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## Fermion Electric Dipole Moments, Muon Polarization in $\eta \rightarrow \mu\bar{\mu}$, $K_L^0 \rightarrow \mu\bar{\mu}$ Decays, and the Scalar-Pseudoscalar Mixing Mechanism


C. Q. Geng and John N. Ng

*Theory Group, TRIUMF, 4004 Wesbrook Mall, Vancouver, British Columbia, Canada V6T 2A3*



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We construct extended Higgs-boson models of soft *CP*-invariance violation arising from scalar-pseudoscalar mixing. Using the electric dipole moment of the neutron as input, we find that the charged-lepton electric dipole moments scale as $(\text{mass})^3$. They are estimated to be $2.1\times10^{-28}$, $4.4\times10^{-22}$, and $1.8\times10^{-19}$ $e$ cm for electron, muon, and $\tau$, respectively. An important test of this source of *CP* violation lies in the longitudinal polarizations of muon in $\eta \rightarrow \mu\bar{\mu}$ and $K_L^0 \rightarrow \mu\bar{\mu}$, which we estimate to be of the order of $10^{-2}$ and $10^{-1}$, respectively.



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In this Letter we discuss the phenomenology of a class of *CP*-violating models that contains both hard and soft *CP*-violation (CPV) terms. Hard *CP* violation is induced by dimension-four terms in the Lagrangian; the best known example is the standard model of three quark families in which the Yukawa terms are explicitly *CP* violating. On the other hand, soft *CP* violation is generated by dimension-three terms which are absent in the standard model. The simplest model of this kind is that constructed by Lee.[1] However, Lee's model contains flavor-changing neutral currents at the tree level and must be suppressed by hand. Manifestation of soft or spontaneous CPV will indicate new physics beyond the standard model.

On the experimental side, recently limits on a nonzero value for the electric dipole moment (EDM) $d_n$ of the neutron were given.[2] This result is preliminary despite its agreement with a previous measurement[3] $d_n = -(1.4 \pm 0.6)\times10^{-25}$ $e$ cm. To be cautious one sets the limit $d_n < 2\times10^{-25}$ $e$ cm.[3] Such a large value cannot arise from the Kobayashi-Maskawa (KM) phase[4] in the standard model since it gives[5] $d_n \sim 10^{-32}$ $e$ cm. However, one cannot rule out the possibility that $d_n$ originates from the term $\theta G_{\mu\nu}\tilde{G}^{\mu\nu}$ in QCD. This implies[6] $\theta \sim 10^{-10}$ and is deemed to be very unnatural. We are reminded that all other measurements on CPV are consistent with the standard KM model. The situation is exciting enough to motivate us to look into the phenomenology of a simple model that incorporates a KM phase, soft CPV, and a Peccei-Quinn (PQ) U(1) symmetry. The PQ symmetry is invoked so that the effective $\theta$ parameter is dynamically driven to zero.[7]

The simplest model we found has the same gauge group as the standard model plus a global $U(1)_{PQ}$ symmetry.[8,9] It employs two doublet and two singlet Higgs fields. Flavor-changing neutral currents are avoided by coupling one Higgs doublet $\phi_1$ to $u_R$ and the other one, $\phi_2$, to $d_R$.[10] The Yukawa couplings are assumed to be complex. After spontaneous symmetry breaking, there are a pair of physical charged Higgs bosons ($H^{\pm}$) and seven physical neutral spin-0 fields. The $H^{\pm}$ fields carry the same KM phase as that of the $W$ bosons in their couplings to the fermions.[11] Hence, the dominant CPV in the kaon system is given by the KM matrix and the additional spin-0 bosons play a negligible role.[12] Of the seven neutral spin-0 fields one is the axion and can be made invisible. The remaining fields $H_k$ ($k=1,2,\ldots,6$) will mix and the scalar-pseudoscalar mixing is the only new CPV source in this model. We emphasize that these spin-0 fields have no direct flavor-changing couplings to the fermions. Hereafter we shall isolate this new CPV mechanism and discuss possible large CPV effects which can show up in places where the KM model gives vanishing or no contributions. We also note that there is no lower bound on the masses of these Higgs bosons.[13]

