Parity Doubling in Baryons and Its Relevance to Hadronic Structure

F. Iachello
Center for Theoretical Physics and A. W. Wright Nuclear Structure Laboratory, Yale University, New Haven, Connecticut 06511
(Received 17 November 1988)

The occurrence of parity doubling in baryon spectra is pointed out and its relevance to hadronic structure is discussed. It is suggested that parity doubling is a consequence of the geometric structure of baryons.

PACS numbers: 14.20.−c, 12.40.Aa, 12.70.+q

The purpose of this Letter is to bring attention to a major feature of baryon spectra that, although extensively investigated in the late 1960s within the context of both chiral symmetry[1] and Regge-pole theory,[2] has, in recent years, been somewhat overlooked. This is the occurrence in the spectra of parity doublets (i.e., two states with the same angular momentum and opposite parity occurring at the same energy). In Fig. 1 the squared masses M^2 of the N ($I=\frac{1}{2}$) baryons are plotted against the angular momentum J. The occurrence of parity doubling at intermediate values of J, $J\approx\frac{5}{2}$, is evident. Plots of the squared masses of the other baryons show that parity doubling is also evident in the Δ ($I=\frac{3}{2}$) spectrum, but less evident in the Σ ($I=1$) and Λ ($I=0$) spectra. Parity doubling *does not* occur for the ground state and gradually disappears at large angular momenta ($J\geq\frac{11}{2}$).

In this Letter I want to suggest an interpretation of parity doubling in terms of the geometric structure of baryons and show that it is an unescapable consequence of baglike or stringlike models.

(a) Baglike models.— Consider an object with a surface $R\equiv R(\theta,\phi)$, expand this surface in multipoles

$$R(\theta,\phi)=R_0\left[1+\sum_\mu \alpha_{1\mu}Y_{1\mu}(\theta,\phi)+\sum_\mu \alpha_{2\mu}Y_{2\mu}(\theta,\phi)+\sum_\mu \alpha_{3\mu}Y_{3\mu}(\theta,\phi)+\cdots\right] \tag{1}$$

and assume, for simplicity, axial symmetry around the z axis, $\mu=0$. For the sake of the discussion below it is also convenient to remove the displacement of the center of mass induced by the deformation and write R as

$$R(\theta)=R_0\{1+\epsilon_2P_2(\cos\theta)+\tfrac{1}{3}\epsilon_3[P_z(\cos\theta)+P_3(\cos(\theta-2\pi/3))+P_3(\cos(\theta-4\pi/3))]+\cdots\}, \tag{2}$$

where the ϵ_l's denote the deformation parameters, $\epsilon_l=\alpha_{l0}[(2l+1)/4\pi]^{1/2}$. Depending on the values of $\epsilon_2,\epsilon_3,\ldots$, the object described by Eq. (2) is either a spherical top, a prolate, or an oblate symmetric top. The spectrum rotational states[3] of a prolate symmetric top is shown in Fig. 2(a). For a spherical or an oblate symmetric top similar spectra are obtained. One observes in Fig. 2 the occurrence of parity doublets, $1^\pm,2^\pm,\ldots$, for all values of the orbital angular momentum, L, except the ground state, $L=0$. The doubling arises from the degeneracy of states with quantum numbers $+K$ and $-K$. The wave functions of the symmetric top are the Wigner functions, $\mathcal{D}^L_{MK}(\theta_1,\theta_2,\theta_3)$. Since these transform under parity as

$$\mathcal{P}[\mathcal{D}^L_{MK}(\theta_1,\theta_2,\theta_3)]=(-1)^{L+K}\mathcal{D}^L_{M,-K}(\theta_1,\theta_2,\theta_3), \tag{3}$$

the parity doublets are

$$\psi^{(\pm)}=\frac{\mathcal{D}^L_{MK}(\theta_1,\theta_2,\theta_3)\pm(-1)^{L+K}\mathcal{D}^L_{M,-K}(\theta_1,\theta_2,\theta_3)}{\sqrt{2}}.$$

