Transverse Spin Diffusion in Polarized Fermi Gases

J. W. Jeon(1) and W. J. Mullin(1,2)

(1)*Laboratory for Low Temperature Physics, University of Massachusetts, Amherst, Massachusetts 01003*
(2)*Laboratoire de Spectroscopie Hertzienne d'Ecole Normale Superieure, 24 rue Lhomond, 75231 Paris, France*

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We solve a recently derived kinetic equation for a dilute quantum system at arbitrary degeneracy and polarization M. From this equation we develop a spin hydrodynamic equation that allows the generalization of the treatment of spin waves and the Leggett-Rice effect to polarized degenerate Fermi gases. We find a transverse spin-diffusion collision time $\tau_\perp$ that is often shorter than the corresponding longitudinal collision time $t_\parallel$. As $T \rightarrow 0$, $\tau_\perp$ approaches a T-independent value, in contrast to $\tau_\parallel \sim 1/T^2$. Spin waves would thus continue to be damped even to $T=0$ K.

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In 1982, Lhuillier and Laloë (LL)[1] introduced a kinetic equation valid for arbitrarily spin-polarized gases. From this equation LL were able to compute polarization-dependent transport coefficients and to derive a spin hydrodynamic equation that described both longitudinal and transverse spin diffusion. The latter equation predicted the existence of transverse spin waves in Bose and Fermi gases obeying Boltzmann statistics.[2] We have been able to derive an equivalent kinetic equation valid for *degenerate* quantum systems at arbitrary polarization. Here we apply this equation to transverse spin phenomena in dilute Fermi systems. We examine the effects of polarization and degeneracy on spin waves, and also consider the generalization of the Leggett-Rice effect[3] to such situations. We find that the collision time governing transverse spin diffusion can be much smaller than that for the longitudinal case confirming speculations made by Meyerovich in 1985.[4] These results lead, for one thing, to the remarkable result that the damping of transverse spin waves becomes a constant at low temperatures, even to $T=0$ K.

The kinetic equation derived by LL describes the time evolution of a matrix distribution function $\mathbf{n}_p(\mathbf{r},t)$ (2×2 for spin $\frac{1}{2}$). The collision integral produces two types of terms—the usual dissipative terms that determine transport coefficients, and a reactive term which plays a role similar to that occurring in the Landau-Silin equation[5] for a degenerate Fermi fluid. This latter feature describes what is called "identical-particle spin rotation" and is the basis for the existence of spin waves in this system.

The Landau-Silin equation for Fermi liquids, first presented by Silin[5] in 1958, includes a spin-rotation term and the generalization of Landau's mean-field terms to the nondiagonal spin case. However, no form of the dissipative collision integral was presented at that time and Silin limited his discussion to "collisionless" spin waves. In order to give a complete treatment of hydrodynamic spin waves and spin-echo phenomena at low T, it is necessary to have a (matrix) collision integral which is a generalization of the LL collision integral to the degenerate polarized case.

In 1971 Silin actually presented a form of such a collision integral in a kinetic-theory textbook[6] in Russian. To rederive, to generalize, and to simplify Silin's very complicated result, we have used the Kadanoff-Baym Green's-function method.[7] (Recently the Kadanoff-Baym technique has also been used by Ruckenstein and Levy[8] to treat polarized paramagnetic fluids.) Our most general result is expressed in terms of proper self-energies. A reduction of this result to useful form is easily made by use of the Born approximation for the self-energies. Generalization of this result beyond Born approximation is greatly complicated for degenerate systems by the dependence of the many-body T matrices on the nonequilibrium distribution functions in a nontrivial way. Fortunately, however, the Born approximation itself, with use of an effective potential, seems applicable to a prime system of interest, dilute solutions of ^{3}He in liquid ^{4}He. Of course, we might well expect that predictions for dilute systems will carry over qualitatively to strongly interacting Fermi systems. We can freely interpret our kinetic equation as applying to quasiparticles with the substitution of Landau's phenomenological interactions into the mean-field energy ϵ_p and into the collision integral.