We shall denote the phenomena of scalar-pseudoscalar

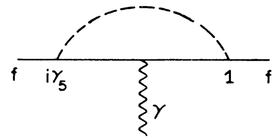


FIG. 1. Feynman diagram for the fermion electric dipole moment due to scalar-pseudoscalar mixing.

mixing by *R-I*, where *R* and *I* represent the real and imaginary parts of the neutral Higgs scalar in the weak-eigenstate basis. The mixing occurs if the Higgs potential has a lowest classical minimum that violates *T* invariance.[8,9,14] The weak eigenstates $R_i$ and $I_j$ and the mass eigenstates $H_k$ are related by a unitarity transformation. This transformation mixes different real and imaginary components of the doublet and singlet Higgs fields, and their coupling to fermions will then contain both scalar (1) and pseudoscalar ($i\gamma_5$) terms which lead to explicit CPV phenomena such as the EDM of fermions $d_f$ and the longitudinal polarization of muons in $\eta \rightarrow \mu\bar{\mu}$ and $K_L^0 \rightarrow \mu\bar{\mu}$ decays. These are depicted in Figs. 1 and 2, respectively. The Lagrangian density of the Yukawa terms involving $R_i, I_j$ fields has the following expression:

$$\mathcal{L}_Y = \frac{R_1}{v_1}\bar{u}M_u u - \frac{I_1}{v_1}\bar{u}M_u i\gamma_5 u + \frac{R_2}{v_2}(\bar{d}M_d d + \bar{e}M_e e) + \frac{I_2}{v_2}(\bar{d}M_d i\gamma_5 d + \bar{e}M_e i\gamma_5 e)\,,$$

which, in terms of the mass eigenstates $H_k$, can be rewritten as follows:

$$\mathcal{L}_Y = (2\sqrt{2}G_F)^{1/2}\sum_k [\alpha_1^k \bar{u}M_u u + \beta_1^k \bar{u}M_u i\gamma_5 u + \alpha_2^k(\bar{d}M_d d + \bar{e}M_e e) + \beta_2^k(\bar{d}M_d i\gamma_5 d + \bar{e}M_e i\gamma_5 e)]H_k\,,$$

where $M_d$, $M_u$, $M_e$ are the fermion mass matrices for *d*-, *u*-type quarks and charged leptons, respectively, $v_i$ ($i=1,2$) are the vacuum expectation values of the Higgs doublets $\phi_i$, and the mixing parameters $\alpha_i^k$ and $\beta_i^k$ depend on the strength of *R-I* mixing. We assume that all these free parameters are of the same order of magnitude.

The contribution to $d_f$ coming from *R-I* mixing is easily calculated to be given by

$$d_f = Q_f \frac{eX_i m_f^3}{4\pi^2 M_0^2 v^2} I\left(\frac{m_F}{M_0}\right), \tag{1}$$

where $Q_f$ is the fermion charge and $v = (v_1^2 + v_2^2)^{1/2} = (\sqrt{2}G_F)^{-1}$. The parameter $X_i$ is the product of $\alpha_i^k$ and $\beta_i^k$ for the lightest $H_k$, denoted by $H_0$, whose mass is $M_0$, and

$$I(Y) = \left[\frac{1}{Y^2} + \frac{1}{2Y^4}\ln Y^2 + \frac{1-2Y^2}{2Y^4(1-4Y^2)^{1/2}} \times \ln\left(\frac{2Y^2}{1-2Y^2-(1-4Y^2)^{1/2}}\right)\right]. \tag{2}$$

The EDM of the neutron is given by

$$d_n = \tfrac{4}{3}d_d - \tfrac{1}{3}d_u \sim -(10^{-21}\,\mathrm{GeV}^4)\left(\frac{X}{M_0^2 v^2}\right) e\,\mathrm{cm}\,, \tag{3}$$

with $m_u \sim 4.2$ MeV, $m_d \sim 7.5$ MeV, and $X \sim X_i$. Here we have assumed that $d_n$ is given by the corresponding free-quark EDM's and the strong QCD effects have not been added.[15] We shall be generous and use Eq. (1) to $M_0 \sim 1$ GeV. Obviously, if the possible signals are interpreted as positive findings, we get

$$\frac{X}{M_0^2 v^2} \sim 2\times(10\,\mathrm{GeV})^{-4}. \tag{4}$$

Otherwise Eq. (4) becomes an upper limit.

