Fourth-Generation Effect on CP Violation in $B \to K\phi$ Decays

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CP-violating asymmetries in the $B \to K\phi$ decays are studied in the four-generation model. The absorptive part in the penguin amplitude leads to the CP-violating asymmetry in the $B_u^{\pm} \to K^{\pm}\phi$ decays, which could be as large as $\pm 15\%$ in the four-generation model, although it is at most a few percent in the three-generation model. The asymmetry between the $B_d^0 \to K_S^0\phi$ and $\bar{B}_d^0 \to K_S^0\phi$ decays is also expected to be $\pm 10\%$ on $\Upsilon(4S)$ through the penguin absorptive part with the contribution of the fourth-generation quark.

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The charmless nonleptonic decays of the B meson are expected to provide a sensitive test of the standard model. In particular, the processes involving three s quarks in the final state are rare decay modes because the dominant processes are the loop-induced $b \to sg$ transitions, the so-called penguin diagrams.[1] One typical mode of these rare decays is the $B \to K\phi$ process, which has been studied by some authors.[2,3] Recently, a numerical study of CP violation in inclusive and exclusive charmless B decays has been done by Gérard and Hou[4] in the framework of the standard model with three generations. CP violations in the B_d^0 meson decays through the tree process such as $B_d^0 \to K_S\psi$ are expected to be large due to the large B_d^0-$\bar{B}_d^0$ mixing in the standard model, as discussed later, and so new physics is unimportant in these decays. On the other hand, the charmless decays through the penguin process have another source of CP violation, which is shown later, and so new physics such as the fourth generation may play an important role in CP violation of these decays. In this paper, we investigate the contribution of the fourth generation to CP violation in the $B \to K\phi$ decay. Since there exist uncalculable final phases through the final-state interactions such as rescatterings, $B \to \bar{D}D + X_s \to K\phi$ and $B \to \psi + X_s \to K\phi$, we concentrate on only the calculable final-state phase in our paper. We analyze CP-violating asymmetries of the $K\phi$ decays of the charged B mesons and the neutral B mesons, and then investigate the asymmetry between the $B_d^0 \to K_S^0\phi$ and $\bar{B}_d^0 \to K_S^0\phi$ processes on $\Upsilon(4S)$, which is advantageous to observe the effect of the fourth generation.

The penguin diagram for the $b \to sg$ transition induces the effective Hamiltonian[1,2]

$$H_{\text{pen}} = \sqrt{2}G_F \frac{\alpha_s}{\pi}\left[\sum V_{ib}V_{is}^* I_i\right]\left[\bar{s}\gamma_\mu(1-\gamma_5)\frac{\lambda^a}{2}b\right]\left[\bar{q}\gamma_\mu\frac{\lambda^a}{2}q\right],$$

where V_{ij} is the Kobayashi-Maskawa (KM) matrix element,[5] I_i is the loop integral, and i runs over up quarks. The loop integral I_i is given by calculating the lowest-order penguin diagrams in the 't Hooft-Feynman gauge as follows:[3]

$$I_i = \frac{M_W^2}{M_W^2 - m_i^2}\left\{\int_0^1 x(1-x)\ln[m_i^2 - k^2x(1-x)]dx - \int_0^1 x(1-x)\ln[M_W^2(1-x) + m_i^2x - k^2x(1-x)]dx\right\}\left[1 + \frac{m_i^2}{2M_W^2}\right],$$

where M_W and m_i are the W-boson mass and the ith-type-quark mass, respectively, and k is the momentum transfer carried by the gluon.

In the case of $M_W^2 \gg m_i^2 \gg k^2$, the integral function I_i reduces to the well known result $I_i \approx \frac{1}{6}\ln(M_W^2/m_i^2)$.[1,2] Now k^2 is not negligible compared with m_c^2 in the B-meson system because the order of k^2 is m_b^2. In the case of timelike k^2 with $k^2 > 4m_i^2$, I_i has an absorptive part derived from the logarithmic integral, which is the source of CP violation of the charmless B-meson decay.[2,6]

