Photon Accelerator

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A novel method of frequency upshifting short (≤ 1 ps) pulses of laser light, which makes use of relativistic plasma waves, is described. This method makes use of the fact that a laser pulse moving in a plasma can be thought of as a packet of photons, each possessing an effective mass of $m_\gamma = \hbar\omega_{pe}/c^2$ and moving with the group velocity of the laser pulse. These photons experience a force acting on them when in the presence of a gradient in the plasma density. By using a relativistic plasma wave (i.e., a moving density gradient) traveling with the photons, the energy of the photons (thus the frequency) can be continuously increased.

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Two recent particle accelerator schemes, the plasma beat-wave accelerator (PBWA)[1] and the plasma wakefield accelerator (PWFA),[2] employ relativistic plasma waves[3] to generate large electric field gradients (~ 1 GeV/cm)[4] for the acceleration of charged particles. A third method (which is actually the proposed method in the original beat-wave article by Tajima and Dawson,[1] and will be implicitly included in our references to the PBWA) relies on a short ($\leq \lambda_p/2$) powerful laser pulse to create the plasma wave. Each scheme relies on using a finite-length, electrical disturbance (in the PBWA, an intense laser pulse; in the PWFA, a relativistic electron beam) propagating through a plasma to set up plasma waves that have a phase velocity of nearly the speed of light in vacuum. Once generated, one then injects a trailing bunch of electrons[2,5] into accelerating phase of the plasma wave, so that energy can be transferred from the wave to the trailing electrons (this is referred to as "beam loading"). In this Letter, we report on the results of replacing this trailing bunch of electrons with a short (less than $\frac{1}{2}$ plasma wavelength) pulse of electromagnetic (EM) radiation. It will be shown that the frequency of the radiation is continuously upshifted, in analogy with the energy gain of the trailing bunch of electrons in the original concept of plasma accelerators.

The idea of "loading" the plasma wave with a laser pulse can be explained conceptually as follows. A light pulse traveling through a plasma leaves behind it a "wake," or density perturbation, of amplitude $\delta n/n \approx e \times E/m_e\omega_p c$. Now consider a second, identical, pulse placed $1\frac{1}{2}$ plasma wavelengths behind the first pulse. This second pulse will create an identical wake that is 180° out of phase with the first wake. The superposition of the two wake fields behind the second pulse results in a lowering of the amplitude of the plasma wave. It is clear that the second laser pulse has absorbed some fraction of the energy stored in the wake created by the first laser pulse. Figure 1 shows the result of a 1D, particle-in-cell (PIC) computer simulation of this loading. Assuming photon conservation, this increase in energy implies that the frequency of the second pulse can be upshifted. This follows from the fact that the energy of the pulse is $U = N\hbar\omega$, N being the total number of photons in the packet.

We now obtain an estimate on the rate of frequency upshift possible in the presence of a relativistic Langmuir wave, of amplitude δn in an otherwise homogeneous plasma with density n_0, that moves with the velocity of the driver (v_p), in the positive x direction. The plasma frequency associated with this density perturbation can therefore be written as

$$\omega_p^2(x,t) = \omega_{p0}^2\{1 - \epsilon \sin[k_p(x - v_p t)]\}\ , \tag{1}$$

where $\epsilon = \delta n/n_0$, $\omega_{p0}^2 = 4\pi n_0 e^2/m_e$, and $k_p = 2\pi/\lambda_p$. We now assume that a laser pulse, of width $\Delta x \leq \lambda_p/2$, and initial frequency $\omega > \omega_p$, has been injected into the accelerating phase of the wave. [The group velocity of the

