Competition between Pairing-Assisted Tunneling and Single-Particle Alignment in the Decay of High-K Isomeric States

T. Bengtsson,[(1)] R. A. Broglia,[(2)] E. Vigezzi,[(2)] F. Barranco,[(3)] F. Dönau,[(4)] and Jing-ye Zhang[(5)]

[(1)]*Department of Mathematical Physics, Lund Institute of Technology, Lund, Sweden*

[(2)]*Dipartimento di Fisica, Università di Milano, Milano, Italy, Istituto Nazionale di Fisica Nucleare, Sezione di Milano, Milano, Italy, and The Niels Bohr Institute, University of Copenhagen, DK-2100 Copenhagen, Denmark*

[(3)]*Escuela Superior de Ingenieros Industriales, Avenue da Reina Mercedes, Sevilla, Spain*

[(4)]*Zentralinstitut für Kernforschung, Rossendorf, 8051 Dresden, PSF 19, East Germany*

[(5)]*Joint Institute of Heavy Ion Research, Holifield Heavy Ion Research Faculty, Oak Ridge, Tennessee 37831*

(Received 25 October 1988)

In an axially symmetric, strongly rotating quadrupole-deformed nucleus, the projection K of the angular momentum along the symmetry axis is a partially conserved quantity. A K isomer ($K=25$) at low excitation energy has been observed in ^{182}Os which decays into the ground-state band ($K=0$). The associated electromagnetic lifetime at spin $I \approx 25$ can be interpreted in terms of tunneling of a system displaying strong pairing fluctuations across a barrier which separates configurations in which the nucleus rotates about the summetry axis and about an axis perpendicular to it. A hindrance factor $\approx 10^{-6}$–10^{-9} for the process is calculated, in overall agreement with the data.

PACS numbers: 21.60.Ev, 21.10.Re, 23.20.Ck, 27.70.+q

The occurrence of collective rotational degrees of freedom may be said to originate in a breaking of rotational invariance which introduces a deformation that makes it possible to specify an orientation of the system. Rotation represents the collective mode associated with such a spontaneous symmetry breaking known as the Goldstone boson.

The orientation of a body in three-dimensional space involves three angular variables, such as the Euler angles, and three quantum numbers are needed in order to specify the state of motion. The total angular momentum I and its component $M=I_z$ on a space-fixed axis provide two of these quantum numbers; the third may be obtained by considering the component of I with respect to an intrinsic, body-fixed, coordinate system, usually denoted $K=I_3$. As a commuting set of angular momentum variables, one may thus choose I^2, I_z, and I_3. The range of values that the K quantum number can take is the same as that associated with the M quantum number.

There is overwhelming experimental evidence that cold, deformed nuclei display axially symmetric quadrupole deformations, and are invariant with respect to a rotation of 180° about an axis perpendicular to the symmetry axis. It can also be inferred from the observed spectra that the deformation is invariant with respect to space and time reflection (cf., e.g., Ref. 1). A variety of consequences ensues from these observations; in particular that the projection of the angular momentum on the body-fixed symmetry axis is a conserved quantity and K is a good quantum number; also, that the single-particle motion in the average deformed potential takes place in twofold-Kramers'-degenerate orbitals. Because of the presence of a pairing component in the two-body nuclear interaction, the pairs of particles moving in time-reversal states condense, giving rise to a superfluid system with $K=0$ quantum number in the case of nuclei with an even number of protons and neutrons. This is evidenced by the fact that the associated moment of inertia is only about half of the value for rigid rotation.

Because K is a good quantum number, excited states with high K values are often isomeric, decaying only by virtue of small admixtures of lower-K components. A consequence of the K selection rule is that the decay from high-K states takes place preferentially stepwise through lower-lying K bands in order to minimize the K forbiddeness. Values for the hindrance factor per degree of K forbiddeness vary from 5 to 100.[2,3]

Recently, the decay of an isomeric state with $I^\pi=25^+$ has been observed[4] in ^{182}Os, directly populating the $I^\pi=24^+$ state of the yrast band ($K=0$), and with a lifetime of the order of 10 μs. One single transition thus changes K dramatically, and with an isomeric lifetime that is relatively short.

