Comment on "Violation of the Weak Equivalence Principle in Theories of Gravity with a Nonsymmetric Metric"

Recently Will[1] has pointed out that gravitational theories with nonsymmetric metrics $g_{\mu\nu}\neq g_{\nu\mu}$ [e.g., the nonsymmetric gravitation theory (NGT) of Ref. 2] can violate the weak equivalence principle (WEP).[1] We comment here that this conclusion is not inevitable by presenting an action for charged test particles in such theories which does not violate the WEP,[3] although it is experimentally distinguishable from the general relativistic case.

The most general action quadratic in both the electromagnetic (EM) field strength $F_{\mu\nu}\equiv A_{[\mu,\nu]}$ (with vector potential A_μ) and the inverse metric is

$$I=I_p-\frac{1}{16\pi}\int d^3x\,dt\,\sqrt{-g}\,\mathcal{F}g^{\mu\alpha}g^{\nu\beta}\times[ZF_{\mu\nu}F_{\alpha\beta}+(1-Z)F_{\alpha\nu}F_{\mu\beta}+YF_{\mu\alpha}F_{\nu\beta}]\,,$$

where

$$I_p=-\sum_a m_a\int(g_{\mu\nu}v_a^\mu v_a^\nu)^{1/2}dt+\sum_a e_a\int A_\mu v_a^\mu\,dt$$

is the action for charged test particles, Y and Z are constants, and $v_a^\mu=dx_a^\mu/dt$. $\mathcal{F}$ is a scalar function which cannot depend on the EM field and which must be unity in the limit $g_{[\mu\nu]}\rightarrow 0$ (the Einstein-Maxwell case), implying that $\mathcal{F}=\mathcal{F}(\sqrt{-g}/\sqrt{-\gamma})$, where $g\equiv\det g_{\mu\nu}$ and $\gamma\equiv\det g_{(\mu\nu)}$.

Consider theories for which a Cartesian coordinate system can be found to allow the static, spherically symmetric metric to be written as $g_{00}=-T(r)$, $g_{ij}=H(r)\times\delta_{ij}$, and $g_{[0i]}=L(r)n_i$, where T, H, and L are functions of $r\equiv|\mathbf{x}|$ and $n_i\equiv x_i/r$. These criteria are satisfied by NGT.[2] For simplicity, we have set $g_{[ij]}=0$, a choice dictated by a certain class of boundary conditions in the spherically symmetric case.[2] Substituting these values into the action and identifying $F_{i0}=E_i$, $F_{ij}=\epsilon_{ijk}B_k$ gives

$$I_{\rm EM}=\frac{1}{8\pi}\int d^3x\,dt\left[\epsilon\mathbf{E}\cdot\mathbf{E}+X\epsilon\alpha(\mathbf{n}\cdot\mathbf{E})^2-\frac{1}{\mu}\mathbf{B}\cdot\mathbf{B}+\frac{\omega}{\mu}(\mathbf{n}\cdot\mathbf{B})^2\right],$$

where

$$\epsilon=\mathcal{F}(H/T)^{1/2}(1-L^2/TH)^{-1/2}\,,$$

$$\mu=\mathcal{F}^{-1}(H/T)^{1/2}(1-L^2/TH)^{1/2}\,,$$

$$\alpha=2L^2/(TH-L^2),\quad \omega=L^2/TH\,,$$

and we have defined $X\equiv 1-Z-Y$.

This action is equivalent to that given by the $TH\epsilon\mu$ formalism,[4] except for the terms proportional to $(\mathbf{n}\cdot\mathbf{E})^2$ and $(\mathbf{n}\cdot\mathbf{B})^2$. Expanding the resultant gravitationally modified Maxwell equations in powers of v/c, we find to lowest order that the $\mathbf{n}\cdot\mathbf{B}$ terms do not contribute but that the $\mathbf{n}\cdot\mathbf{E}$ terms do, causing deviations from Coulomb's law. These deviations are proportional to X and so we shall choose $X=0$. The action then reduces to that investigated by Will[1] except for the presence of the scalar function $\mathcal{F}$. The results of the $TH\epsilon\mu$ formalism[3] then apply: WEP violations do not occur if $\epsilon=\mu=(H/T)^{1/2}$.

Requiring $\epsilon=\mu$ uniquely yields $\mathcal{F}=(1-L^2/TH)^{1/2}$ implying $\mathcal{F}(\sqrt{-g}/\sqrt{-\gamma})=\sqrt{-g}/\sqrt{-\gamma}$. This choice necessarily implies $\epsilon=(H/T)^{1/2}=\mu$ and

$$I_{\rm EM}=\frac{1}{8\pi}\int d^3x\,dt\left[\epsilon E^2-\frac{1}{\mu}[B^2-\omega(\mathbf{n}\cdot\mathbf{B})^2]\right],$$

yielding no WEP violations due to electromagnetic binding effects; any other choice[1,5] will necessarily lead to WEP violations. This result is independent of the gravitational field equations governing the evolution of $g_{\mu\nu}$.

This action differs from that of Einstein's theory because different field equations determine the functions H and T and because of the presence of the $(\mathbf{n}\cdot\mathbf{B})^2$ term. Such a term could produce perturbations in the energy levels of atomic systems which are in principle measurable.[1] The possible extent of such repercussions is presently being investigated.

This work was supported by the Natural Sciences and Engineering Research Council of Canada.

R. B. Mann and J. H. Palmer
Guelph-Waterloo Program for Graduate Work in Physics
Department of Physics
University of Waterloo
Waterloo, Ontario, Canada N2L 3G1

J. W. Moffat
Department of Physics
University of Toronto
Toronto, Ontario, Canada M5S 1A7

Received 6 March 1989
PACS numbers: 04.80.+z, 04.50.+h

[1] C. M. Will, Phys. Rev. Lett. **62**, 369 (1989).

[2] J. W. Moffat, Phys. Rev. D **19**, 3554 (1979); **19**, 3562 (1979); **35**, 3733 (1987); **39**, 474 (1989); J. W. Moffat and E. Woolgar, Phys. Rev. D **37**, 918 (1988).

[3] WEP violations could be trivially avoided by coupling only $g_{(\mu\nu)}$ to the EM field.

[4] A. P. Lightman and D. L. Lee, Phys. Rev. D **8**, 364 (1973).

[5] R. B. Mann and J. W. Moffat, Can. J. Phys. **59**, 1730 (1981).