The analytic form of I_i is given as follows:[3]

$$I_i = \frac{M_W^2}{M_W^2 - m_i^2}(S - F_1 + F_2)\left[1 + \frac{m_i^2}{2M_W^2}\right], \tag{1}$$

where the function S is given as, in the case of $0 < k^2 < 4m_i^2$,

$$S = \frac{1}{6}\ln m_i^2 - \frac{5}{18} - \frac{2m_i^2}{3k^2} + \frac{r}{2}\left[1 + \frac{r^2}{3}\right]\arctan\left[\frac{1}{r}\right],$$

and in the case of $k^2 \geqq 4m_i^2$,

$$S = \frac{1}{6}\ln m_i^2 - \frac{5}{18} - \frac{2m_i^2}{3k^2} - \frac{r}{12}\left[1 + \frac{2m_i^2}{k^2}\right]\ln\left[\frac{r-1}{r+1}\right]^2 - i\frac{\pi}{6}r\left[1 + \frac{2m_i^2}{k^2}\right],$$

where $r = |1-4m_i^2/k^2|^{1/2}$, and the functions F_1 and F_2 are given as, in the case of $k^2 < (M_W - m_i)^2$,

$$F_1 = \frac{1}{2}\ln m_i^2 - 1 - \frac{\delta}{2} - \frac{R}{4}(1+\delta)\ln\left[\frac{R+\delta+1}{R+\delta-1}\right]^2 - \frac{1}{8}(1+\delta-R)^2\ln\frac{m_i^2}{M_W^2},$$

$$F_2 = \frac{1}{3}\ln m_i^2 - \frac{2}{9} - \frac{1}{6}(1+\delta)(3+2\delta) + \frac{2M_W^2}{3k^2} + \frac{R}{6}\left[(1+\delta)^2 - \frac{M_W^2}{k^2}\right]\ln\left[\frac{R+\delta+1}{R+\delta-1}\right]^2 - \frac{1}{24}(1+\delta-R)^3\ln\frac{m_i^2}{M_W^2},$$

where $R = |(1+\delta)^2 - 4M_W^2/k^2|^{1/2}$ and $\delta = (M_W^2 - m_i^2)/k^2$. The above forms of S, F_1, and F_2 are shown only for the k^2 region where CP-violating asymmetries are calculated in our paper.

Now we begin with calculating the asymmetries of the $B_u^\pm \to K^\pm\phi$ processes, which are caused by the timelike penguin process and the spacelike one. Since the spacelike penguin amplitude is suppressed compared with the timelike one because of the form-factor effect in the annihilation structure of the associated diagram,[2,3] we neglect the spacelike penguin amplitude in the following analysis.

The penguin amplitudes for $B_u^\pm \to K^\pm\phi$ are given as follows:[2,3]

$$T(B_u^+ \to K^+\phi) = \left[\sum V_{ib}^* V_{is} I_i\right] A,$$
$$T(B_u^- \to K^-\phi) = \left[\sum V_{ib} V_{is}^* I_i\right] A, \qquad (2)$$

where A is the amplitude with the KM matrix elements and the loop integrals factored out. Then, the asymmetry parameter $A^\pm$ is given using Eq. (2) as follows:

$$A^\pm = \frac{|T(B_u^+ \to K^+\phi)|^2 - |T(B_u^- \to K^-\phi)|^2}{|T(B_u^+ \to K^+\phi)|^2 + |T(B_u^- \to K^-\phi)|^2} = \frac{|\sum V_{ib}^* V_{is} I_i|^2 - |\sum V_{ib} V_{is}^* I_i|^2}{|\sum V_{ib}^* V_{is} I_i|^2 + |\sum V_{ib} V_{is}^* I_i|^2}. \qquad (3)$$

Since I_u and I_c $(k^2 > m_c^2)$ have the absorptive part owing to $k^2 \sim m_b^2$, we obtain a nonzero value of $A^\pm$ in Eq. (3). Namely, the interfering amplitudes required for CP violation come from the penguin diagrams with different internal quarks, which consequently give rise to the different weak phases, and so these absorptive parts give a final phase. The value of k^2 should be given definitely in this two-body decay, but for now we do not have knowledge enough of the hadron to estimate k^2 exactly, and so we show our results for typical values of $k = |k^2|^{1/2} = 2.5$, 3.5, and 5.0 GeV.