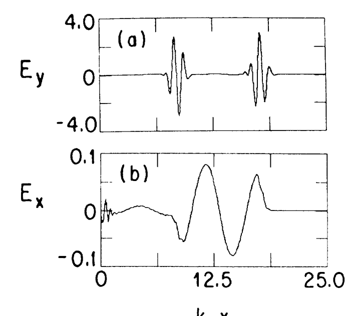

FIG. 1. Superposition of wakes from two identical laser pulses spaced $1\frac{1}{2}$ plasma wavelengths apart, shown at $t = 15.0 \times \omega_p^{-1}$. (a) Electric field components of the laser pulses. (b) Longitudinal electric field showing partial absorption of plasma-wave energy by the second pulse. Both laser pulses have frequencies of $\omega = 6\omega_p$ and were injected (with a delay between pulses) into a homogeneous plasma from the left-hand side of the simulation box.

pulse is $v_g = c(1-\omega_p^2/\omega^2)^{1/2}$.] The plasma dispersion relation for the EM wave in the plasma is

$$\omega^2 = \omega_p^2(x,t) + c^2k^2 . \tag{2}$$

The variation of this is simply

$$2\omega\delta\omega - 2c^2k\delta k = \delta\omega_p^2 . \tag{3}$$

If we transform into the plasma-wave frame (i.e., the primed frame) we obtain $\omega' = \gamma_p(\omega - v_p k)$. Using this, and the fact that in the wave frame ω' is constant, we can rewrite Eq. (3) as

$$\delta\omega_p^2 = 2\omega\delta\omega(1 - c^2k/\omega v_p) . \tag{4}$$

In the laboratory frame, the variation of $\delta\omega_p^2$ is

$$\delta\omega_p^2 = \frac{\partial\omega_p^2}{\partial x}(\delta x - v_p\delta t) , \tag{5}$$

where ω_p^2 is assumed to be a function of $x - v_p t$. Using Eq. (1) and the fact that $\delta x/\delta t$ is the velocity of the pulse (v_g), Eqs. (4) and (5) can be combined to give

$$\delta\omega/\delta x = \omega_{pe}^2 \epsilon k_p / 2\omega . \tag{6}$$

It is found that this increase in frequency of the accelerated laser packet exactly accounts for the loss in the energy of the accelerating plasma wave.

For the laser packet to gain energy as given by Eq. (6) requires the photons to feel the same gradient for the entire time they are accelerated. However, since the accelerating wave has a velocity $v_p < c$ (similar to the way the accelerating wave in a linac must be less than c) which we take to be constant, we see that as the photons accelerate, they phase slip with respect to the wave, and cannot continue to gain energy indefinitely. To estimate the limit on the frequency upshift possible using PWFA- or PBWA-generated Langmuir waves in a homogeneous plasma due to this phase-slippage effect, we now make a Lorentz transformation to a frame moving with the phase velocity of the plasma wave. Since the dispersion relation for an electromagnetic wave in a plasma is invariant under a Lorentz transformation, it is obvious that by going to this frame

$$c^2k'^2 = \omega'^2 - \omega_{p0}^2[1 - \epsilon\sin(k_p'x')] , \tag{7}$$

where primes denote moving-frame quantities. In order to find the maximum frequency upshift of the laser pulse, we note that if the pulse is injected into the trough of the plasma wave at $t=0$, then the maximum increase in k' will be when the pulse is just turned around at the peak density of the wave, and then travels back until it reaches the trough once more. The point at which it will be turned around (the peak of the density) is also the point where $k'=0$, or equivalently $\omega' = \omega_{p0}(1+\epsilon)^{1/2}$. This will necessarily be the frequency of the pulse in the moving frame for all times because nothing in the wave frame is changing in time, except the position in the wave. From these considerations and Eq. (7), we find that the initial wave number in the moving frame is given by $k_i' = -(\omega_{p0}/c)(2\epsilon)^{1/2}$. In the photon picture, this is equivalent to the photons initially traveling backwards in the wave frame. They are gradually accelerated as they slip back in the wave, until they are reflected at the peak, and when they reach the bottom of the trough, the photon momentum has simply changed sign to $k_f' = +(\omega_{p0}/c)(2\epsilon)^{1/2}$. By Lorentz transforming the frequency back to the laboratory frame, we find that this change in k' gives a new frequency of