The kinetic equation that results from our derivation[9] is a matrix equation for $\underline{n}_p(\mathbf{r},t)$ valid for both bosons and fermions. The 2×2 version for fermions is

$$\frac{\partial \underline{n}_{p_1}}{\partial t}+\frac{1}{2}[\nabla_{p_1}\underline{\epsilon}_{p_1},\nabla_r \underline{n}_{p_1}]_+-\frac{1}{2}[\nabla_r\underline{\epsilon}_{p_1},\nabla_{p_1}\underline{n}_{p_1}]_++\frac{i}{\hbar}[\underline{\epsilon}_{p_1},n_{p_1}]_-$$
$$=\frac{(2\pi)^2}{h^7}\int d\mathbf{p}_2 d\mathbf{p}_3 d\mathbf{p}_4\,\delta(\mathbf{p}_1+\mathbf{p}_2-\mathbf{p}_3-\mathbf{p}_4)\delta(\epsilon_{p_1}+\epsilon_{p_2}-\epsilon_{p_3}-\epsilon_{p_4})$$
$$\times\tfrac{1}{2}\big([V(\mathbf{p}_1-\mathbf{p}_3)]^2\{[\underline{n}_3,\underline{\tilde{n}}_1]_+\mathrm{Tr}(\underline{\tilde{n}}_2\underline{n}_4)-[\underline{\tilde{n}}_3,\underline{n}_1]_+\mathrm{Tr}(\underline{n}_2\underline{\tilde{n}}_4)\}$$
$$-V(\mathbf{p}_1-\mathbf{p}_3)V(\mathbf{p}_1-\mathbf{p}_4)\{[\underline{n}_3\underline{\tilde{n}}_2\underline{n}_4,\underline{\tilde{n}}_1]_+-[\underline{\tilde{n}}_3\underline{n}_2\underline{\tilde{n}}_4,\underline{n}_1]_+\}\big)\,. \tag{1}$$

Here $\underline{\epsilon}_p(\mathbf{r},t)$ is an energy matrix given by

$$\underline{\epsilon}_p(\mathbf{r},t)=\epsilon_p\underline{I}-\tfrac{1}{2}\hbar\gamma\mathbf{B}\cdot\underline{\boldsymbol{\sigma}}+\frac{1}{h^3}\int d\mathbf{p}'\{V(0)\underline{I}\,\mathrm{Tr}[\underline{n}_{p'}(\mathbf{r},t)]-V(\mathbf{p}-\mathbf{p}')\underline{n}_{p'}(\mathbf{r},t)\}\,. \tag{2}$$

In the above $V(\mathbf{p})$ is the Fourier transform of the potential, ϵ_p is $p^2/2m^*$, m^* is the particle mass, $\mathbf{B}$ is the magnetic field, and $\underline{\boldsymbol{\sigma}}$ is a Pauli matrix. We also use $\underline{\tilde{n}}_p=\underline{I}-\underline{n}_p$. The notation $\underline{n}_1$, etc., is short for $\underline{n}_{p_1}$. Details of the derivation of the kinetic equation are given in Ref. 9.

The left-hand side of this equation contains terms familiar from the Landau-Silin equation. The commutator ($[,]_-$) is the spin-rotation term and the anticommutators ($[,]_+$) include the mean-field terms. In the limit of Boltzmann statistics, the collision integral on the right-hand side of Eq. (1) reduces to the Born-approximate version of LL's collision integral. When all spins are quantized along the same axis (the longitudinal case) the collision integral becomes the conventional Uehling-Uhlenbeck form.[10] It is easy to show that the collision integral satisfies the usual conservation laws of particle number, momentum, energy, and spin.

The ordinary Landau equation with diagonal collision integral, is adequate to describe longitudinal spin diffusion.[11,12] However, in order to discuss spin waves or spin-echo experiments, we use Eq. (1) to develop a magnetization hydrodynamic equation for the spin current, defined by

$$J_j(\mathbf{m})=\frac{1}{h^3}\int d\mathbf{p}\,v_{pj}\,\mathrm{Tr}(\underline{\boldsymbol{\sigma}}\underline{n}_p)\,, \tag{3}$$

where $\mathbf{m}=m\hat{\mathbf{e}}$ is the magnetization in the local direction $\hat{\mathbf{e}}(\mathbf{r},t)$, v_{pj} is a particle velocity, and $j=x,y,z$.

We proceed with a linearization and variational solution of Eq. (1). We linearize about the local equilibrium distribution function $\underline{n}_p^0$ by writing $\underline{n}_p=\underline{n}_p^0+\delta\underline{n}_p$. This form is inserted into the left-hand side of Eq. (1) and the second and third terms are approximated by dropping $\delta\underline{n}_p$. We cannot drop $\delta\underline{n}_p$ from the spin-rotation term since that provides its leading contribution. The appropriate local equilibrium function is diagonal in a reference frame with z axis along $\hat{\mathbf{e}}$ and has diagonal elements n_{p+} and n_{p-}, where $n_{p\sigma}=[\exp\beta(\epsilon_p-\mu_\sigma)+1]^{-1}$. Here $\sigma=\pm1$ and μ_σ the chemical potential for spin species σ. For simplicity, we consider the s-wave approximation $V(\mathbf{p})\cong V(0)$.