Parity doubling does not occur for $k=0$. The shape given by Eq. (2) appears to be a natural shape for baryons at intermediate angular momenta. Using arguments similar to those presented by Johnson and Thorn[4] for mesons, one expects the three quarks forming the baryon to sit, for low angular momenta, at the three lobes of an octupole shape, $\epsilon_2=0$, $\epsilon_3\neq0$, as shown schematically in Fig. 3(c). This configuration is different from that of mesons, quadrupole shape ($\epsilon_1\neq0$, $\epsilon_3=0$), Fig. 3(a). For the latter shape, parity doubling of the type discussed above does not occur. The rotational spectrum of an object as in Fig. 3(a) is shown[3] in Fig. 2(b). It consists of a Regge trajectory with angular momenta and parities $L^P=0^+,1^-,2^+,3^-,\ldots$.

The occurrence in some states and nonoccurrence in others of parity doubling appears thus to be a consequence of the geometric structure of hadrons. However, one must in the case of hadrons proceed further since, in addition to a geometric structure, hadrons have an internal structure. The presence of the internal symmetry imposes some conditions on the allowed states. For example, the total wave function of baryons must be antisymmetric under interchange of all constituent variables. If one assumes that the internal degrees of freedom are described by $SU_c(3)\otimes SU_{sf}(6)$, where $SU_c(3)$ denotes the color algebra and $SU_{sf}(6)$ the spin-flavor

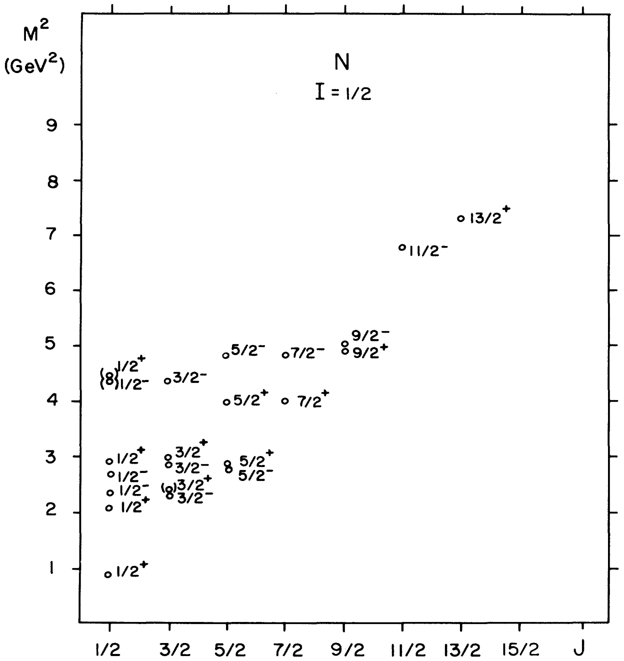

FIG. 1. Experimental spectrum of the N ($I=\frac{1}{2}$) baryons (Ref. 10). The square of the mass M^2 is plotted against the angular momentum J.

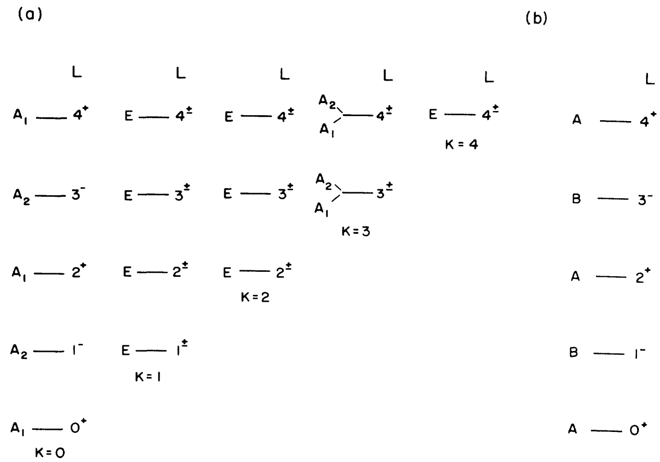

FIG. 2. (a) Schematic representation of a portion of the spectrum of rotational states of a prolate symmetric top. K denotes the projection of the angular momentum on a body-fixed axis. The transformation properties of the states under S_3 (A_1, A_2, and E species) are also shown. The vertical scale represents energy. For nonrelativistic tops, $M(L,K)=M_0+CL(L+1)+DK^2$. For baryons, $M^2(L,K)$ is given by Eq. (4). (b) Schematic representation of a portion of the spectrum of rotational states of a linear object (symmetry $C_{\infty v}$). The transformation properties of the states under S_2 (A and B species) are also shown.