$$\omega_f \approx \omega_i[1 + 2(2\epsilon)^{1/2} + 4\epsilon + O(\epsilon^{3/2})] , \tag{8}$$

where it has been assumed that $(1+\epsilon)^{1/2} \approx 1 + \epsilon/2$ and $\beta_\phi \approx 1$ (i.e., the plasma-wave phase velocity is roughly c). Note that this maximum final frequency can be achieved only if the wave phase velocity is chosen such that

$$\gamma_{\text{wave}} \le (\omega_i/\omega_p)[(1+\epsilon)^{1/2} - (2\epsilon)^{1/2}]^{-1} .$$

This can be found from the above condition on ω' at the turning point and the Lorentz transformation $\omega_i = \gamma_{\text{wave}}(\omega_i' + \beta k_i')$.

For realistic density perturbations ($\delta n/n_0 \sim 0.1$–0.5), Eq. (8) predicts no more than about a factor of 5 increase in frequency. However, this restriction simply arises from the dephasing of the accelerated photons. One possible solution to this is to continuously change the phase velocity of the plasma wave as the photons are accelerated so that they remain at the same phase, in the same manner that wigglers are tapered to keep the electrons in the deaccelerating phase for a longer distance. A ramped plasma density can accomplish this, provided that the density-gradient scale length is roughly[6]

$$L_n = \left(\frac{1}{n}\frac{dn}{dx}\right)^{-1} \approx m\lambda_p\gamma_{\text{wave}}^2\frac{1+\sqrt{\epsilon}}{\epsilon} , \tag{9}$$

where m is the number of wavelengths behind the back of the driver (in this case, a relativistic electron beam with peak density ϵ and energy γ_{wave}) the packet sits. (The fact that driver is slowing down, and ϵ is decreasing, is neglected in this estimate.) If density ramps such as this could be produced, continuous upshifts at the rate given by Eq. (6) would result, allowing for upshifts of factors of 10 or more. However, various limiting factors other than phase slippage become important before these wavelengths can be achieved. We will examine some of these limiting factors, as they apply to two examples discussed below. It is found that frequency upshifts of from 2–10 require only modest extrapolation of the recent experimental achievements in coherent plasma-wave generation.[7]

Without ramping the density we would expect slightly less upshift than that predicted by Eq. (8) because we have assumed that the pulse reflects off of the peak of the density, and then propagates back down the wave (in the wave frame). However, due to the finite extent of the laser packet, the "turning point" must be chosen sufficiently far from the peak of the density so that tunneling of the packet out the back of the wave cannot occur. (This is analogous to the quantum-mechanical tunneling of a particle through a finite-height potential well.) It is found that as long as the back of the pulse is half of a collisionless skin depth ($\sim c/\omega_p$), in the wave frame, from the density maximum, leakage of the pulse out the back of the wave should not be a problem. However, another effect continuously acts on the pulse that makes tunneling more likely to occur for long propagation distances. This effect is the well-known spreading of a wave packet in a dispersive medium.[8] This spreading can be minimized by choosing a sufficiently large number of wavelengths in the packet; however, the upshift is more dramatic (i.e., occurs in a shorter distance) for cases where the packet is only ~10–50 (laser) wavelengths long. Therefore, spreading due to plasma dispersion can be a limiting factor in acceleration length. The characteristic distance over which this spreading occurs (in the absence of a density gradient) is usually given as $L_{\rm disp} \sim 2\omega^3 L_{\rm pulse}^2/\omega_{p0}^2 c$.[8] The final limitation we choose to address here is the diffraction length, which is a measure of how far the pulse can propagate in vacuum before the diameter of the beam becomes unacceptably wide. For the simple case of a laser pulse focused into a plasma in the absence of a plasma wave, this length is roughly given by $L_{\rm diff} \sim \pi r_0^2/\lambda$, where r_0 is the laser spot size at the focus. We actually expect this limitation to be less restrictive when the plasma wave is present, due to a region of both focusing and accelerating in the plasma wave.[2,3]

