we can have lasing even if only a small fraction of the atoms are in the excited state $|a\rangle$.

In the V-type laser [see Fig. 1(b)] we have a similar equation

$$\frac{d}{dt}\bar{n}=\alpha\rho_{aa}^{0}(1-\cos\phi)(\bar{n}+1)-\alpha\rho_{cc}^{0}\bar{n}\,. \tag{2}$$

Only now, we see that for $\phi=0$, the population of the upper level ρ_{aa}^{0} undergoes no spontaneous or stimulated radiative decay into the mode in question.

In the following paragraphs we analyze the Λ and V degenerate quantum-beat lasers. The equations for the time rate of change of the photon number as given above are derived, and a physical discussion of the effect is given.

We first consider the Λ quantum-beat laser. In the present model the atoms enter the lasing region with an injection rate r where they spend a time τ. The atoms have one upper level $|a^j\rangle$ with energy $\hbar\omega_a$ and two lower levels $|b^j\rangle$ and $|c^j\rangle$ with energies $\hbar\omega_b$ and $\hbar\omega_c$, respectively, as shown in Fig. 1(a). The superscript j indicates the jth atom, which enters the lasing region at time t_j with initial populations ρ_{aa}^{0} and ρ_{cc}^{0}. The transitions between $|a^j\rangle\rightarrow|b^j\rangle$ and $|a^j\rangle\rightarrow|c^j\rangle$ are assumed dipole allowed. The dipole-forbidden transition $|b^j\rangle\rightarrow|c^j\rangle$ is induced by the microwave field of frequency ν_μ and the corresponding Rabi factor is $\Omega e^{-i\phi}$, where Ω is the Rabi frequency and ϕ is the phase of the microwave field. The interaction Hamiltonian for the system in the interactive picture is[11-13]

$$V_I=\hbar\sum_j[N(t,t_j)(g_1a^\dagger\sigma_1^je^{-i\Delta t}+g_2a^\dagger\sigma_2^je^{i\Delta t})+\tfrac{1}{2}\Omega e^{i\phi}\sigma_\mu^j]+\text{adj}\,, \tag{3}$$

where a ($a^\dagger$) is the annihilation (creation) operator for the light field with frequency ν, which induces the transition $|a\rangle\rightarrow|c\rangle$ and $|a\rangle\rightarrow|b\rangle$, $\sum_j$ stands for a sum over all atoms, $\sigma_1^j=|c^j\rangle\langle a^j|$, $\sigma_2^j=|b^j\rangle\langle a^j|$, and $\sigma_\mu^j=|c^j\rangle\times\langle b^j|$, $N(t,t_j)$ is a notch function which is equal to 1 within t_j to $t_j+\tau$ and to 0 otherwise. In Eq. (3) $\omega_{ac}+\omega_{ab}=2\nu$ ($\omega_{\alpha\beta}=\omega_\alpha-\omega_\beta$) and $\omega_{bc}=\nu_\mu$ have been assumed, and $\Delta=\frac{1}{2}\omega_{bc}$. We transform the Schrödinger equation, $i\hbar\dot{\psi}=V_I\psi$, via a unitary transformation $\psi=U\psi'$, which allows us to treat the microwave field to all orders. We find

$$\begin{aligned}U&=\exp\left[-\frac{i}{2}\Omega\sum_j(e^{i\phi}\sigma_\mu^j+e^{-i\phi}\sigma_\mu^{j\dagger})t\right]\\&=\prod_j\left[|a^j\rangle\langle a^j|+(|c^j\rangle\langle c^j|+|b^j\rangle\langle b^j|)\cos\frac{\Omega}{2}t-i(e^{i\phi}\sigma_\mu^j+e^{-i\phi}\sigma_\mu^{j\dagger})\sin\frac{\Omega}{2}t\right].\end{aligned} \tag{4}$$

We then have the equation of motion for ψ', $i\hbar\dot{\psi}'=V_I'\psi'$, where

$$V_I'=\sum_jU^\dagger N(t,t_j)(\hbar g_1a^\dagger\sigma_1^je^{-i\Delta t}+\hbar g_2a^\dagger\sigma_2^je^{i\Delta t}+\text{adj})U\,. \tag{5}$$

We adjust the Rabi frequency Ω so that $\Omega=\omega_{bc}=2\Delta$, and V_I' becomes

$$V_I'=\tfrac{1}{2}\hbar\sum_jN(t,t_j)a^\dagger[G_1\sigma_1^j+G_2\sigma_2^j]+\text{adj}\,, \tag{6}$$

where $G_1=g_1-g_2\exp(i\phi)$, $G_2=g_2+g_1\exp(-i\phi)$, and high-frequency terms, going as $\exp(i2\Delta t)$, have been dropped.