If we write

$$\delta\underline{n}_p=\tfrac{1}{2}(\delta f_p\underline{I}+\delta\boldsymbol{\sigma}_p\cdot\underline{\boldsymbol{\sigma}}) \tag{4}$$

then the magnetization portion $D\boldsymbol{\sigma}_p/Dt$ of the drift term becomes

$$\frac{D\boldsymbol{\sigma}_p}{Dt}=\frac{\partial\boldsymbol{\sigma}_p}{\partial t}-\sum_i\hat{\mathbf{e}}v_{pi}\frac{\partial m}{\partial r_i}\sum_{\sigma=\pm1}\left[\frac{\partial n_{p\sigma}}{\partial\epsilon_p}t_\sigma\right]+\sum_i\frac{\partial\hat{\mathbf{e}}}{\partial r_i}v_{pi}\sum_{\sigma=\pm1}\sigma n_{p\sigma}-\frac{1}{\hbar}\boldsymbol{\sigma}_p\times[\hbar\gamma\mathbf{B}+V(0)\mathbf{m}]\,. \tag{5}$$

The factor t_σ is a quantity depending on temperature and polarization whose value is given in Ref. 9.

We note that all longitudinal terms (those proportional to $\hat{\mathbf{e}}$) contain the factor $\partial n_{p\sigma}/\partial\epsilon_p$, which, for a degenerate system, constrains them to the Fermi surfaces. By contrast the transverse term, in $\partial\hat{\mathbf{e}}/\partial r_i$, depends on $\sum_\sigma\sigma n_{p\sigma}$ which is nonzero for all momenta *between* the Fermi surfaces. We know that the solution $\delta\boldsymbol{\sigma}_p$ must strongly overlap the drift terms so that the former should include a term in $\sum_i v_i(\partial\hat{\mathbf{e}}/\partial r_i)\sum_\sigma\sigma n_{p\sigma}$. Such a term results physically from the fact that spins tipped transversely away from local equilibrium constitute a disturbance everywhere in momentum space where there is a net magnetization, that is, between the two Fermi spheres. Thus we take as a trial function

$$\delta\boldsymbol{\sigma}_p=\delta\boldsymbol{\sigma}_p^{\parallel}+\delta\boldsymbol{\sigma}_p^{\perp}\,, \tag{6}$$

where the longitudinal part is

$$\delta\boldsymbol{\sigma}_p^{\parallel}=q\sum_i v_{pi}\frac{\partial m}{\partial r_i}\hat{\mathbf{e}}\left[\sum_\sigma\frac{1}{n_\sigma}\frac{\partial n_{p\sigma}}{\partial\epsilon_p}\right] \tag{7}$$

and the transverse part is

$$\delta\boldsymbol{\sigma}_p^{\perp}=A\sum_i v_{pi}\hat{\mathbf{g}}_i\left[\sum_\sigma\sigma n_{p\sigma}\right]. \tag{8}$$

Here q and A are variational parameters and the unit vectors $\hat{\mathbf{g}}_i$ in the transverse part are perpendicular to the local magnetization direction $\hat{\mathbf{e}}$, i.e., $\hat{\mathbf{g}}_i = x\, \partial\hat{\mathbf{e}}/\partial r_i + y\hat{\mathbf{e}}\times\partial\hat{\mathbf{e}}/\partial r_i$, where x and y are constants.

The form chosen for $\delta\sigma_p^{\parallel}$ corresponds to that used in our previous work[11,12] and will lead to the same collision time $\tau_{\parallel}$ and the same diffusion constant $D_{\parallel}$. However, $\delta\sigma_p^{\perp}$ is new and leads to a transverse collision time $\tau_{\perp}$ that differs markedly from $\tau_{\parallel}$. It will reduce to a form localized on the Fermi surfaces, like Eq. (7), only if T is finite and M is sufficiently small.