In the case of the three-generation model, Gérard and Hou[4] have found $A^\pm$ to be at most 1%. In the case of the four-generation model, we show that $A^\pm$ could be as large as $\pm 15\%$ if the KM parameters of the fourth generation have relevant values. Following the parametrization of the 4×4 KM matrix by Botella and Chau,[7] the relevant products of the KM matrix elements are written as follows: $V_{cb}V_{cs}^* = s_y - s_x s_y^2 s_z e^{-i\phi_1}$, $V_{tb}V_{ts}^* = -s_y - s_x \times s_z e^{-i\phi_1} - s_u s_v e^{-i\phi_3}$, $V_{ab}V_{as}^* = s_u s_v e^{-i\phi_3}$, and $V_{ub}V_{us}^* = s_x s_z e^{-i\phi_1}$, where u, c, t, and a denote the four up quarks, and $s_x = 0.22$, $s_y = 0.044$, and $s_z/s_y \leqq 0.14$.[8] These parametrizations satisfy the unitarity relation in the order $s_x s_y^2 s_z$. We take $\pi/2 \leqq \phi_1 \leqq 5\pi/6$ based on the recent analyses[8] of ϵ'/ϵ and the B_d^0-$\bar{B}_d^0$ mixing;[9] on the other hand, s_u, s_v, and ϕ_3 are unknown parameters associated with the fourth-generation quark. In our analysis, we take $\phi_3 = \pm\pi/2$ in order to estimate the maximum contribution of the fourth generation. We change the product $s_u s_v$ in the region $0 < s_u s_v < 5\times 10^{-2}$ because we expect at most $V_{ab}V_{as}^* \approx \lambda^2$ with $\lambda = 0.22$ in the analyses of the B_d^0-$\bar{B}_d^0$ mixing in the four-generation model.[10] In order to calculate the values of I_i $(i = u,c,t,a)$ in Eq. (1), we take the quark masses $(m_b, m_c, m_u) = (5, 1.4, 0.005)$ GeV, and we tentatively fix the masses of the t quark and the a quark as $m_t = 40$ GeV and $m_a = 200$ GeV. Our numerical results of the asymmetries are insensitive to the values in the ranges $m_t = 40$–60 GeV and $m_a = 100$–300 GeV.

Now, we show in Fig. 1 the calculated values of $A^\pm$ versus the product $s_u s_v$ for $k = 2.5$, 3.5, and 5.0 GeV in the case of $s_z/s_y = 0.14$ and $\phi_1 = \pi/2$. Here $A^\pm$ at $s_u s_v = 0$ corresponds to the predicted value in the three-generation model, and $A^\pm$ at $s_u s_v > 0$ (<0) to $\phi_3 = \pi/2$ $(-\pi/2)$ in the four-generation model. From Fig. 1, we find that the asymmetry in the three-generation model is at most 3% even if we take maximum values of s_z/s_y and

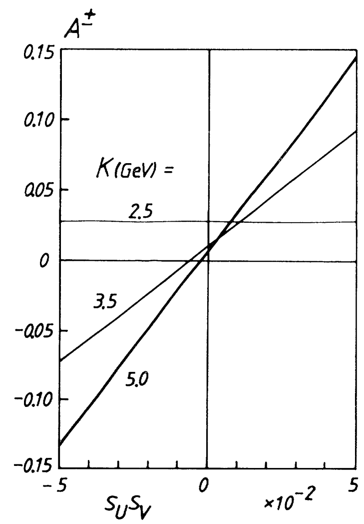

FIG. 1. Asymmetry of $B_u^\pm \to K^\pm\phi$ in the case of $s_z/s_y = 0.14$ and $\phi_1 = \pi/2$. Here $s_u s_v > 0$ (<0) corresponds to the case of $\phi_3 = \pi/2$ $(-\pi/2)$.

ϕ_1 such as $s_z/s_y=0.14$ and $\phi_1=\pi/2$. If the value of $s_u s_v$ is 5×10^{-2}, the asymmetry amounts to $\pm15\%$, whose sign depends on the sign of $\phi_3=\pm\pi/2$. Thus, the asymmetry of $B_u^\pm\to K^\pm\phi$ gives us a very good chance to observe the fourth-generation effect. Since the first three-generation effect is very small, the results in Fig. 1 are insensitive to the values of s_z/s_y and ϕ_1.

Here we comment on the asymmetry $A^\pm$ at $k=2.5$ GeV, which is almost independent of the fourth-generation parameter $s_u s_v$. The absorptive part of I_c ($k>2m_c$) is crucial for CP violation in the four-generation model, but I_c is real at $k=2.5$ GeV, and then the fourth-generation effect on CP violation almost vanishes. The sizable difference between the $A^\pm$'s at $k=2.5$ and 3.5 GeV in the three-generation model (at $s_u s_v=0$) is due to the u-quark and the c-quark contributions. In the case of $k>2m_c$, the CP-violating phase is canceled partially between I_c and I_u.