By using Eq. (6), the equation of motion for n is obtained,[13]

$$\dot{n}=\frac{i}{2}\sum_jN(t,t_j)\{[G_1^*\sigma_1^{j\dagger}a+G_2^*\sigma_2^{j\dagger}a]-[G_1a^\dagger\sigma_1^j+G_2a^\dagger\sigma_2^j]\}\,. \tag{7a}$$

In order to obtain an equation of motion involving only n, we seek a, σ_1^j, σ_2^j, and their adjoints. The equations of motion for a, σ_1^j, and σ_2^j are

$$\dot{a}=-\frac{i}{2}\sum_jN(t,t_j)[G_1\sigma_1^j+G_2\sigma_2^j]\,, \tag{7b}$$

$$\dot{\sigma}_1^j=\frac{i}{2}N(t,t_j)[G_1^*a(\sigma_a^j-\sigma_c^j)-G_2^*a\sigma_\mu^j]\,, \tag{7c}$$

$$\dot{\sigma}_2^j=\frac{i}{2}N(t,t_j)[G_2^*a(\sigma_a^j-\sigma_b^j)-G_1^*a\sigma_\mu^{j\dagger}]\,. \tag{7d}$$

We note that atomic decay terms can be added to (7c) and (7d) without changing our basic conclusions. However, we concentrate here on the simpler "maserlike" transit-time broadened case in order to simplify the physics. The effects of radiative and collision broadening will be given elsewhere. Integrating Eqs. (7b)-(7d), substituting the results into (7a), noting that n is slowly varying so that $n(t_j) \cong n(t)$, and taking expectation values, we obtain

$$\frac{d}{dt}\bar{n} = \frac{r\tau^2}{4}(G_1G_1^* + G_2G_2^*)\rho_{aa}^0(\bar{n}+1) - \frac{r\tau^2}{4}(G_1G_1^*\rho_{cc}^0 + G_2G_2^*\rho_{bb}^0)\bar{n} , \quad (8)$$

where $\bar{n}$ is the average photon number, and the expectation values of atomic operators are properly replaced by their initial values, e.g., $\langle\sigma_a\rangle = \rho_{aa}^0$, $\langle\sigma_\mu\rangle = 0$ (since we are interested in the linear gain to lowest order in g^2), and the sum $\sum_j$ has been replaced by an integration $\int_{-\infty}^{t} r\,dt_j$, where r denotes the rate with which atoms enter the lasing region.

If $g_1 = g_2 = g$, Eq. (8) becomes

$$\frac{d}{dt}\bar{n} = \alpha\{\rho_{aa}^0(\bar{n}+1) - \tfrac{1}{2}[\rho_{bb}^0(1+\cos\phi) + \rho_{cc}^0(1-\cos\phi)]\bar{n}\} , \quad (9)$$

where the linear gain parameter $\alpha = rg^2\tau^2$. In (9) the first term (proportional to ρ_{aa}^0) represents stimulated and spontaneous emission due to the population in the upper level, while the second term describes absorption due to the population of the lower levels. For example, when $\phi = 0$ and $\rho_{bb}^0 = 0$ the second term will be zero, that is to say, the population in the lower levels has no absorptive effect. Any amount of population in the upper level (which can now be much smaller than the population in the lower levels) will lead to net gain, i.e., lasing without population inversion.

Physically, the lack of absorption in the Λ quantum-beat laser is a manifestation of quantum interference phenomena. When an atom makes a transition from the upper level to the two lower levels, the total transition probability is the sum of the $a \to b$ and $a \to c$ probabilities. However, transition probabilities from the two lower levels to the single upper level are obtained by squaring the sum of the two probability amplitudes. When there is coherence between the two lower levels this can lead to interference terms yielding a null in the transition probability corresponding to photon absorption.

From the above it is clear that if we "turn off" the microwaves, no net gain will exist when $2\rho_{aa}^0 < \rho_{bb}^0 + \rho_{cc}^0$. It is the microwave field that coherently "drives" the lower states. However, if we initially prepared the atoms in a coherent superposition of levels b and c, noninversion lasing is still possible, but now without the microwaves. This case will be presented elsewhere.