An immediate consequence of knowing $d_n$ is the prediction of EDM for leptons, $d_l$. Now we have a one-to-one correspondence between $d_l$ and $d_n$. Explicitly, we find that $d_l$ is $1.8\times10^{-19}$, $4.4\times10^{-22}$, and $2.1\times10^{-28}$ $e$cm, respectively, for $\tau$, $\mu$, and $e$. This is in contrast to the case where $d_n$ could arise from the QCD via the strong $\theta$ parameter as mentioned before. Measurements of the EDM of the muon or electron will indicate that QCD may not be the only source of $d_n$. Furthermore, a measurement of $d_e/d_\mu \simeq (m_e/m_\mu)^3 \sim 10^{-7}$ will be a clear indication that Higgs-boson exchanges play an important role.

Another interesting test of the *R-I* mixing mechanism for CPV is given by the measurement of the longitudinal polarization asymmetry $P_L$ of the outgoing muon or an-

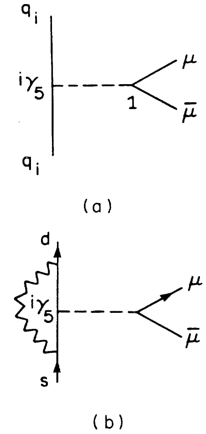


FIG. 2. Contribution to $P_L$ in (a) $\eta \rightarrow \mu\bar{\mu}$ and (b) $K_L^0 \rightarrow \mu\bar{\mu}$ decays due to scalar-pseudoscalar mixing.

timuon in the following decays:[16]

$$\eta \to \mu^+\mu^- , \tag{5a}$$

$$K_L^0 \to \mu^+\mu^- . \tag{5b}$$

The polarization $P_L$ is defined by $P_L=(N_R-N_L)/(N_R+N_L)$, where $N_R$ ($N_L$) is the number of right-handed (left-handed) $\mu^-$ in the final state.

In general the matrix element for the decays of Eq. (5) is given by

$$M = a\bar{u} i\gamma_5 v + b\bar{u}v , \tag{6}$$

where $u$ ($v$) is the usual Dirac spinor (antispinor) for the muon and $P_L$ is calculated to be

$$P_L = \frac{M_p r^2 \mathrm{Im}(ba^*)}{4\pi\Gamma} , \tag{7}$$

where $M_p$ is the mass of the decaying meson, $r=(1-4m_\mu^2/M_p^2)^{1/2}$, and $\Gamma$ is the width into muon pairs. The dominant contribution of the imaginary part in Eq. (7) is due to the two-photon intermediate state. A nonvanishing $P_L$ will come from effective CPV neutral-current interactions that contribute to $b$.

The effective interaction for the $\eta$ decay involving $R$-$I$ mixing is given by

$$\mathcal{L}(\eta \to \bar{\mu}\mu) = \frac{iG}{\sqrt{2}} \left\{ -\frac{1}{2}\bar{s}\gamma_\mu\gamma_5 s \bar{\mu}\gamma^\mu\gamma_5\mu + \frac{m_\mu X}{M_0^2}[m_d(\bar{u}i\gamma_5 u + \bar{d}i\gamma_5 d) + m_s \bar{s}i\gamma_5 s](\bar{\mu}i\gamma_5\mu + \bar{\mu}\mu) \right\} . \tag{8}$$

The first term of Eq. (8) denotes the contribution from the $Z^0$ exchange and the rest comes from spin-0 exchange with $R$-$I$ mixing (see Fig. 2). The unknown parameters in Eq. (8) are fixed by $d_n$, and we obtain

$$|P_L(\eta \to \mu^+\mu^-)| \le 1.5\times 10^{-2} . \tag{9}$$

Equation (8) also allows us to calculate the $H_0$ contribution to $\Gamma(\eta \to \bar{\mu}\mu)$ to be $5\times 10^{-6}$ eV. This is much smaller than the two-photon unitarity contribution.