The asymmetry between the $B_d^0\to K_S^0\phi$ and $\bar{B}_d^0\to K_S^0\phi$ processes is very different from that of $B_u^\pm\to K^\pm\phi$ processes because of the large B_d^0-$\bar{B}_d^0$ mixing.[11] The time-dependent rate for an initially pure B_d^0 to decay to the $K_S\phi$ state is given as[12,13]

$$\Gamma(B_d^0(t)\to K_S\phi)=|T|^2 e^{-\Gamma_B t}\left[\left(\cos\frac{\Delta mt}{2}\right)^2+|\lambda|^2\left(\sin\frac{\Delta mt}{2}\right)^2-2\,\mathrm{Im}\left(\lambda\cos\frac{\Delta mt}{2}\sin\frac{\Delta mt}{2}\right)\right], \quad (4)$$

and $\Gamma(\bar{B}_d^0(t)\to K_S\phi)$ for $\bar{B}_d^0$ is given by replacing T and λ with $\bar{T}$ and $\bar{\lambda}$ in Eq. (4), respectively, where $T=\langle K_S\phi|B_d^0\rangle$, $\bar{T}=\langle K_S\phi|\bar{B}_d^0\rangle$, $\lambda=(q/p)(\bar{T}/T)$, and $\bar{\lambda}=(p/q)(T/\bar{T})$ with $q/p=M_{12}^*/|M_{12}|$, M_{12} being the off-diagonal matrix element in the B_d^0-$\bar{B}_d^0$ system. In Eq. (4), Δm and Γ_B are the mass difference and the decay width of the B mesons, respectively. Since the amplitudes T and $\bar{T}$ are proportional to $\sum V_{ib}^* V_{is} I_i$ and $\sum V_{ib} V_{is}^* I_i$, respectively, the time-integrated asymmetry parameter A^0 defined as

$$A^0=\frac{\Gamma(B^0_{\rm phys}\to K_S\phi)-\Gamma(\bar{B}^0_{\rm phys}\to K_S\phi)}{\Gamma(B^0_{\rm phys}\to K_S\phi)+\Gamma(\bar{B}^0_{\rm phys}\to K_S\phi)}$$

is given by integrating the time-dependent rates over t as follows:

$$A^0=\frac{(2+x_{B_d}^2)(|T|^2-|\bar{T}|^2)+x_{B_d}^2(|T\lambda|^2-|\bar{T}\bar{\lambda}|^2)-2x_{B_d}(|T|^2\mathrm{Im}\lambda-|\bar{T}|^2\mathrm{Im}\bar{\lambda})}{(2+x_{B_d}^2)(|T|^2+|\bar{T}|^2)+x_{B_d}^2(|T\lambda|^2+|\bar{T}\bar{\lambda}|^2)-2x_{B_d}(|T|^2\mathrm{Im}\lambda+|\bar{T}|^2\mathrm{Im}\bar{\lambda})}. \quad (5)$$

In contrast with the tree process such as $B_d^0\to K_S\psi$,[12,13] the first two terms in the numerator of Eq. (5) are significant in addition to the last term, because of $|T|^2\neq|\bar{T}|^2$ in the penguin process. Thus, the asymmetry A^0 has two origins, from the B_d^0-$\bar{B}_d^0$ mixing and from the loop-integral absorptive part of the penguin process. We show the calculated values of A^0 versus the product $s_u s_v$ for $k=2.5$, 3.5, and 5.0 GeV in the case of $s_z/s_y=0.14$ and $\phi_i=\pi/2, 5\pi/6$ in Fig. 2, where we used the observed value of $x_{B_d}=0.7$.[11] The predicted value of A^0 is large even in the three-generation model (at $s_u s_v=0$) due to the large B_d^0-$\bar{B}_d^0$ mixing, so our results depend strongly on the value of s_z/s_y and ϕ_1. As seen in Fig. 2, if the asymmetry is observed to be 20%–40%, it will be impos-

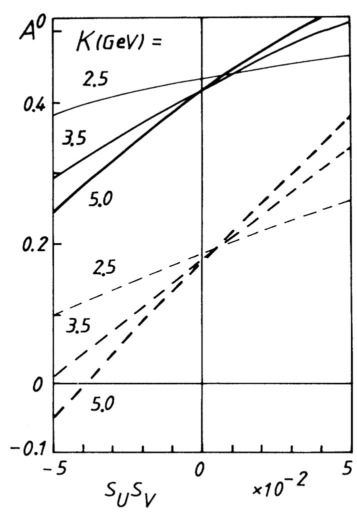

FIG. 2. Asymmetry between $B_d^0\to K_S^0\phi$ and $\bar{B}_d^0\to K_S^0\phi$ in the case of $s_z/s_y=0.14$ and $\phi_1=\pi/2, 5\pi/6$, where solid lines (dashed lines) denote the case of $\phi_1=\pi/2$ ($5\pi/6$).