On the basis of these data it was proposed in Ref. 4 that the one-step process could be viewed as a tunneling between a configuration in which the prolate-deformed nucleus rotates along the symmetry axis (rotation-aligned K isomer), and another where it rotates perpendicular to this axis (deformation-aligned, yrast configuration), thus involving the γ degree of freedom, which measures the departure of the nuclear shape from axial symmetry. This picture is quite analogous to that used in the description of electromagnetic decay of shape isomers in the actinide nuclei.[5] However, in this case, the initial and final states have different shapes. In the proposed K-isomer decay in ^{182}Os, the initial and final states have similar shapes but with the angular momentum parallel and perpendicular to the symmetry axis, respectively. The path related to such a process is quite unusual, being associated with a shape degree of freedom which is not usually connected with large-amplitude motion. In the present paper we calculate the tunneling of the K isomer along the γ path. It will be concluded that this path can indeed lead to an overall account of the data.

The lifetime of the K isomer ($I^\pi=25^+$, $K=25$) is equal to 1.3×10^{-7} (cf. Ref. 4), of which 2% is associated with the population of the $I^\pi=24^+$ member of the yrast band ($K=0$). The corresponding lifetime of 0.7×10^{-5} s can be compared with the Weisskopf estimate for $M1$ decay [$\tau_{sp}(M1)\approx0.3\times10^{-13}$ s], leading to a hindrance factor

$$F_H\approx10^{-8}. \quad (1)$$

In keeping with the discussion above, this quantity can be identified with the tunneling probability between the rotation-aligned and the deformation-aligned configuration.

In what follows we calculate this probability, making use of the model of large-amplitude motion proposed in Ref. 6. The Hamiltonian describing the process is

$$\left[-\frac{\hbar^2}{2D}\frac{d^2}{d\xi^2}+V(\xi)\right]\psi_n(\xi)=E_n\psi_n(\xi). \quad (2)$$

The collective variable ξ corresponds, in the present case, to the angle γ describing the triaxiality of the quadrupole-deformed nucleus. It varies in the range $-120°\le\gamma\le0°$, leading to a change of ξ in the range $0\le\xi\le1$. Following Ref. 4, in the initial configuration ($\xi=0$), neutrons and protons are assigned spin and parity $I^\pi=15^+$, $I^\pi=10^+$, respectively.

The potential energy as a function of ξ displays two minima, located at about $\xi=0$ and $\xi=1$. These minima are separated by a barrier of about 2.6 MeV (cf. Fig. 1). This energy barrier has been obtained from the total energy surface calculated for ^{182}Os at spin and parity $I^\pi=25^+$, as described in Ref. 7. The barrier is thus minimized with respect to quadrupole and hexadecapole deformations. These deformations do not change considerably along the path, being close to $\epsilon_2=0.2$ and $\epsilon_4=0.02$–0.04.

In heavy nuclei such as ^{182}Os, pairing correlations play an important role, as attested by the reduced moment of inertia observed at low spins, and by the sizable dealignments observed at high rotational frequencies. These phenomena are associated with the fact that heavy nuclei are, as a rule, superfluid in their ground state, and that they display strong pairing vibrations at rotational frequencies above the critical frequency ω_c, at which the static pairing gap collapses.

As befitting a condensed many-body system, or one close to condensation, the nuclear wave function is a superposition of many Hartree-Fock configurations. Pairing allows the system to jump from one Hartree configuration to another, eventually reaching states whose shapes differ considerably from the initial one.