As an example of this upshift, Eq. (6) is plotted in Fig. 2 for two different lasers. The initial laser pulses come from KrF ($\lambda = 0.26$ μm) and a Nd^{3+}:glass, or yttrium-aluminum-garnet ($\lambda = 1.06$ μm) lasers that are injected into a plasma with density $n_0 = 10^{18}$ cm^3 containing a 30% plasma wave ($\epsilon = 0.3$). Figure 2 shows the amount of wavelength decrease (corresponding to frequency upshift) that can be expected if the plasma density is tailored according to Eq. (9). (This corresponded to a 2% density increase in the case of KrF, and a 28% increase for Nd:glass.) The pulse lengths of both were chosen to be $\lambda_p/4 = 1.4$ μm. The characteristic length for dispersion to becomes a problem is $L_{\rm disp} \approx 5$ m for KrF, and $L_{\rm disp} \approx 0.9$ m for Nd:glass. Diffraction of the pulses becomes a problem when $L_{\rm diff} \approx 3$ m for KrF, and $L_{\rm diff} \approx 0.74$ m for Nd:glass (where a spot size radius of 5 mm was used). As mentioned aboved, 2D effects should relax this constraint. Notice that as the distance the pulse travels through the plasma approaches ~1 m, the final wavelength becomes essentially independent of the initial wavelength.

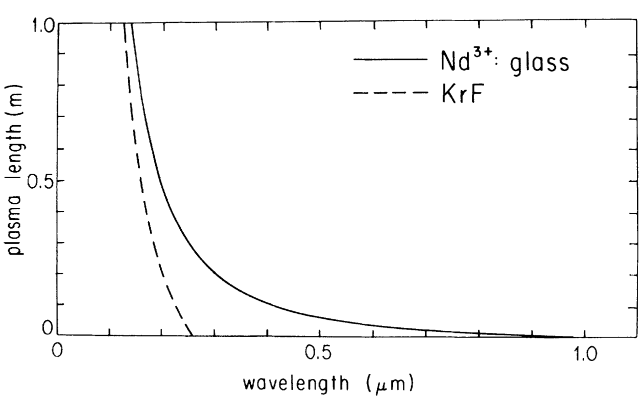

FIG. 2. Amount of upshift possible for two different types of lasers (KrF and Nd:glass) assuming appropriately ramped densities.

Computer simulations have been performed with the plasma simulation codes WAVE[9] and ISIS, to test these predictions. Figure 3 shows the results of a 1- and $\frac{2}{2}$-dimensional, fully relativistic, electromagnetic PIC simulation designed to show frequency upshift. The plasma wave was set up by a wake-field driver[2] with initial $\gamma = 22$. The density perturbation, $\epsilon \sim 0.25$, set up by this driver produced a wake into which was injected a laser pulse of width $L_{\rm pulse} \approx 2.5c/\omega_p$, and initial frequency $\omega_i = 18\omega_p$. After a distance of $237c/\omega_p$, we found that there was an upshift of 10% in the laser pulse, or a final frequency of $\approx 19.8\omega_p$. Equation (6) predicts an upshift to $\approx 19.5\omega_p$. This slight difference is attributed to the fact that even at density perturbations of 25%, the wake shows nonlinear wave steepening. This implies a larger electric field (or equivalently, a larger density gradient) than expected for a density perturbation of 25%, which the simple model above assumed to be linear. Thus, this nonlinearity would cause the rate at which the pulse is upshifted to be slightly higher than what the linear theory predicts.