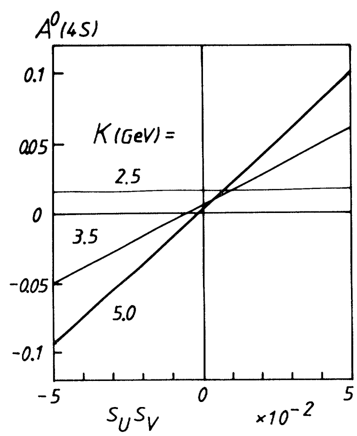

FIG. 3. Asymmetry between $B_d^0\to K_S^0\phi$ and $\bar{B}_d^0\to K_S^0\phi$ on $\Upsilon(4S)$.

sible to detect the fourth-generation effect, unless the values of s_z/s_y and ϕ_1 are determined precisely.

The fourth-generation effect on the asymmetry between the $B_d^0 \to K_S^0\phi$ and $\bar{B}_d^0 \to K_S^0\phi$ processes may be screened by the large B_d^0-$\bar{B}_d^0$ mixing in contrast with the case of $B_u^\pm \to K^\pm\phi$. However, its effect will be observed clearly on $\Upsilon(4S)$. As is well known, the *CP*-violating asymmetry in the neutral-*B*-meson decays originating from the B_d^0-$\bar{B}_d^0$ mixing vanishes on $\Upsilon(4S)$ due to the Bose statistics in the case $|q/p|=1$, which is also satisfied in the KM model with four generations,[10] but the asymmetry through the loop integral of the penguin amplitude does not vanish even on $\Upsilon(4S)$ as shown below.

The probability of finding $B_d^0 \to l^+$ (a charged lepton) at t_1 and $\bar{B}_d^0 \to K_S\phi$ at t_2 on $\Upsilon(4S)$ is given as follows:[12,14]

$$P(l^+,t_1;K_S\phi,t_2) = \tfrac{1}{2}|T^+|^2|\bar{T}|^2 e^{-\Gamma_B(t_1+t_2)}\{1+|\bar{\lambda}|^2+(1-|\bar{\lambda}|^2)\cos[\Delta m(t_1-t_2)]-2(\mathrm{Im}\bar{\lambda})\sin[\Delta m(t_1-t_2)]\}\,; \quad (6)$$

the probability $P(l^-,t_1;K_S\phi,t_2)$ of $\bar{B}_d^0 \to l^-$ at t_1 and $B_d^0 \to K_S\phi$ at t_2 is given by replacing T^+, $\bar{T}$, and $\bar{\lambda}$ with T^-, T, and λ in Eq. (6), respectively, where $T^\pm$ denote $\langle l^+X|B_d^0\rangle$ and $\langle l^-X|\bar{B}_d^0\rangle$. Since *CPT* invariance implies $|T^+|=|T^-|$, the time-integrated asymmetry parameter $A^0(4S)$ is given with only x_{B_d}, T, and $\bar{T}$ by integrating the time-dependent probabilities over t_1 and t_2 as follows:

$$A^0(4S) = \frac{P(l^-,K_S\phi)-P(l^+,K_S\phi)}{P(l^-,K_S\phi)+P(l^+,K_S\phi)} = \frac{|T|^2-|\bar{T}|^2}{|T|^2+|\bar{T}|^2}\frac{1}{1+x_{B_d}^2}\,. \quad (7)$$

In the limit of vanishing B_d^0-$\bar{B}_d^0$ mixing, Eq. (7) reduces to Eq. (3), and in the limit of $|T|=|\bar{T}|$, $A^0(4S)$ vanishes as is expected.

We show in Fig. 3 the calculated values of $A^0(4S)$ vs $s_u s_v$ in the case of $s_z/s_y=0.14$ and $\phi_1=\pi/2$ for $k=2.5$, 3.5, 5.0 GeV. The results are similar to that in Fig. 1, since $A^0(4S)$ of Eq. (7) differs from $A^\pm$ of Eq. (3) only in the factor $(1+x_{B_d}^2)^{-1}$. We have found that the asymmetry $A^0(4S)$ could be $\pm 10\%$ in the fourth-generation model.

We have already calculated the branching ratio of $B \to K\phi$ to be 1×10^{-5} in Ref. 3. Consequently a 3σ effect would require at least 10^{10} $\Upsilon(4S)$'s if the asymmetry is of the order of 1%,[4] but 10^7–10^8 $\Upsilon(4S)$'s if it is $\pm 15\%$. Since 10^6–10^7 $\Upsilon(4S)$'s are attainable in the upgrade of CLEO II at the Cornell Electron Storage Ring and the proposed Paul Scherrer Institute *B*-meson factory in Switzerland, it is expected that the fourth-generation effect will be observed in the $B_u^\pm \to K^\pm\phi$, $B_d^0 \to K_S\phi$, and $\bar{B}_d^0 \to K_S\phi$ processes on $\Upsilon(4S)$.

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