At this point it may be well to mention that Raman "lasers" also operate without inversion. However, our noninversion laser scheme is very different from a Raman device which parametrically transfers energy from one pump frequency to the Stokes or anti-Stokes line, and not from the atoms to the field. In our system the energy of the lasing field comes from the atoms as in a normal laser, and not via parametric conversion from another radiation field.

The most obvious application of a Λ quantum-beat laser would be to realize laser operation for those wavelengths where population inversion is difficult. However, we note that the noninversion laser will enjoy reduced spontaneous emission phase noise. In the case of a normal laser noise comes from spontaneous emission from the upper level. The phase diffusion coefficient is

$$D_{\phi\phi} = \frac{\alpha}{4\bar{n}}\rho_{aa} . \quad (10)$$

The suppression of the stimulated absorption leads to a reduction of the population of the upper level needed to reach threshold. We now have

$$\rho_{aa}^{(\Lambda)} = \rho_{aa}^{(n)} - \rho_{cc}^{(n)} , \quad (11)$$

where (Λ) and (n) denote the noninversion (Λ laser) and (normal) inversion lasers, respectively. Hence, the ratio of phase diffusion coefficients is

$$\frac{D_{\phi\phi}^{(\Lambda)}}{D_{\phi\phi}^{(n)}} = \frac{\rho_{aa}^{(\Lambda)}}{\rho_{aa}^{(n)}} = \frac{\rho_{aa}^{(n)} - \rho_{cc}^{(n)}}{\rho_{aa}^{(n)}} . \quad (12)$$

For example, if the inversion is 10% for a normal laser, the ratio will be 0.18. We also note that the Λ quantum-beat laser can show a complete quenching of spontaneous emission noise under certain conditions; this will be discussed elsewhere.

In the above, we discussed the Λ quantum-beat laser, and found gain without population inversion. Via a similar analysis, we obtain the equation of motion for the V quantum-beat laser; in this case we find

$$\frac{d}{dt}\bar{n} = \tfrac{1}{2}\alpha[\rho_{aa}(1-\cos\phi) + \rho_{bb}^0(1+\cos\phi)](\bar{n}+1) - \alpha\rho_{cc}^0\bar{n} . \quad (13)$$

If $\rho_{bb} = 0$, $\phi = 0$, the first term in Eq. (13) is equal to zero, and the excited atoms will remain in the excited level without radiation decay; i.e., no stimulated or spontaneous emission into the lasing mode will take place.

We note that it would be difficult to observe such "population trapping" in a laser since decay into other modes of the radiation field will take place. However, in the case of an atom placed in a high-Q cavity, such that the density of states of the radiation field is very small except at the frequency of the "V" laser-to-maser transition, we anticipate that there could be a persistence of excited population. See, for example, the proposed experiment sketched in Fig. 1(b). There we see an atomic beam (of Rydberg atoms) which is excited by a train of short pulses. The laser pulses are arranged to preferentially excite only one of the two upper states, say the $|a\rangle$ state from some far removed ground state. This can be arranged by controlling the polarization, but not the frequency, of the laser since the spectrum of laser frequencies would "cover" both the $|a\rangle$ and $|b\rangle$ levels. In this way the atoms are injected into the cavity at very specific times, relative to the microwave field. For example, we may arrange that the phase of the microwave field is zero when the atoms enter the cavity. Atoms then transverse the microwave cavity and, in the absence of microwave radiation coupling the $|a\rangle$ and $|b\rangle$ states, we could easily ensure that there is a high probability for the atoms to decay to the $|c\rangle$ state. On the other hand, if the microwave radiation is present, the atoms would not decay when the microwave field satisfies the conditions discussed in conjunction with Eq. (13).

In order to monitor the state of the atoms leaving the cavity we could use the technique of Leuchs, Smith, and Walther[14] in which excited ($|a\rangle,|b\rangle$) atoms are photoionized but $|c\rangle$ state atoms are not. Thus, one would observe an ionization current in the presence of the microwaves but no ionization current in their absence. Such an experiment would demonstrate "inversion without lasing."

In conclusion, we see that it is possible to have laser gain even when only a small fraction of the atomic population is in the upper level, via a Λ-type quantum-beat laser. Furthermore, it is possible to "lock" the atom into an upper level which is radiatively pumped from a lower level. That is, it is possible to arrange for the absorption of light without stimulated or spontaneous emission into the mode in question, when using a V-type quantum-beat laser.

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