Correlation Length and Order of the Deconfining Phase Transition

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We analyze SU(2) and SU(3) lattice gauge theory on $L_t \times L^2 \infty$ lattices ($L_t \leq L$). By β_c we denote the critical coupling at $L=\infty$. In the neighborhood of the deconfining phase transition, at appropriately defined coupling constants $\beta(L,L')$, $L>L'$ [with $\beta(L,L') \to \beta_c$ for $L' \to \infty$], the correlation length ξ scales $\sim L/L'$ for second- *and first*-order transitions ($\xi = 1/E_1$ with E_1 the energy of one unit of 't Hooft electric flux). Linearization around the couplings $\beta(L,L')$ allows the calculation of critical exponents. Numerical results ($L_t=4$) support a second-order transition for SU(2), but not for SU(3).

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Extensive numerical investigations of the SU(3) deconfining phase transition exist in the literature and the first-order of the transition seems to be well established.[1,2] However, recently this wisdom was challenged by the APE Collaboration.[3] They observed $\xi \sim L$ near the critical point and took this as evidence for a second-order transition. To resolve this apparent contradiction is the purpose of this Letter. Our considerations are carried out for the deconfining phase transition. Obviously, they are more general and apply as well for magnetic systems (say 3D Ising model versus 3D three-state Potts model). The bottom line is to clarify the finite-size behavior of the correlation length for a first- versus a second-order phase transition.

Following Refs. 4 and 5 we consider $L_t L^2 \infty$ ($L_t \leq L$) lattices with sites labeled by t, x, y, and z. Let us close Polyakov loops in the t direction and sum over the (x,y) position (zero momentum). The asymptotic falloff of their (disconnected) correlations,

$$\langle \mathcal{P}(0)\mathcal{P}(z)\rangle \sim \exp(-E_1 z) \quad (z \to \infty)\,, \tag{1}$$

defines the energy E_1 of one unit of 't Hooft electric flux and $\xi = 1/E_1$ is the relevant correlation length for the deconfining phase transition. (E_1 is the analog of the mass gap in magnetic spin systems.) In the following we keep L_t finite and fixed. Let us first comment on the infinite-volume system (i.e., $L=\infty$). We denote by $\beta_c = \beta_c(L_t)$ its critical coupling. The electric flux behaves pronouncedly different for first- and second-order phase transitions and is qualitatively depicted in Fig. 1. We shall see that for $L \neq \infty$ the situation is considerably altered by finite-size effects.

For $\beta \neq \beta_c$, the large-L behavior of the 't Hooft electric flux is given by

$$E_1(L,\beta) = E_1(\infty,\beta)[1+O(e^{-c_-(\beta)L^2})]\,, \tag{2a}$$

$$c_-(\beta) > 0, \text{ for } \beta < \beta_c$$

and

$$E_1(L,\beta) = O(e^{-c_+(\beta)L^2}),\ c_+(\beta)>0, \text{ for } \beta > \beta_c\,. \tag{2b}$$

The functions $c_\pm(\beta)$ are monotonically increasing in $|\beta-\beta_c|$ for β in a sufficiently close neighborhood of β_c. Equations (2) are valid for first-order as well as for second-order transitions and they reflect that, away from the critical point, finite-size corrections are exponentially small in L. The finite electric flux for $\beta > \beta_c$ is due to tunneling. In the case of a second-order transition we have the additional equation

$$E_1(L,\beta_c) = \frac{\text{const}}{L} + O\left(\frac{1}{L^2}\right) \tag{3}$$

[or equivalently $\xi(L,\beta_c) \sim L$].

Equation (3) follows in the limit $\beta \to \beta_c$ [$\xi(\infty,\beta) \sim |\beta-\beta_c|^{-\nu}$] from the finite-size scaling equation[6]

$$\frac{\xi(L,\beta)}{\xi(\infty,\beta)} = f\left(\frac{L}{\xi(\infty,\beta)}\right), \tag{4}$$

and the fact that for L finite, $\xi(L,\beta)$ is a regular function of β even at β_c.