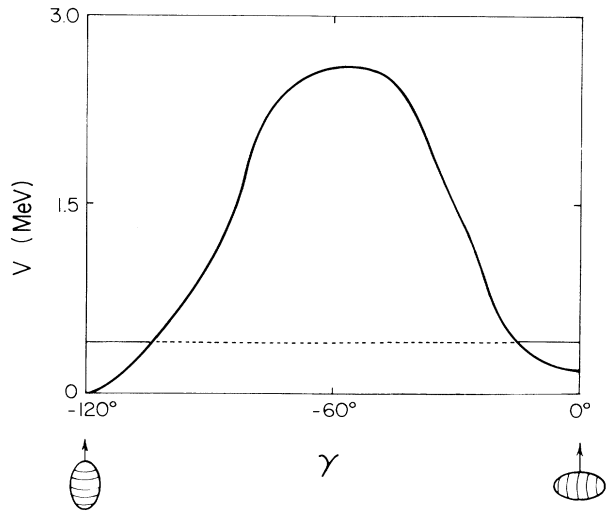

FIG. 1. A cut through the potential-energy surface of ^{182}Os at $I^\pi=25^+$ as a function of γ, as calculated according to Ref. 7. At $\gamma=-120°$ the nucleus is aligned along the symmetry axis, while at $\gamma=0°$ it rotates collectively around an axis perpendicular to the symmetry axis. The zero-point energy in the first well is indicated.

In keeping with this discussion, the inertial mass appearing in Eq. (2) is strongly dependent on the value of the pairing gap or of the zero-point amplitude associated with the pairing vibrations. In fact,[8]

$$D=D_n+D_p \quad (3)$$

is the sum of the neutron (n) and the proton (p) inertias, given by

$$\frac{D_i}{\hbar^2}=\frac{2G_i}{\delta_i^2}\left(\frac{dn_i}{d\xi}\right)^2 \quad (i=n,p), \quad (4)$$

where $\delta_i=G_i(\langle\psi_i|P^\dagger P|\psi_i\rangle)^{1/2}$ is the value of the "pairing gap" associated with the pairing force $H_p=G_iP^\dagger P$. The operator $P^\dagger=\sum_\nu a_\nu^\dagger a_{\bar\nu}^\dagger$ creates two particles in time-reversal states, and G_i is the pairing coupling constant. The quantities δ_i were determined along the γ path, by carrying out a self-consistent calculation of the pairing interaction, within the framework of the number projection formalism, minimizing the energy after projection[9] (for more details, cf. Refs. 10 and 11). The strengths of the neutron and proton components of the pairing force used in the calculations are $G_n=18/A$ MeV and $G_p=21.4/A$ MeV. The resulting average values along the γ path are $\delta_n=0.37$ MeV and $\delta_p=0.52$ MeV, respectively. Similar results were obtained making use of the BCS plus random-phase-approximation formalism as described in Ref. 12.

The quantity $dn_i/d\xi$, whose square appears in Eq. (4), is the density of configuration changes along the deformation path, when the system is adiabatically deformed under the influence of pairing. When $\omega=0$, time-reversal invariance insures that each single-particle orbital is twofold degenerate. For finite values of ω, where Kramers' degeneracy in the single-particle motion is lifted, the above prescription to calculate $dn_i/d\xi$ is not applicable, and has to be extended. In our case, a change of configuration taking place at $\gamma=\gamma_c$ is associated with the emptying of the orbitals (a_1,a_2) in favor of the single-particle states (a_3,a_4), under the following conditions:[13]

$$e_{a_1}(\gamma_c)+e_{a_2}(\gamma_c)=e_{a_3}(\gamma_c)+e_{a_4}(\gamma_c)\,, \tag{5a}$$

$$\pi_1\pi_2=\pi_3\pi_4=1\,, \tag{5b}$$

$$\alpha_1+\alpha_2=\alpha_3+\alpha_4=0\,. \tag{5c}$$

The set of quantum numbers $a_i\equiv(\pi_i,\alpha_i)$ and the energy $e_{a_i}(\gamma)$ in the rotating frame of reference (routhian[14]) identify each orbital. This is because the only symmetries left in the single-particle motion are those associated with reflection operations on a plane perpendicular to the symmetry axis which contains the origin of coordinates (parity π), and with rotations of 180° around an axis perpendicular to the symmetry axis[15] (signature, α). Following this prescription we obtain $dn_n/d\xi=7$, as shown in Fig. 2, where the single-neutron orbitals are displayed which follow the maximum overlap of the wave functions.[11] A similar calculation for protons leads to $dn_p/d\xi=2$.

Using the values discussed above, we obtain

$$D\approx 75\hbar^2\,\text{MeV}^{-1}\,. \tag{6}$$

We are now in possession of all the elements needed to calculate the hindrance factor (1) which, in the present discussion, is the probability of the K-isomer state, peaked around the maximum at $\xi=0$ ($\gamma=-120°$), to have a component in the well around $\xi=1$ ($\gamma=0°$).

