Coherence Effects in Long-Wavelength Infrared Synchrotron Radiation Emission

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We report measurements of synchrotron radiation emission in the wavelength region from 30 to 400 μm where coherent enhancement was predicted. The power ratio relative to a blackbody source exhibits no such enhancement and can be accounted for by the incoherent emission theory. This discrepancy is explained by showing that the coherent enhancement is reduced when proper account is taken of the relative phases of all emitting particles.

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Enhancement of synchrotron radiation in the far infrared due to coherence effects was considered theoretically by Michel[1] who suggested that electron bunches in synchrotron storage rings might actually serve as models which might help in understanding the behavior of pulsars. In addition to the astronomical applications, however, such an enhancement would have a big impact on solid-state physics by providing an intense photon source in the far-infrared region from room temperature wavelengths (40–50 μm) up to 100 μm where broad-band, blackbody sources are very weak. Quantitative understanding of such coherence effects would also add new insight into the use of relativistic electrons in free electron lasers. Further, these studies may help understand questions relating to the effects of thermal radiation from the beam on the beam itself, which becomes increasingly important on larger rings.

Experimental quantitative tests for coherent emission are especially difficult because of the unusually long-wavelength region involved. To overcome diffraction effects a very large vertical aperture (≈ 100 mrad) is required for beam extraction. (10 mrad is the value typically found on vacuum ultraviolet storage rings.) Early, pioneering measurements in the far-infrared region (100–1000 μm) by Yarwood *et al.*[2] were inconclusive and suggested that such enhancement may be present.

We report here a careful quantitative study of synchrotron radiation (absolute intensity dependence on wavelength and electron current) in the region (30–400 μm) where coherence effects would be expected theoretically from the arguments of Ref. 1 applied to the National Synchrotron Light Source (NSLS) ring. Our results clearly rule out any such enhancement mechanism and can be simply explained by classical incoherent radiation theory.[3,4] We show, in addition, that if proper account is taken of the relative phases of all the emitting electrons within each bunch, coherence effects should in fact be unimportant in the region investigated but should be manifest at wavelengths approaching the bunch length at which point-diffraction and chamber-screening effects[5] would interfere with measurements.

The data were measured at the National Synchrotron Light Source, Brookhaven National Laboratory, on the 750-MeV, 50-m circumference electron storage ring. A specially constructed beam line[6] with an f10 opening aperture was used. Synchrotron light was collimated and then analyzed with a modified Nicolet 20F rapid-scan Michelson interferometer. Light was detected with a liquid-helium–cooled bolometer from Infrared Laboratories with suitable filters.

Because of difficulties in absolute calibration, the synchrotron radiation was compared to a well characterized blackbody source. A mirror arrangement allowed for a rapid and reproducible exchange of the blackbody and synchrotron sources into the 20F interferometer and detection system.

The results of the measurements are shown in Fig. 1(a) (solid and dashed lines in the main figure), where we show spectra in the region 25–400 cm^{-1} (30–400 μm). The synchrotron spectrum (curve A) was measured with 750 mA of stored current in the ring. The overall shape of the spectrum arises from the interferometer beam-splitter efficiency, the detector response, the filtering used, and the response of the optical elements (mirrors, windows). The blackbody spectrum (curve B) was measured with the Nicolet 20F standard silicon-carbide source. Beams from both sources passed through all the same optics for the input mirrors between the NSLS and the interferometer system.

A signature of coherence effects should be sought in (a) the wavelength dependence, (b) the dependence on ring parameters such as beam current and bunch length