algebra, the statement above implies that the geometric wave functions must be combined with the internal wave functions in such a way that the total wave function is antisymmetric. Since the color wave function must be antisymmetric this implies that the product of the $\mathrm{SU_{sf}}(6)$ and the $\mathcal{R}$ (geometric) wave function must be totally symmetric. The $\mathrm{SU_{sf}}(6)$ representations that appear in baryons are of three types, as shown in Table I. In order to see which states survive, we must consider the property of the geometric wave function under permutation of the three lobes (quark permutations). Since the permutation group S_3 is isomorphic to the point group C_{3v}, it is relatively easy to see what are the transformation properties of the rotational states.[3] C_{3v} has one two-dimensional representation called E, and two one-dimensional representations called A_1 and A_2 (Table I). The states in Fig. 2(a) are labeled by these representations. The argument given above then says that A_1 must be combined with **56** of $\mathrm{SU_{sf}}(6)$, A_2 with **20** of $\mathrm{SU_{sf}}(6)$, and E with **70** of $\mathrm{SU_{sf}}(6)$. Thus, the ground state of baryons belonging to the representation **56** of $\mathrm{SU_{sf}}(6)$ should *not* be parity doubled. Only those states belonging to the representation **70** are expected to be parity doubled. Although the experimental evidence is not complete since not all states are known and, in addition, some of them may be overlapping because of the large width, it appears that parity doubling rules, as provided by the combination of representations of S_3 and $\mathrm{SU}_{sf}(6)$, are satisfied.

It is worthwhile mentioning here two additional points. First, exact parity doubling relies on baryons behaving as spherical or symmetric tops. This can only occur if the masses of the three quarks are identical. If one of the masses is different, as in the case of strange baryons, one has instead an *asymmetric* top. In asymmetric tops, K-type parity doubling is broken.[3] The amount of asymmetry depends on how large the mass difference is between strange (s) and nonstrange (u,d) quarks. Indeed it appears that parity doubling is somewhat broken in the Λ and Σ baryons. The second observation is that, using arguments similar to those of Ref. 4, one can show that as the angular momentum increases there is a tendency for one of the quarks to move farther and farther away from the other two, thus producing an elongated shape similar to that of mesons (a diquark transforms as $\bar{3}$ under color), Fig. 3(b). Since, as discussed above, this shape does not have parity doubling, one expects that this feature gradually disappears as L increases. This again appears to be consistent with observation. Indeed, at large angular momenta baryons look very similar to mesons and their rotational spectra (Regge trajectories) become identical.[4,5]

(b) Stringlike models.— Arguments similar to those presented above for the bag model will apply to stringlike models (Nambu-Goto strings). Stringlike configurations of the type shown in Fig. 3(g)[6] also have C_{3v} invariance and one can characterize their rotational states with representations of this group.