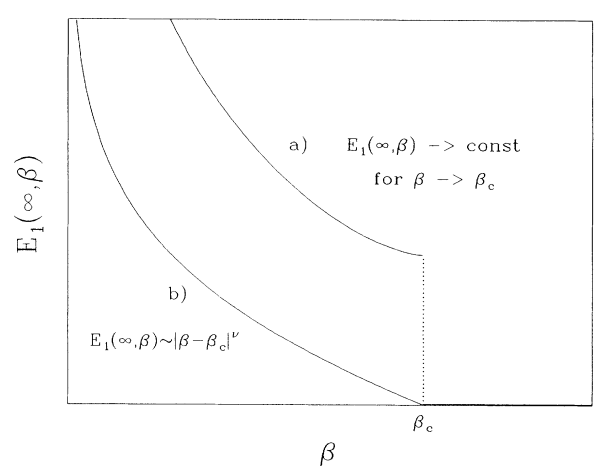

FIG. 1. $E_1(\infty,\beta)$ for first-order (curve a) and second-order (curve b) phase transitions. With our definition (1), $E_1(\infty,\beta) = 0$ for $\beta > \beta_c$.

In numerical applications Eq. (3) does not allow us to distinguish a first- from a second-order transition. In the case of a first-order transition, one can find β_0 values in an arbitrarily small neighborhood of β_c such as that at $\beta=\beta_0$ a behavior like (3) is true (for sufficiently large values of L). Consider two lattices of size L and L', with $L>L'$ (and, as always, L_t fixed). Equations (2) imply the existence of a coupling $\beta(L,L')$ such that $LE_1(L,\beta) > L'E_1'(L',\beta)$ for $\beta<\beta(L,L')$ and $LE_1(L,\beta)<L'E_1'(L',\beta)$ for $\beta>\beta(L,L')$. Consequently,

$$\frac{\xi(L,\beta_0)}{\xi(L',\beta_0)}=\frac{L}{L'} \text{ for } \beta_0=\beta(L,L')$$

$$\text{and } \lim_{L'\to\infty}\beta(L,L')=\beta_c\,. \qquad (5)$$

For instance, one may choose $L=2L'$ and find at $\beta(L,L')$ an increase of the correlation length by a factor of 2 when one compares results from lattices with sizes L and L'.

In a neighborhood of β_0 we define the transformation

$$\beta\to\beta' \text{ by requesting } LE_1(L,\beta)=L'E_1(L',\beta')\,, \qquad (6)$$

which has β_0 as a fixed point. [If this transformation is unclear, find out from Fig. 2(a) how it works.] In the case of a second-order transition an equation for the critical exponent ν is obtained by linearization around β_0.[7] Let $\Delta\beta=\beta-\beta_0$, $\Delta\beta'=\beta'-\beta_0$; we have

$$E_1(L,\beta)=E_1(L,\beta_0)+\Delta\beta\left[\frac{dE_1(L,\beta)}{d\beta}\right]_{\beta=\beta_0},$$

and

$$E_1(L',\beta')=E_1(L',\beta_0)+\Delta\beta'\left[\frac{dE_1(L',\beta')}{d\beta'}\right]_{\beta'=\beta_0}.$$

Using Eqs. (5) and (6) this gives

$$\Delta\beta L\left[\frac{dE_1(L,\beta)}{d\beta}\right]_{\beta=\beta_0}=\Delta\beta' L'\left[\frac{dE_1(L',\beta')}{d\beta'}\right]_{\beta'=\beta_0}.$$

So far our considerations are true in general. Finally, let us assume that the transition is of second order. Then Eq. (6) and the finite-size scaling equation (4) combine to give

$$\frac{L}{L'}=\frac{\xi(\infty,\beta)}{\xi(\infty,\beta')}=\left[\frac{\Delta\beta}{\Delta\beta'}\right]^{-\nu}$$

$$\Longrightarrow \ln\left[\frac{\Delta\beta'}{\Delta\beta}\right]=+\frac{1}{\nu}\ln\left[\frac{L}{L'}\right] \text{ and } \beta'<\beta<\beta_c\,.$$

Putting everything together we arrive at

$$\nu(L,L')=\ln\left[\frac{L}{L'}\right]\ln\left[\frac{[dLE_1(L,\beta)/d\beta]_{\beta=\beta_0}}{[dL'E_1(L',\beta')/d\beta']_{\beta'=\beta_0}}\right]^{-1}\to\nu \qquad (7)$$

for a second-order phase transition. As far as we can judge, Eq. (7) is meaningless in the case of a first-order transition. Using it (formally) nevertheless we would not expect the $\nu(L,L')$ to converge towards a meaningful value (although we are not aware of a proof of such a statement).