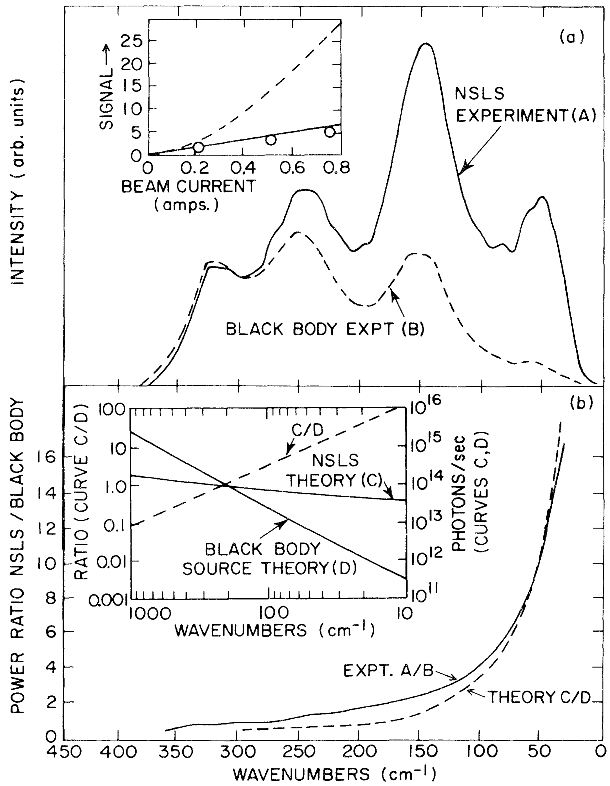

FIG. 1. (a) Measured spectra from globar (dashed curve) and synchrotron radiation source (solid line), $I=750$ mA. The vertical scale is arbitrary but the same for both sources. Inset: The dependence of signal (arbitrary scale) on beam current obtained by integration of the whole spectrum (open circles). The solid line shows a linear (incoherent) dependence and the dashed curve shows a quadratic dependence which would apply to coherent behavior. (b) The solid line in the main figure is the measured ratio (A/B) of the output from the NSLS and the globar source. The dashed line (C/D) is the ratio derived from the theoretical calculations. Inset: The two intensity calculations, for the NSLS (C) and for the blackbody (D), from which the ratio C/D is derived.

(i.e., rf, voltage, and beam energy), and (c) the absolute intensity. From Michel's calculations,[1] the intensity, if coherence effects dominate, is expected to be proportional to $\lambda^{-1/3}$ in the far-infrared region. On the other hand, classical calculations[4] neglecting coherence lead to a power dependence as follows:

$$P_{\text{synch}}(\lambda)=\text{const}\, i\theta\rho^{1/3}\lambda^{-7/3}\,, \tag{1}$$

where i is the electron current, θ is the horizontal collection angle, and ρ is the radius of curvature of the bending magnet.

For a blackbody, the emitted power is given by Planck's formula which simplifies to the Rayleigh-Jean law in the far infrared:

$$P_{bb}(\lambda)=\text{const}\,\lambda^{-4}\,, \tag{2}$$

leading to a wavelength dependence of the ratio given by

$$P_{\text{synch}}/P_{bb}\approx\lambda^{5/3} \tag{3}$$

in the long-wavelength region. The experimental ratio P_{synch}/P_{bb} (curve A/curve B), is shown in Fig. 1(b) (solid line). It is compared with the dotted line (C/D) which was derived from the individual theoretical calculations for the NSLS[4,6] (C) and globar[6] (D) shown in the inset. These calculations are for a 100-mm^2, 1000-K globar source with f2 optics (0.5 sr) and for a 3-mm^2 effective synchrotron source size with f10 optics (0.1 sr). Clearly, the frequency dependence is well reproduced and could not be accounted for if coherence effects dominated. The relative intensity is also well accounted for without involving coherence. Since its measure depends on the knowledge of physical quantities (i.e., source size and acceptance angle) which can be determined within $\approx 10\%$, the absolute intensity is determined mostly by the accuracy with which the blackbody-source intensity is calculated from its nominal temperature. Overall, the accuracy is well within a factor of 2 and sufficient to exclude any coherence effects.

Finally, the dependence on the electron current was measured by integration of the overall spectrum [Fig. 1(a)] for a range of currents between 250 and 750 mA. The results, which are shown in the inset of Fig. 1(a), clearly show good agreement with the solid line depicting a linear dependence. Agreement with the dashed line showing a quadratic dependence (expected from coherent behavior) can clearly be ruled out. Small discrepancies of order 10% are attributable to known changes in emittance with current.[7]

The lack of observation of coherence effects that were originally predicted[1] for the frequency region investigated warrants a reexamination of the assumptions made in those theoretical considerations. Notably, Michel assumes a cubic bunch (i.e., isotropic) of uniform electron density. For wavelengths less than the bunch length, the enhancement is proportional to the number of electrons in a cube of side λ. In electron storage rings such as the NSLS at Brookhaven, the circulating electrons are confined to bunches typically 30 cm long by 0.05 cm wide and distributed in a Gaussian fashion.

