Nambu–Jona-Lasinio Model and Charge Independence

E. M. Henley and G. Krein[a]
Institute for Nuclear Theory, Department of Physics, FM-15, University of Washington, Seattle, Washington 98195
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For different up and down current quark masses, charge independence follows in the Nambu–Jona-Lasinio model from chiral-symmetry breaking; however, chiral symmetry is restored at high densities. The dependence of the constituent quark masses and the neutron-proton mass difference on the density are examined. The effect of the up-down-quark mass difference on the neutron-proton mass difference is large and in the right direction to explain the Nolen-Schiffer anomaly.

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QCD is thought to be the basic theory of hadronic forces. Within QCD, the current quark masses (m) of the up and down quarks are quite different[1] and their ratio is about $\frac{1}{2}$. The masses are of the order of $m_u \approx 5$ MeV and $m_d \approx 10$ MeV.[1] Within QCD, a number of arguments have been given as to why charge independence should be a good symmetry for hadronic forces: It is generally agreed that the QCD scale is of the order of several hundred MeV and that, at this scale, a mass difference of the order of 5 MeV is a perturbation. The constituent quark mass (M) is ~ 330 MeV, and if $M_d - M_u \approx 5$ MeV, then charge independence should hold to about 1%. Because the long-range confining force is flavor independent, the fact that $M_d - M_u \approx m_d - m_u$ might be expected.[2] However, many of these arguments are unsupported by detailed calculations. In this Letter, we examine charge independence quantitatively within the Nambu–Jona-Lasinio (NJL) model.[3]

The origin of the current quark masses is not known; they are thought to be caused by a Higgs mechanism or spontaneous symmetry breaking. They cannot be predicted, but can be deduced from mass differences of strange baryons and meson decays.[1] The QCD Lagrangian is chirally symmetric, as is that of the NJL model. The latter is a nonlinear field theory, which has received considerable recent attention because it allows one to examine chiral symmetry and its breaking as observed in low-energy hadronic phenomena, and to study the effects of chiral-symmetry restoration at high density or temperature.[4]

In the NJL model with zero-mass fermions (current quarks) chiral symmetry is dynamically broken and the fermions acquire a (constituent quark) mass M. We study this model for three colors and two flavors, up and down (u and d), of different nonzero current quark masses m_u and m_d. In this model a small chiral-symmetry breaking is thus introduced in the Lagrangian density[3]

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi + G[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\tau\psi)^2]\,, \tag{1}$$

where m is a diagonal matrix in flavor space with elements m_u and m_d and we include only scalar and pseudoscalar couplings. Vector and axial-vector couplings can be obtained by a Fierz transformation.[3]

In this case of two flavors of different masses and three colors, the NJL model yields two coupled equations for the constituent quark masses,[3]

$$M_u = m_u + 3M_u I(M_u) + \tfrac{7}{2} M_d I(M_d)\,, \tag{2a}$$

$$M_d = m_d + 3M_d I(M_d) + \tfrac{7}{2} M_u I(M_u)\,, \tag{2b}$$

where

$$I(M) = \frac{2G}{\pi^2}\int_{k_F}^{\Lambda} dk\, k^2/(k^2+M^2)^{1/2}\,. \tag{2c}$$

In Eq. (2c), Λ is a noncovariant cutoff (appropriate for this case of a non-Lorentz-invariant Fermi sea) and k_F is the Fermi momentum related to the quark denstiy ρ_q by

$$\rho_q = 2k_F^3/\pi^2\,. \tag{3}$$

It is easily seen from Eq. (2c) that as $k_F \to \Lambda$, the effects of dynamical chiral-symmetry breaking vanish and $M \to m$.

In order to study the density dependence of the nucleon mass we adopt the model of Isgur and collaborators,[5] which has been very successful in describing the mass splittings of baryons and mesons. In this model the proton-neutron mass difference, $\Delta M_{pn} \equiv M_p - M_n$, can be expressed in terms of the mass difference $\delta M = M_d - M_u$ and of the average mass $M = \frac{1}{2}(M_d + M_u)$ as follows (for details see Ref. 5):

$$\Delta M_{pn} = (-1 + AM^{-3/2} + BM^{-9/4} + 0.08)\delta M + CM^{1/4} - DM^{-5/4}\,, \tag{4}$$

where the various terms have the following origin: $A = (3K/4)^{1/2}$, where K is the spring tension for quarks confined by harmonic forces; it comes from the zero-point energy shift. $B = 5A^{3/2}\alpha_s/9\sqrt{\pi}$ is due to the color-hyperfine interaction, where α_s is the strong coupling constant. $C = 2A^{1/2}\alpha_{em}/3\sqrt{\pi}$ and $D = AC/3$, where α_{em} is the electromagnetic fine-structure constant, come from the electric and magnetic energy differences, respectively. The last term in the parentheses in Eq. (4) is a second-order color-hyperfine effect and the first one has an obvious origin.