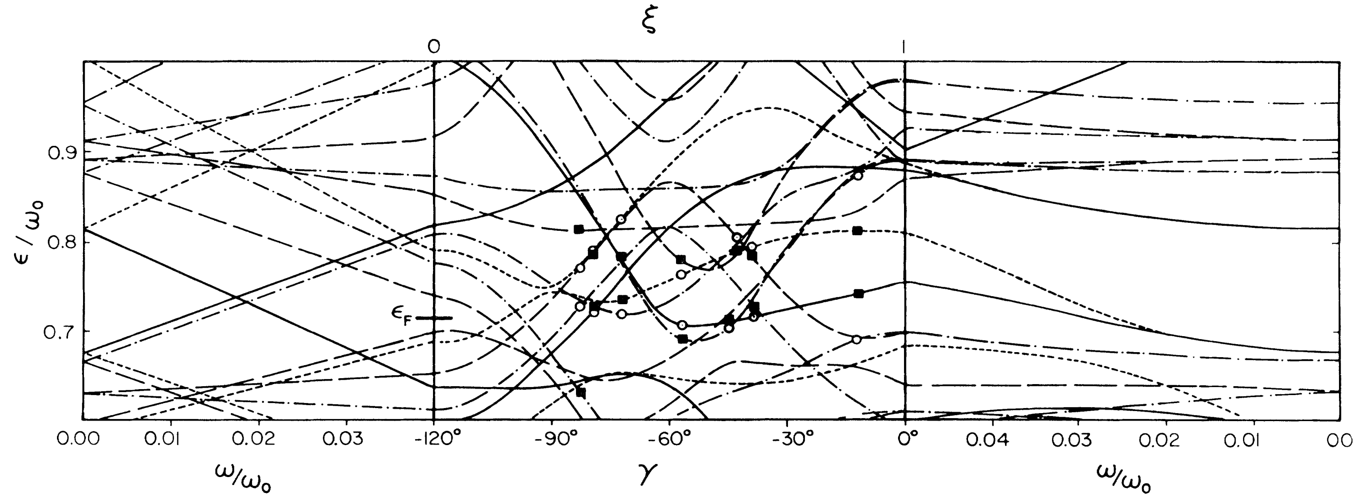

FIG. 2. The single-particle neutron levels used for counting the number of crossings. Levels of positive parity, having signature equal to $\frac{1}{2}$ or to $-\frac{1}{2}$, are drawn by solid and dashed lines, respectively; levels of negative parity having signature equal to $\frac{1}{2}$ or to $-\frac{1}{2}$ are drawn by dash-dotted and long-dashed lines, respectively. On the left-hand side of the figure, the levels are drawn as a function of the rotational frequency ω, at the fixed value $\gamma=-120°$ (aligned configuration, corresponding to rotation about the symmetry axis) starting from $\omega=0$, and ending at $\omega=0.04\omega_0$, where the configuration assigned to the K isomer under study is found to have spin and parity $I^\pi=15^+$ (the corresponding proton configuration has $I^\pi=10^+$). The quantity ω_0 is the frequency of the single-particle harmonic-oscillator potential ($\hbar\omega_0\approx 41A^{-1/3}$ MeV). Because the spin of the neutron configuration is odd, its signature is equal to 1. On the right-hand side of the figure, the levels are instead followed as a function of the rotational frequency at the fixed value $\gamma=0°$ (corresponding to collective rotation about an axis perpendicular to the symmetry axis), starting from $\omega=0$ and ending up to $\omega=0.05\omega_0$, at which value the lowest configuration of parity and signature $(+,1)$ is found to have the value $I^\pi=15^+$. The two configurations, having the same spin, parity, and signature, but different γ and ω, are then connected by a path in the $\{\omega,\gamma\}$ plane. We let ω vary linearly as a function of γ between $\omega=0.04\omega_0$ and $\omega=0.05\omega_0$ along the path, and the resulting single-particle levels are shown as a function of γ in the central part of the figure. The orbitals are drawn to follow the development of the wave functions and therefore levels with the same parity and signature might cross. The Fermi energy ϵ_F associated with the K-isomer configuration at $\gamma=-120°$ is indicated. When a change in configuration takes place along the path, as explained in the text, two single-particle levels become occupied, and two become empty at the same time. Levels becoming occupied are marked by filled squares, and levels becoming empty are marked by open circles, at the γ value γ_c where the configuration change takes place. Constant quadrupole and hexadecapole deformations, $\epsilon_2=0.202$ and $\epsilon_4=0.032$, are used throughout the figure.