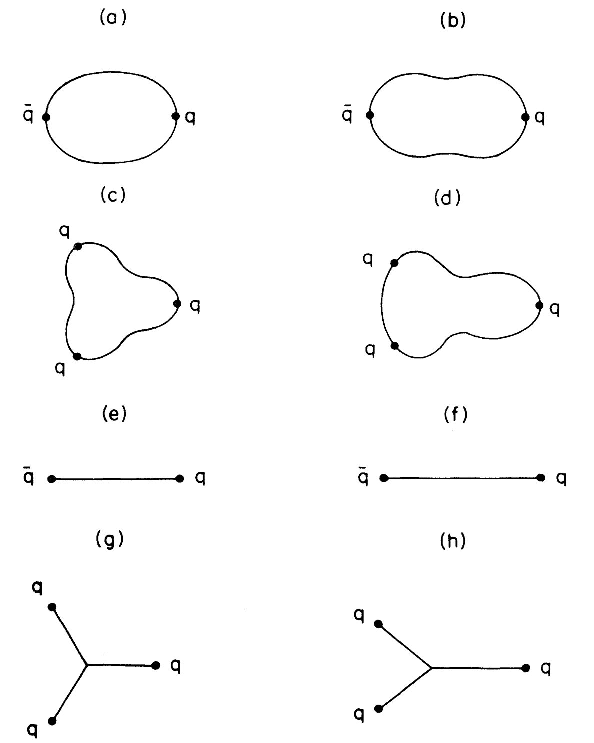

FIG. 3. Schematic representation of the geometric structure of hadrons: (a)–(d) bag model; (e)–(h) string model. As the angular momentum increases hadrons elongate [parts (b),(d) and (f),(h)]. The longitudinal length (z axis) is expected to increase (Ref. 4) as $\sqrt{L}$. Since in mesons nonidentical particles, q and $\bar{q}$, sit at the end points, the geometric group is $C_{\infty v}$ (and not $D_{\infty h}$) and remains the same for all angular momenta. The geometric group of baryons is instead C_{3v} at low angular momenta, $L \approx 1$, part (c) and gradually changes into $C_{\infty v}$ at large angular momenta, part (d).

TABLE I. Top: SU(6) representations that appear in baryons; bottom: S_3 representations that appear in baryons.

Young tableau	$[\lambda_1,\lambda_2,\lambda_3,\lambda_4,\lambda_5]$	Dim.
□ □ □	(3,0,0,0,0)	56
□ □ □	(2,1,0,0,0)	70
□ □ □	(1,1,1,0,0)	20
S_3	C_{3v}	Dim.
□ □ □	A_1	1
□ □ □	E	2
□ □ □	A_2	1

All the arguments presented in the previous paragraphs have been of a general nature. One may inquire how a detailed analysis of baryonic spectra can be done which fully exploits the inherent symmetry of the problem (parity doubling). I would like to suggest the use of algebraic methods for treating the geometric structure of hadrons, similar to those employed in other fields of physics.

(a) Baglike models.—Shapes of the type of Eq. (1) have been treated[7] by considering the algebra of $\mathcal{R} \equiv U(16)$, obtained by quantizing the classical variables $\alpha_{3\mu}$ ($\mu = \pm 3, \pm 2, \pm 1, 0$), $\alpha_{2\mu}$ ($\mu = \pm 2, \pm 1, 0$), and $\alpha_{1\mu}$ ($\mu = \pm 1, 0$) together with the monopole variable α_{00}. U(16) is a compact algebra with finite-dimensional representations and is appropriate for nuclei. In hadrons, in view of confinement, one could use the noncompact extension U(15,1) and consider its discrete, infinite-dimensional series.

(b) Stringlike models.—Planar stringlike configurations, such as those given in Figs. 3(b) and 3(h), have been treated[8] by considering the algebra $\mathcal{R} = U_1(4) \otimes U_2(4)$, where the indices 1 and 2 refer to variables describing the relative location of particles in the string. This is again a compact algebra appropriate for molecules. In hadrons one could use the corresponding noncompact algebra $U_1(3,1) \otimes U_2(3,1)$. If the string is aplanar, more complex algebraic constructs are needed. It should be pointed out that in any case, $\mathcal{R} \equiv U(16)$ or $U_1(4) \otimes U_2(4)$ for more complex constructions, one must insist, in order to obtain rotational spectra as in Fig. 2(a), that the subalgebras $O(4) \supset O(3) \supset O(2)$ be contained in $\mathcal{R}$, as discussed in detail in Ref. 8 for the rotational spectra of molecules. If one uses the noncompact algebras U(15,1) or $U_1(3,1) \otimes U_2(3,1)$, the corresponding noncompact extension $O(3,1) \supset O(3) \supset O(2)$ should be contained in $\mathcal{R}$.