From the linearized kinetic equation and its variational solution we derive a hydrodynamic equation for the spin current.[9] Here we quote only the solution:

$$J_j(\mathbf{m}) = -D_{\parallel}\frac{\partial m}{\partial r_j}\hat{\mathbf{e}} - \frac{D_{\perp}}{1+(\mu m/n)^2}\left[m\frac{\partial\hat{\mathbf{e}}}{\partial r_j} + \frac{\mu m^2}{n}\hat{\mathbf{e}}\times\frac{\partial\hat{\mathbf{e}}}{\partial r_j}\right], \qquad (9)$$

where μ is the spin-rotation parameter given by $\mu = -nV(0)\tau_{\perp}/\hbar$, and n is the density. Longitudinal spin diffusion is described by the first term with diffusion constant $D_{\parallel} = \alpha_{\parallel}\tau_{\parallel}$, where $\alpha_{\parallel}$ is a polarization-dependent parameter.[9,11]

The effective transverse diffusion constant is given by the ratio in front of the square bracket in Eq. (9), with $D_{\perp} = \alpha_{\perp}\tau_{\perp}$. ($\alpha_{\perp}$ is a different polarization-dependent parameter.[9]) Equation (9) is a generalization of the Leggett equation[3] to arbitrarily polarized systems. Meyerovich[4] reasoned that a relaxation approximation to the collision integral ought to contain two separate relaxation times, $\tau_{\parallel}$ and $\tau_{\perp}$. He also arrived at an equation like Eq. (9) and our results verify his surmise. In addition, we have found an explicit expression for $\tau_{\perp}$ from our variational solution:[13]

$$\frac{1}{\tau_{\perp}} = \frac{4\pi^2}{h^{10}}\frac{\sinh(\Delta/2)V(0)^2}{\alpha_{\perp} m m^{*2}}\int d\mathbf{p}_1 d\mathbf{p}_2 d\mathbf{p}_3 d\mathbf{p}_4\,\delta(\mathbf{p}_1+\mathbf{p}_2-\mathbf{p}_3-\mathbf{p}_4)$$
$$\times\delta(\epsilon_1+\epsilon_2-\epsilon_3-\epsilon_4)(\mathbf{p}_1\cdot\hat{\mathbf{a}})(\mathbf{p}_1-\mathbf{p}_2)\cdot\hat{\mathbf{a}}\{e^{\Delta/2}n_{1-}n_{2-}+e^{-\Delta/2}n_{1+}n_{2+}\}\tilde{n}_{3+}\tilde{n}_{4-}\,. \qquad (10)$$

In this $\hat{\mathbf{a}}$ is a constant unit vector; $\tilde{n}_{p\sigma} = 1 - n_{p\sigma}$; μ_σ, the chemical potential for spin species σ, is determined to give the correct local particle density n and spin density m; and $\Delta = \beta(\mu_+ - \mu_-)$. A similar expression has been derived for the longitudinal collision time[11] which we reproduce here for comparison:

$$\frac{1}{\tau_{\parallel}} = \frac{4\pi^2}{h^{10}}\frac{n\beta V(0)^2}{n_+ n_- m^*}\int d\mathbf{p}_1 d\mathbf{p}_2 d\mathbf{p}_3 d\mathbf{p}_4\,\delta(\mathbf{p}_1+\mathbf{p}_2-\mathbf{p}_3-\mathbf{p}_4)\delta(\epsilon_1+\epsilon_2-\epsilon_3-\epsilon_4)(\mathbf{p}_1\cdot\hat{\mathbf{a}})(\mathbf{p}_1-\mathbf{p}_3)\cdot\hat{\mathbf{a}}\,n_{1+}n_{2-}\tilde{n}_{3+}\tilde{n}_{4-}\,. \qquad (11)$$

Note that unusual forms, e.g., $n_{1+}n_{2+}(1-n_{3+})(1-n_{4-})$, appear in the expression for $\tau_{\perp}$. This peculiarity arises from the fact that we are concerned here with transverse spins which can be described as a superposition of up and down components. In particular the variational form, Eq. (8), which contains the factor $n_{p+} - n_{p-}$, leads directly[9] to the forms appearing in $\tau_{\perp}$. The combinations of $n_{p\sigma}$ factors in $\tau_{\perp}$ result in an integrand which does not confine the scatterings to the Fermi surfaces. We thus have the possibility of $\tau_{\perp} \ll \tau_{\parallel}$.