This quantity is given by

$$F_H = \int_a^b |\psi_0(\xi)|^2 d\xi , \qquad (7)$$

where $\psi_0(\xi)$ is the wave function associated with the lowest eigenstate of Eq. (2), that is peaked around $\xi=0$. The limits of integration a and b are the boundaries in the well around $\xi=1$ associated with this state, whose zero-point energy is ≈ 0.4 MeV (cf. Fig. 1). One obtains

$$F_H \approx 10^{-6}\text{–}10^{-9}. \qquad (8)$$

The error ascribed to the calculated F_H is associated with the uncertainties connected with the potential energy around the second well, and with the fluctuations of the "pairing gap" δ_i along the γ path. It is noted that assuming the system to display, in the K-isomer configuration, the same pairing gap as in the ground state ($\delta_i \approx 1$ MeV), the decay will proceed with lifetimes which are 5–6 orders of magnitude shorter.

We conclude that the K-isomer decay of ^{182}Os taking place at $I \approx 25$ can be viewed as a tunneling in the γ degree of freedom, strongly renormalized by the presence of pairing vibrations, between a configuration where the nucleus essentially carries all the angular momentum in single-particle motion ($\gamma=-120°$) to the situation in which the system rotates as a whole ($\gamma=0°$).

[1]A. Bohr and B. R. Mottelson, *Nuclear Structure* (Benjamin, Reading, MA, 1975), Vol. 2.

[2]J. Pedersen *et al.*, Phys. Rev. Lett. **54**, 306 (1985).

[3]J. Pedersen *et al.*, Z. Phys. A **321**, 267 (1985).

[4]R. Chowdhury *et al.*, Nucl. Phys. **A485**, 136 (1988).

[5]S. Bjørnholm and E. Lynn, Rev. Mod. Phys. **52**, 725 (1980).

[6]F. Barranco, R. A. Broglia, and G. F. Bertsch, Phys. Rev. Lett. **60**, 507 (1988); G. F. Bertsch, F. Barranco, and R. A. Broglia, in *Windsurfing the Fermi Sea,* edited by J. Speth and T. Kuo (Elsevier, New York, 1987); G. F. Bertsch, in *Frontiers and Borderlines in Many-Particle Physics,* International School of Physics "Enrico Fermi," Course CIV, edited by R. A. Broglia and J. R. Schrieffer (North-Holland, Amsterdam, 1988).

[7]T. Bengtsson and I. Ragnarsson, Nucl. Phys. **A436**, 14 (1985).

[8]F. Barranco, R. A. Broglia, and E. Vigezzi, in *The Twenty-Seventh International Winter Meeting on Nuclear Physics, Bormio, 1988,* edited by I. Iori (Univ. Milan Press, Milan, 1988).

[9]A. Faessler *et al.*, Nucl. Phys. **A256**, 106 (1976); J. L. Egido and P. Ring, Nucl. Phys. **A388**, 388 (1982).

[10]R. Bengtsson and J.-ye Zhang, Phys. Lett. **135B**, 358 (1984).

[11]T. Bengtsson, University of Lund Report No. 88/17, 1988 (to be published).

[12]Y. R. Shimuzi *et al.*, Rev. Mod. Phys. **61**, 131 (1989).

[13]Equations (5b) and (5c) are in keeping with the fact that the pairing interaction connects states with the same parity and opposite signature, and furthermore conserves total signature and parity.

[14]R. Bengtsson and S. Frauendorf, Nucl. Phys. **A314**, 27 (1979); **A327**, 139 (1979).

[15]B. R. Mottelson, in *High Angular Momentum Properties of Nuclei,* edited by N. Johnson (Harwood, New York, 1982), p. 1; A. Goodman, Nucl. Phys. **A230**, 466 (1974).