In the $K_L^0$ decay depicted in Eq. (5b) there are two CPV sources. This comes from the fact that $K_L^0 = K_2 + \epsilon K_1$, where $K_2$ and $K_1$ are, respectively, odd- and even-$CP$ eigenstates. Experimentally $|\epsilon| \sim 2\times 10^{-3}$, and we shall neglect it and consider $K_2$ decays only. In the standard KM model $P_L$ arises from the usual KM phase via the induced $s$-$d$-$H$ coupling, where $H$ is the standard-model Higgs boson.[17,18] This polarization can be large if $H$ is light, for example, if its mass $M_H$ is less than 1 GeV. Explicitly one finds[17,19]

$$|P_L^{\mathrm{SM}}(K_L \to \mu^+\mu^-)| \approx 1.8\times 10^6 \frac{3g^2 m_\mu m_s m_t^2}{64\pi^2 M_H^2 M_W^2} A^2\lambda^5 \rho \sin\delta \simeq 0.001\text{-}0.86 , \tag{10}$$

where we have taken $A=1.0$ in the Wolfenstein parametrization[20] of the KM matrix and $\lambda=0.22$. The unknown parameters are $\rho$, $m_t$, $M_H$, and the KM phase $\delta$. Notice that the size of the $CP$-violation effect is also of the order of $\lambda^5$, which is characteristic of the standard-model KM mechanism. For example, the value $|P_L| \sim 0.12$ is obtained for $m_t \sim 180$ GeV, $M_H = 1$ GeV, and allowed favorable values of $\rho \sim 0.9$ and $\delta \sim 0.7$.[21] The upper bound on $P_L$ which we obtained above is less than the value given in Ref. 14 due to our use of the recent measurement of $K_L^0 \to \gamma\gamma$ to extract Im$a$.

In the $R$-$I$ mixing mechanism we are advocating, the Higgs-boson-exchange process does not change flavor at the tree level. Hence, a nonvanishing $P_L$ can only occur at the one loop level for Eq. (5b). However, there is an important difference in that it takes place at the level of $\lambda$ associating with a virtual charm-quark exchange [see Fig. 2(b)]. Explicitly, we estimate

$$|P_L^{RI}(K_L \to \mu^+\mu^-)| \approx 1.8\times 10^6 \frac{g^2 X m_s m_\mu}{16\pi^2 M_0^2} \left[ \frac{m_c^2}{M_W^2} f\left(\frac{m_c^2}{M_W^2}\right)\lambda + \frac{m_t^2}{M_W^2} f\left(\frac{m_t^2}{M_W^2}\right) A^3\lambda^5 \sin\delta \right] , \tag{11}$$

where $f(Y) \sim 1$ and the unknown parameters involving the $R$-$I$ term are again fixed by $d_n$, leaving us only the same unknown parameters of the standard model. Using the largest value of $\rho$ and $\delta$ as obtained in analyses of $\epsilon$ and $B_d^0$-$\bar{B}_d^0$ mixing[21] we find that

$$P_L \sim 0.1\text{-}0.86 . \tag{12}$$

Here the large value of $P_L$ does not require a heavy $t$ quark.

In conclusion, we have analyzed the possibility of $R$-$I$ mixing as an additional source of CPV arising from the Higgs potential of extended Higgs-boson models of electroweak interactions. A manifestation of this is a nonzero value of $d_n$. For reasonable values of the mixing parameter $X$ we find that a relatively light $H_0$ with $M_0 \sim 1$ GeV can accommodate $d_n \sim 10^{-25}$ $e$ cm. This model predicts that the EDM of charged leptons scales like (mass)$^3$. However, the neutrinos do not pick up EDM by this mechanism. Interestingly, this mechanism gives rise to large longitudinal polarization of final-state muons in both flavor-conserving and flavor-changing decays of pseudoscalar meson decays into muon pairs. These are important phenomena that deserve more careful consideration, both theoretically and experimentally.

Finally, we remark that an approved BNL experiment[22] which will improve the current limit on $d_\mu$ by a factor of 20 is not far from reaching the value predicted here. For the decay Eq. (5b), experiments at both KEK and BNL are now under way to collect $100\bar{\mu}\mu$ events. To measure $P_L$ to the level of (20–30)% would require 1000 events and this is now being actively pursued. Furthermore, measuring $P_L$ in the decay of Eq. (5a) is also under consideration.[23]

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