The evaluation of Eq. (10) is not straightforward. However, it is fortunate that analytic and numerical techniques similar to those used in Ref. 11 are applicable to the present problem. This method allows us to reduce $\tau_{\perp}$ to a twofold integral which can be evaluated for all temperatures and polarizations. The numerical results for $\tau_{\perp}$ and $D_{\perp}$ are shown in Fig. 1. In the Boltzmann limit the transverse and longitudinal quantities coincide. Here, even if M is not small, we find $\tau_{\perp} = \tau_{\parallel}[1+O(n\lambda_T^3 M^2)]$, $D_{\perp} = D_{\parallel}[1+O(n\lambda_T^3 M^2)]$, where λ_T is the thermal wavelength. The equality of $D_{\perp}$ and $D_{\parallel}$ in this regime is consistent with the theory of Ref. 1.

In the degenerate limit, for small polarization $M = m/n$, $\tau_{\perp}$ appears to coincide with $\tau_{\parallel}$, and $D_{\perp}$ with $D_{\parallel}$. For larger M and $T < T_F$, $\tau_{\perp}$ falls below $\tau_{\parallel}$. In fact, as $T \to 0$, $\tau_{\perp}$ ultimately *always* approaches a finite limit that depends on M, although for very small M this limit may be approached only at unreachably low temperatures. The temperature T_b at which $\tau_{\perp}$ "breaks away" from $\tau_{\parallel}$, although not sharply defined, is given approximately by $\Delta \approx 1$, or

$$T_b = [(1+M)^{2/3} - (1-M)^{2/3}]T_F \approx MT_F\,,$$

where the latter approximation holds for small polarization. When $\Delta \ll 1$, as will be the case when the system is degenerate but at temperatures above T_b, it is easy to show that $\tau_{\perp} = \tau_{\parallel}[1+O(\Delta^2)]$, and $D_{\perp} = D_{\parallel}[1+O(\Delta^2)]$.

Analytic solutions for the zero-temperature limiting behavior of $\tau_{\perp}$ and $D_{\perp}$ can be derived. We find

$$\tau_{\perp} = \frac{21}{8}A\left(\frac{1+d^3}{2}\right)^{4/3}\frac{1-d^5}{f(d)}\,, \qquad (12)$$

$$D_{\perp} = \frac{21}{80}\frac{2AkT_F}{m^*}\frac{1+d^3}{1-d^3}\left(\frac{2}{1+d^3}\right)^{1/3}\frac{(1-d^5)^2}{f(d)}\,, \qquad (13)$$

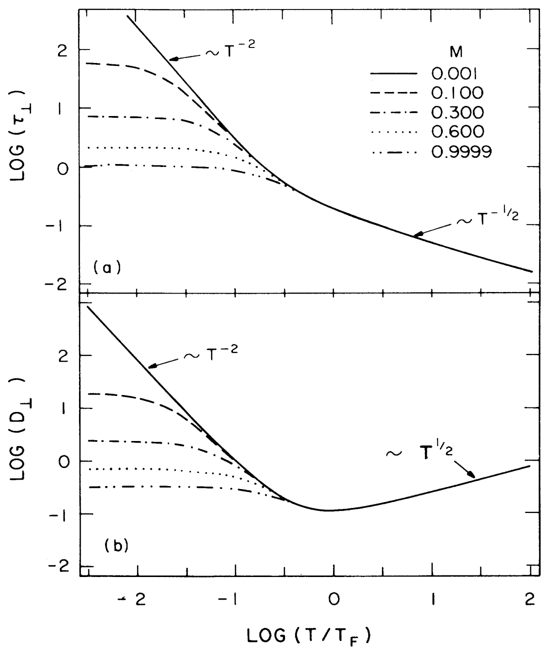

FIG. 1. (a) Transverse spin-diffusion collision time $\tau_\perp$ {in units of $A=[3/8\pi^2V(0)kT_F]^2(h^7/m^{*3})$} vs T/T_F, where T_F is the Fermi temperature of the unpolarized gas. Polarization is $M=m/n$. (b) Transverse spin-diffusion constant $D_\perp$ (in units of $2AkT_F/m^*$) vs T/T_F. At high T both $\tau_\perp$ and $D_\perp$ behave as their longitudinal counterparts $\tau_\parallel$ and $D_\parallel$. At $T<T_F$, $\tau_\perp$ may coincide with $\tau_\parallel$ and $D_\perp$ with $D_\parallel$ for a range of temperature, but both ultimately cease behaving as T^{-2} below a certain temperature and approach finite limits depending on M. Note that for $M=0.001$, $\tau_\perp$ and $D_\perp$ finally diverge from the T^{-2} behavior at $\log_{10}T\sim -3$.

