Akutsu, Akutsu, and Yamamoto Reply: In the preceding Comment,[1] Saam gives another derivation of the universal Gaussian curvature jump (UGCJ) of the equilibrium crystal shape (ECS).[2] His argument is correct and is important because, combined with ours, it leads to a deeper understanding of the UGCJ and associated fluctuation properties of vicinal surface. His approach relies on the correct capillary-wave description[3,4] of the rough surface which requires a relationship between interface stiffness, ECS curvature, and height-height correlation function. To give further support for this approach, we investigated the squared interface width[4,5] which is also related to the Gaussian curvature K. The capillary-wave Hamiltonian for an $L\times L$-sized system is written as

$$H=\frac{1}{2}\int\int dx^2\sum_{ij}f^{ij}\frac{\partial z}{\partial x^i}\frac{\partial z}{\partial x^j}\,, \tag{1}$$

where f^{ij}, $i,j=1,2$ is the stiffness tensor. Denoting the eigenvalues of f^{ij} by f_1 and f_2, we obtain the following asymptotic expressions for the height-height correlation function $G(x,y)=\langle[z(x,y)-z(0,0)]^2\rangle$ and the squared interface width $W^2=\langle[z(0,0)-\langle z(0,0)\rangle]^2\rangle$:

$$G(x,y)=4\eta\ln R'(x,y)\,,\ R'(x,y)=\left(\frac{x^2}{f_1}+\frac{y^2}{f_2}\right)^{1/2}; \tag{2}$$

$$W^2=2\eta\ln L\,, \tag{3}$$

with [see Eq. (4) of Ref. 2]

$$\eta=\frac{1}{4\pi}k_BT\frac{1}{(1+|\mathbf{p}|^2)[\det(f^{ij})]^{1/2}}=\frac{k_BT}{4\pi}\frac{1}{\lambda}\sqrt{K} \tag{4}$$

where $\mathbf{p}$ is the surface gradient, and the second equality is Eq. (4) of Ref. 1. Recalling that in Ref. 2, the "universal" free-fermion picture of the coarse-grained vicinal surface is essential, we calculated the left-hand sides of (2) and (3) from the free-fermion Hamiltonian to yield $\eta=1/4\pi^2$ ($|\mathbf{p}|\to 0$), in agreement with the sine-Gordon result[6] cited by Saam. This calculation not only provides a rederivation of UGCJ but also proves the universality of η implicitly assumed in Ref. 1. It also explains the fact that the sine-Gordon model becomes a free-fermion model at the facet edge ($k_c\to 0$ limit[6]).

Note that $\eta=1/4\pi^2$ holds for *any* free-fermion system,[7] irrespective of parameters in the Hamiltonian. At first sight, this is peculiar because, in Ref. 2, the crucial role is played by a relation between the fermion mass and the step stiffness. This relation originates from the one[8] between the scaled interface width σ and the interface stiffness $\gamma+\gamma''$ in *one dimension,*

$$\sigma(\theta)^{-2}=\beta[\gamma(\theta)+\gamma''(\theta)]\,, \tag{5}$$

which is the source of the universal relation for the coefficients

$$B(\theta)=\pi^2(k_BT)^2/6[\gamma_s(\theta)+\gamma_s''(0)]\,, \tag{6}$$

appearing in the vicinal-surface free-energy $f(\mathbf{p})=f(\mathbf{0})+\gamma_s(\theta)|\mathbf{p}|+B(\theta)|\mathbf{p}|^3$. Recall that the Gaussian curvature jump ΔK calculated from $f(\mathbf{p})$ involves these coefficients. Hence the requirement that ΔK be universal leads to (6), and *further* to (5). We thus see that different levels of arguments, ours and Saam's, are completely consistent: Fluctuation properties of the one-dimensional interface govern those of the two-dimensional interface, and *vice versa.*

To conclude, two derivations of UGCJ, ours and Saam's, look the same from opposite directions. At the center lies the universal free-fermion picture of the vicinal surface. As an important implication drawn from Saam's Comment, we should note that UGCJ can be tested by measuring η by scattering experiments.[9] We should also note that experimental detection of nonuniversal behavior might allow us to determine the strength of excess interactions, including those discussed by Saam, which cause the nonuniversal behavior.

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