With $A \simeq (199.4\ \text{MeV})^{3/2}$, $\alpha_s \simeq 1.63$, $M = 0.33$ GeV, and $\delta M = 6$ MeV, Isgur obtains an excellent fit to the observed baryon isomultiplet splittings. Here we fix the parameter G of the NJL model for a cutoff of $\Lambda = 800$ MeV and for $m_u = 4.0$ MeV and $m_d \simeq 10$ MeV such that $M \simeq 333$ MeV. This is obtained for $G\Lambda^2 \simeq 1.85$.

The variation of quark mass with density is considered explicitly in Eq. (4). However, we have assumed $\alpha_s = \text{const}$ with density, and it can be argued that the hyperfine interaction, in particular, is sensitive to this assumption. We note, first, that the hyperfine interaction term B in Eq. (4) is a small part of δM. Moreover, we have examined the effect of a decrease of α_s with ρ under two extreme assumptions: (a) a linear dependence for which $\alpha_s = \alpha_s(0)(1 - \rho/5\rho_0)$, where ρ_0 is normal nuclear-matter density, and (b) $\alpha_s \propto M^{9/4}$, so that the B term is independent of mass and the pion mass remains small for all M. Even then, the effect is negligibly small for our numerical results, but is included in one of our figures. Indeed, the more rapid decrease of α_s with density gives results in still better agreement with the experimental data. The electromagnetic coupling is independent of density and the confinement parameter K is expected to be much less sensitive to density than α_s.

In Fig. 1 we show M and δM as a function of ρ. In Fig. 2 we show the average nucleon mass, $M_N = \frac{1}{2} \times (M_p + M_n)$, and the mass difference ΔM_{pn} as a function of the nuclear density (which we define as being equal to one-third of the quark density). Comparing the two curves of Figs. 1 and 2 we can see that, although the density effect is small on the average quark and nucleon masses as well as on δM, it is a major effect on the neutron-proton mass difference.

This NJL model's effect on the proton-neutron mass difference is in the right direction and order of magnitude to explain the Nolen-Schiffer anomaly.[6,7] The mass difference of mirror states (Z,N) and $(Z'=N, N'=Z)$ with $Z = (A+1)/2$, $N = (A-1)/2$ can be written as

$$M(Z,N) - M(Z',N') = \Delta M_{\text{em}} + \Delta M_{pn} , \tag{5}$$

where ΔM_{em} is the difference of electromagnetic self-energies. The experimental mass differences are larger than the calculated ones. Clearly, as ΔM_{pn} increases, the discrepancy decreases. As examples, we have examined the cases of $A = 16$ and 40. We define the average density as that of a sphere of uniform density with the same root-mean-square (rms) radius as that obtained with harmonic-oscillator wave functions with several nodes

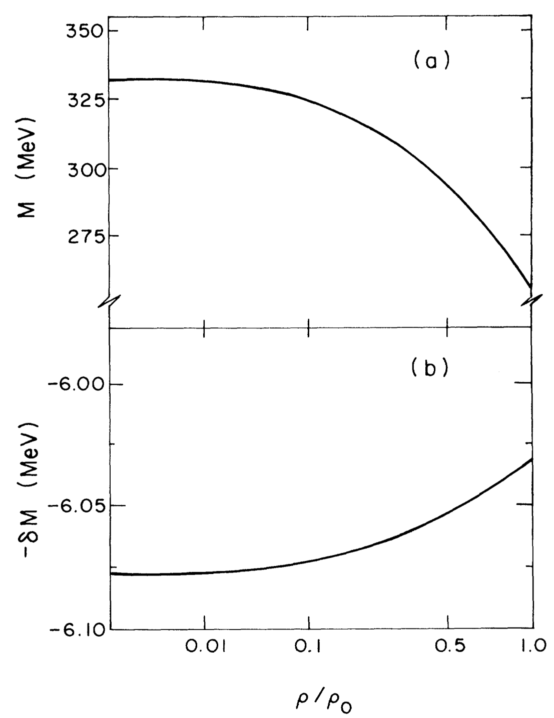

FIG. 1. (a) The average quark mass $M = \frac{1}{2}(M_n + M_d)$ and (b) the mass difference $-\delta M = M_u - M_d$ as functions of the ratio of the average nuclear density to that of nuclear matter, $\rho_0 = 0.17\ \text{fm}^{-3}$.