where

$$A=\left(\frac{3}{8\pi^2V(0)kT_F}\right)^2\frac{h^7}{m^{*3}},\quad d=[(1-M)/(1+M)]^{1/3}$$

and

$$f(d)=1-\tfrac{21}{64}d^3(1-d^2)(5+2d^2+17d^4)$$
$$-\tfrac{1}{8}\epsilon(2d^2-1)^{7/2}(1+7d^2)\,,$$

with

$$\epsilon=\begin{cases}0, & d\leq 1/\sqrt{2}\,,\\ 1, & d>1/\sqrt{2}\,.\end{cases}$$

Corrections to Eqs. (12) and (13) are of order $(T/T_F)^2$, where T_F is the Fermi temperature of zero polarization.

If we combine Eq. (9) with the continuity equation for magnetization, we can show that for small tip angles, the components of magnetization transverse to the external field direction satisfy[1-3]

$$\partial m_+/\partial t+i\gamma Bm_+=D\nabla^2m_+\,, \qquad (14)$$

where $m_+=m_x+im_y$. Here D is a complex diffusion constant given by $D=D_\perp/(1-i\mu m/n)$. Equation (14) is valid at small tip angle either for a spin-echo experiment or for a spin-wave experiment. [Of course, it is possible to derive equations, analogous to (14), valid for any tip angle.] Thus one can measure $\tau_\perp$ in either of these two types of experiment. A particularly remarkable feature of Eq. (14) is that, because $\tau_\perp$ goes to a constant as $T\to 0$, rather than to infinity, spin-wave damping persists even to $T=0$ K.

Previous spin-echo experiments[14] on polarized dilute solutions showed anomalously low values of μ and $D_\perp$ as T approached T_F from above. Since our results show deviations from longitudinal behavior only for T somewhat less than T_F, our theory may not explain those anomalous results.

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[1] C. Lhuillier and F. Laloë, J. Phys. (Paris) **43**, 197 (1982); **43**, 225 (1982).

[2] Spin waves in Boltzmann gases were predicted independently by E. P. Bashkin, Pis'ma Zh. Eksp. Teor. Fiz. **33**, 11 (1981) [JETP Lett. 33, 8 (1981)]; Zh. Eksp. Teor. Fiz. **87**, 1948 (1984) [Sov. Phys. JETP **60**, 1122 (1984)]; an early thorough treatment was given by A. E. Ruckenstein and L. P. Levy, Phys. Rev. Lett. **55**, 1427 (1985).

[3] A. J. Leggett, J. Phys. C **3**, 448 (1970).

[4] A. E. Meyerovich, Phys. Lett. **107A**, 177 (1985); in *Progress in Low Temp Physics,* edited by D. F. Brewer (North-Holland, Amsterdam, 1987), Vol. 11, Chap. 1, p. 1.

[5] V. P. Silin, Zh. Eksp. Teor. Fiz. **33**, 1227 (1958) [Sov. Phys. JETP **6**, 945 (1958)].

[6] V. P. Silin, *Vvedenie v Kineticheskuyu Teoriyu Gasov* (Nanka, Moskow, 1971), Chap. 8.

[7] L. P. Kadanoff and G. Baym, *Quantum Statistical Mechanics* (Benjamin, New York, 1962).

[8] A. E. Ruckenstein and L. P. Levy (to be published).

[9] J. W. Jeon and W. J. Mullin, J. Phys. (Paris) **49**, 1691 (1988).

[10] E. A. Uehling and G. E. Uhlenbeck, Phys. Rev. **43**, 552 (1933).

[11] J. W. Jeon and W. J. Mullin, J. Low Temp. Phys. **67**, 421 (1987).

[12] W. J. Mullin and K. Miyake, J. Low Temp. Phys. **53**, 479 (1983).

[13] In Eq. (1), we have neglected a term which has been denoted as I_2 in the literature. This term is of order a/Λ, where a is the scattering length and Λ is the smaller of the interparticle spacing and the thermal wavelength. I_2 is a small contribution to the spin-rotation term and has nothing whatsoever to do with the relaxation times $\tau_\perp$ and $\tau_\parallel$. Cf. E. P. Bashkin, Phys. Rev. Lett. **55**, 1426 (1985).

[14] W. J. Gully and W. J. Mullin, Phys. Rev. Lett. **52**, 1810 (1984).