In our calculation,[8] the emitted intensity is calculated from the total electric field evaluated exactly by summing the contribution from each electron, paying attention to the relative phases. Using classical electrodynamics and the notation of Jackson,[9] we can write that the intensity of the radiation emitted from a collection of N particles (electrons) of charge e, into a solid angle Ω, at a frequency ω, is given by

$$\frac{d^2I}{d\omega\, d\Omega}=\frac{e^2\omega^2}{4\pi^2c}\left|\sum_{j=1}^{N}e^{i\omega\mathbf{n}\cdot\mathbf{r}_j/\beta_j c}\right|^2\left|\int_{-\infty}^{\infty}\mathbf{n}\times(\mathbf{n}\times\boldsymbol{\beta}_j)e^{i\omega[t-\mathbf{n}\cdot\mathbf{r}(t)/c]}\,dt\right|^2, \tag{4}$$

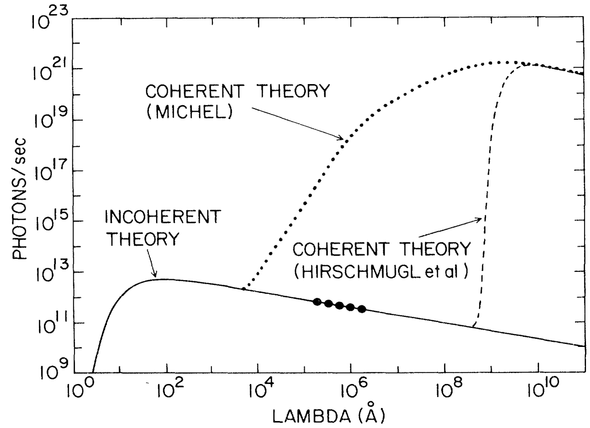

FIG. 2. Radiation output spectra from the NSLS. The solid line is the classical incoherent calculated spectrum, the dashed line is the coherent enhancement described in Ref. 1, and the dotted line is the enhancement described in Refs. 5 and 6. The data are shown as solid circles.

where β_j is the ratio of the jth particle velocity to the velocity of light and r_j is the position of the jth particle. This is equivalent to the formula for a single particle except for the summation term. This sum is not a continuous function because of statistical variations in the particle distribution function and thus cannot be converted into an integral. We have solved this expression both analytically, using the formalism of Nodvick and Saxon,[5] and by Monte Carlo methods to verify the approximations involved in the analytical technique. Analytically,[5] Eq. (4) can be rewritten in the following form:

$$\frac{d^2I}{d\omega\, d\Omega} = [N + N(N-1)f(\omega)]P(\omega)\,, \tag{5}$$

where $P(\omega)$ is the power radiated by a single electron and $f(\omega)$ is a form factor given by

$$f(\omega) = \left| \int_{-\infty}^{\infty} e^{i\omega \mathbf{n}\cdot\mathbf{r}/c} S(r)\, dr \right|^2 , \tag{6}$$

where $S(r)$ is a single-particle distribution function. The form of Eq. (5) allows us to see on quick inspection that in the incoherent limit $[f(\omega)=0]$ the total power from N electrons is simply N times the result for a single electron whereas in the coherent limit $[f(\omega)=1]$ the total power is N^2 times the result for a single electron. Equation (6) yields the important general result that $f(\omega)$ (which determines the coherence) is the Fourier transform of $S(r)$. Thus for a Gaussian distribution of particles within a circulating bunch, the coherent enhancement of the emission is itself Gaussian and of the form $e^{-2\pi^2\sigma^2/\lambda^2}$, where σ is the Gaussian parameter describing the bunch length. Therefore the coherence effects will be negligible if σ is small compared to λ. Indeed it is the bunch length, and not the smaller bunch width or height, that establishes the wavelength at which coherence effects become important. The results of these analyses are shown in Fig. 2.

In summary, our data conclusively rule out coherence effects in the 30–400-μm region. By taking proper account of the phase correlation between adjacent particles in our calculations, we find that no such coherence effects should be detectable for bunch lengths of 30 cm. However, coherence effects should be observed at much shorter wavelengths at the Advanced Light Source at Berkeley, for example, where the bunch length is almost 2 orders of magnitude shorter.

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[1]F. Curtis Michel, Phys. Rev. Lett. **48**, 580 (1982).

[2]J. Yarwood, T. Shuttleworth, J. B. Hasted, and T. Namba, Nature (London) **312**, 742 (1984).

[3]Julian Schwinger, Phys. Rev. **75**, 1912 (1949).

[4]W. D. Duncan and G. P. Williams, Appl. Opt. **22**, 2914 (1983).

[5]J. S. Nodvick and D. S. Saxon, Phys. Rev. **96**, 180 (1954).

[6]Gwyn P. Williams, Int. J. Infrared Millimeter Waves **5**, 829 (1984).

[7]J. Galayda, Nucl. Instrum. Methods Phys. Res., Sect. A **239**, 106 (1985).

[8]C. J. Hirschmugl, M. J. Sagurton, and G. P. Williams, unpublished.

[9]J. D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1975).