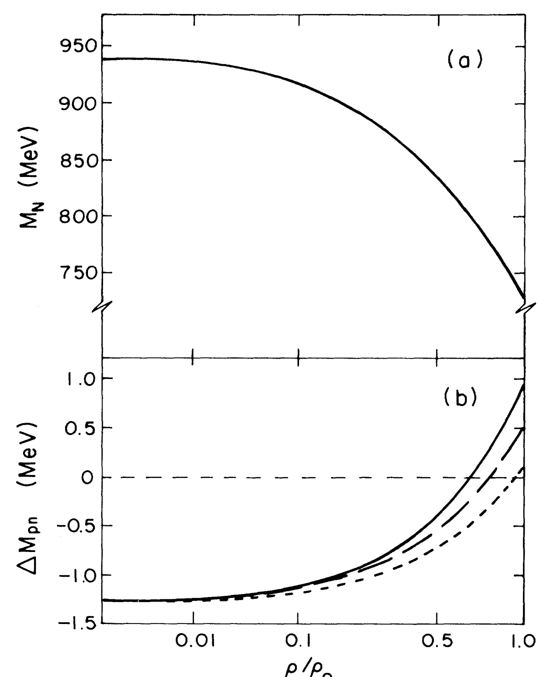

FIG. 2. (a) The average nucleon mass $M_N = \frac{1}{2}(M_p + M_u)$ and (b) the mass difference $\Delta M_{pn} = M_p - M_n$ as functions of the ratio of the average nuclear density to that of nuclear matter, $\rho_0 = 0.17\ \text{fm}^{-3}$. The long-dashed curve shows the effect of a linear variation of α_s with density, and the short-dashed curve is for $\alpha_s \propto M^{9/4}$ (see text).

TABLE I. Discrepancies and ΔM_{pn} for nuclei with masses in the regions of $A=16$ and $A=40$. ρ_{av} is the average density defined in the text.

Nuclei	Discrepancy with $\Delta M_{pn}=-1.3$ MeV	ΔM_{pn} at ρ_{av}	Discrepancy with NJL correction
$A=16$, $\rho_{av} \simeq 0.09$ fm^{-3}	0.3	−0.4	−0.6
$A=40$, $\rho_{av} \simeq 0.10$ fm^{-3}	0.7	−0.12	−0.5

which fit the experimental rms radii. The first column of Table I presents a typical value of the discrepancy between experiment and theory for the nuclear-mass difference between mirror nuclei.[7] The second column shows ΔM_{pn}; the third one lists the effect predicted after correction for the NJL-computed neutron-proton mass difference. The effect is not only in the right direction to remove the anomaly, but also of the right order of magnitude. We believe that the overestimate of the NJL calculation stems from several sources: (1) We have not included kinetic-energy corrections due to ΔM_{pn}.[7] (2) Our relation of the nuclear density to the quark density and our use and definition of average density are only approximate. (3) The NJL model does not incorporate quark confinement; it has been shown by Henley and Muether[8] that the incorporation of confinement slows the mass changes with density, which would help agreement. We believe that the important point is not the numerically calculated correction, but the direction and order of magnitude.

In conclusion, we have shown that approximate charge independence follows in the NJL model as a result of chiral-symmetry breaking. In addition, we have used the model to calculate the effect of the up- and down-current-quark mass difference on the masses of the constituent quarks and of the nucleons. The effect of increasing nuclear density on the neutron-proton mass difference is dramatic—even the sign reverses. We have shown that this difference is of the right sign and order of magnitude to explain the Nolen-Schiffer anomaly. As also argued by Brown *et al.*,[9] we believe that the NJL model effects should be taken seriously in nuclear physics. We intend to apply the mechanism to other charge-independence-breaking phenomena. Also, a proper QCD calculation of the quark-mass generation[10] is being undertaken.

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[a]On leave from Departamento de Física, Universidade Federal de Santa Maria, Rio Grande do Sul, Brazil.

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