Because of the fact that our simulation program differenced the electromagnetic field using the leap-frog method, numerical dispersion caused some spreading (in real space) of the pulse. The effect of this spreading was

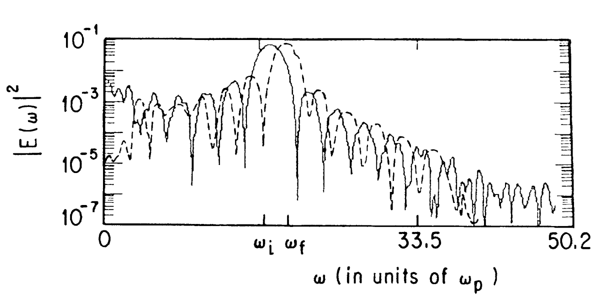

FIG. 3. Frequency spectrum of laser packet showing frequency upshift. The initial pulse had a frequency of $\omega_i = 18\omega_p$ (solid line), and the final pulse (dotted line) has a frequency of $\omega_f \approx 19.8\omega_p$.

"delocalization" of the wave packet in space, decreasing the efficiency of the upshift somewhat. This effect was minimized by choosing $c\Delta t/\Delta x$ as close to the light cone as possible.[10] A rough estimate for the amount of spreading, analogous to the spreading due to real linear dispersion, yields a characteristic length of $L_{\rm num} \sim 4 \times L_{\rm pulse}^2/c^2 k \Delta t^2$. For our simulation parameters, the real dispersion length was $L_{\rm disp} \approx 73\,000 c/\omega_p$, whereas for the numerical dispersion $L_{\rm num} \approx 680 c/\omega_p$. Thus, the numerical spreading was already causing our pulse to spread into the deaccelerating regions of the plasma wave, even for our relatively short propagation distances.

Insight into this peculiar frequency upshifting can be gained by first considering the more common wave-wave interaction occurring in the Raman effect. In the form most closely associated with our model, consider a high-frequency laser pulse ($\lambda_{\rm laser} \ll \lambda_p$), which is many plasma wavelengths long, injected into a relativistic plasma wave. After allowing for sufficient time for these waves to interact, we find that the frequency of the laser is no longer a single peak at ω_i, but has a series of peaks spaced at intervals of ω_p both above and below the original frequency. The analogous case for the electron beam would be to inject a dc electron beam into a similar plasma wave and see the beam bunch up around the stable points of the accelerating wave; some gaining energy, others losing energy, depending on the initial phase of the wave at the points where they were injected. The main difference between the electron and the photon bunching is that each electron's spatial extent is so small, relative to the wavelength of the bunching wave, that it is essentially a point particle in the accelerating wave. The photons, on the other hand, are some fraction of the accelerating wavelength. Once they span more than one plasma wavelength, wave-wave interactions dominate, and they gain (and lose) energy in quanta of the plasma-wave energy. Once we localize the photons to a small fraction of the accelerating wave, they gain energy continuously. In this regime the laser packet is no longer sampling many plasma wavelengths, and therefore cannot contribute to a wave-wave interaction.

Both the estimate of the acceleration gradient and the computer simulations have been based on using a relativistic electron plasma wave as the mechanism for accelerating the photons. However, the fundamental cause of acceleration is just the interaction of a very localized packet of photons with a density gradient in the plasma for an extended period of time. This requires that the density gradient move with $v \approx c$; a requirement that plasma waves conveniently satisfy. Therefore, although the use of Langmuir waves for photon acceleration has been discussed in this Letter, any density gradient satisfying the above requirements should produce frequency upshift.

In conclusion, we have presented a novel method of continuously upshifting short ($L_{\rm pulse} \lesssim \lambda_p/2$) pulses of electromagnetic radiation by use of relativistic electron plasma waves. By choosing the length of time the packet interacts with the plasma wave, any frequency between the initial frequency and the final frequency [Eq. (8), for a homogeneous plasma] can be obtained. It is found that if the plasma density profile satisfies Eq. (9), the frequency can be increased by almost a factor of 10. PIC simulations were presented and demonstrated this effect.

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