In the paragraphs above I have discussed only rotational excitations. A similar discussion can be presented for vibrational excitations either of the bag or of the string. To see which vibrations survive when combined with the internal symmetry, one must perform an analysis of the vibrational states similar to that given above for rotational states. Both vibrational and rotational states can be included in the algebraic approaches U(15,1) or $U_1(3,1) \otimes U_2(3,1)$ suggested here. In addition, these approaches allow one to construct mass formulas describing the rotational and vibrational excitations. These mass formulas can be obtained by the usual method of expanding operators in terms of Casimir invariants. Mass formulas which embody the features of hadronic spectra can be conjectured from the well-known formulas for nonrelativistic tops[3] and taking into account the elongation of the bag (or string) with angular momentum.[4] In the case of identical particles at the lobes of the bag (or at the ends of the string) one obtains

$$M^2(L,K) = M_0^2 + [\alpha L(L+1) + \beta L(L+1)K^2]^{1/2}$$
$$\underset{L \gg 1}{\approx} M_0^2 + L(\alpha + \beta K^2)^{1/2}; \qquad (4)$$
$$K = L, L-1, \ldots, -L+1, -L.$$

For nonidentical particles, C_{3v} invariance is broken, and the mass formula is modified accordingly. Similar mass formulas can be constructed for vibrational excitations. A full account will be given in a longer paper. Finally, when comparing with the experimental spectra, the combination of internal and geometric degrees of freedom must be carried out explicitly. For example, the orbital angular momenta and parities shown in Fig. 2 must be combined with the intrinsic spin, S, and parity, P, of the states. This implies that for mesons the parities given in Fig. 2(b) must be reversed (since $q\bar{q}$ has negative intrinsic parity) and that each state becomes a multiplet when $S \neq 0$. In conclusion, I have emphasized the occurrence of parity doubling in baryon spectra and associated it with the geometric structure of hadrons (C_{3v} invariance). This geometric interpretation is not in conflict with the Nambu-Goldstone realization of chiral symmetry[1] and appears to be consistent with the present data. Because of its discrete nature C_{3v} invariance is similar to the Z invariance of Dashen,[9] although it has a different physical origin. The geometric interpretation arises naturally in collective models of hadrons (such as the bag and string models) and is linked to the fact that the color group is SU(3). It does not arise naturally in single-particle models of hadrons (such as the nonrelativistic quark model with harmonic-oscillator potentials) where parity doubling is accidental, since states of opposite parity have different numbers of oscillator quanta. Indeed, these two classes of models differ somewhat in their spectroscopic properties, especially those related to the $N(1440)$, $N(1540)$, $\Delta(1550)$, and $\Delta(1600)$ states. These differences could be resolved by new measurements such as those planned at the Continuous Electron Beam Accelerator Facility (CEBAF) presently under construction.

This work was performed in part under U.S. DOE Contract No. DE-AC-02-76ER03074. I wish to thank A. Leviatan for discussions on the degeneracies or rotational states in molecules, and A. Chodos and F. Gürsey for bringing Ref. 9 to my attention.

[1] H. Pagels, Phys. Rep. **16**, 219 (1975), and references therein.

[2] P. D. B. Collins and E. J. Squires, in *Regge Poles in Particle Physics,* edited by G. Höhler, Springer Tracts in Modern Physics Vol. 45 (Springer-Verlag, Berlin, 1968).

[3] See, for example, G. Herzberg, in *Infrared and Raman Spectra of Polyatomic Molecules* (Van Nostrand, New York, 1945), Chap. 1.

[4] K. Johnson and C. B. Thorn, Phys. Rev. D **13**, 1394 (1976).

[5] See, for example, S. Catto and F. Gürsey, Nuovo Cimento **86A**, 201 (1985).

[6] I. Bars and A. J. Hanson, Phys. Rev. D **13**, 1744 (1976).

[7] J. Engel and F. Iachello, Phys. Rev. Lett. **54**, 1126 (1985).

[8] F. Iachello and R. D. Levine, J. Chem. Phys. **77**, 3066 (1982); O. van Roosmalen, F. Iachello, R. D. Levine, and A. E. L. Dieperink, J. Chem. Phys. **79**, 2515 (1983).

[9] R. Dashen, Phys. Rev. **183**, 1245 (1969).

[10] Particle Data Group, M. Aguilar-Benitez *et al.*, Phys. Lett. **170B**, 1 (1986).