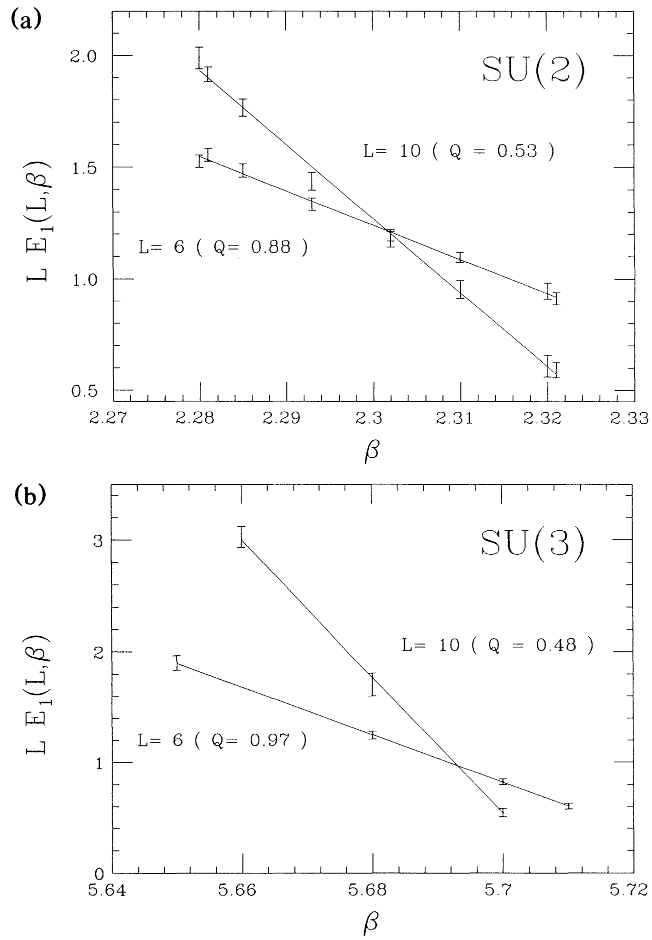

FIG. 2. (a) SU(2), $LE_1(L,\beta)$ vs β. The lines are the least-squares fits of our $L=6$ and $L=10$ data. (b) SU(3), otherwise as in (a).

Our numerical results are now easily presented. Along the lines of Ref. 4 we have performed high-statistics simulations (100000–120000 sweeps per data point) on $4\times L^2\infty$ lattices ($\infty=64$–66) with $L=4$, 6, 8, and 10. In the neighborhood of β_c we adjusted β values such that linear regression of $LE_1(L,\beta)$ is self-consistent. More precisely, for the data used the subroutine FIT[8] gives us Q values which are rather uniformly distributed in the range $0.16<Q<0.98$. Figure 2(a) depicts the SU(2) fits for $L=10$ and $L'=6$, and Fig. 2(b) depicts the corresponding SU(3) fits. The derivatives in (7) and their statistical errors are then obtained from the slopes as evaluated by the subroutine FIT and the β_0 values are determined by the crosspoints of the straight lines. Taking error propagation properly into account we arrive at the β_0 results

$$\beta(8,4)=2.2937\pm0.0008\,,\qquad \beta(10,6)=2.3106\pm0.0010 \tag{8a}$$

for SU(2), and

$$\beta(8,4)=5.6854\pm0.0008\,,\qquad \beta(10,6)=5.6930\pm0.0010 \tag{8b}$$

for SU(3). These numbers may be compared with various "infinite-volume" estimates for the critical coupling constant β_c, namely, around $\beta_c=2.298\pm0.004$ for SU(2) (Ref. 4) and $\beta_c=5.696\pm0.004$ for SU(3).[9]

Our numerical results for the critical exponents $\nu(L,L')$ are

$$\nu(8,4)=0.603\pm0.022\,,\qquad \nu(10,6)=0.661\pm0.044 \tag{9a}$$

for SU(2), and

$$\nu(8,4)=0.334\pm0.018\,,\qquad \nu(10,6)=0.487\pm0.028 \tag{9b}$$

for SU(3).

In the case of SU(2) both results are consistent with one another and may hence be averaged to

$$\nu=0.615\pm0.020\,. \tag{10}$$

This agrees with 3D Ising-model estimates, which center around $\nu=0.629$,[10] and is a direct numerical confirmation of the often exploited analogy[11] between the SU(2) deconfining phase transition and the phase transition of the 3D Ising model. Previously,[12] the exponents α, β, and γ were found to be consistent with Ref. 11 and $\nu=0.61\pm0.03$ was reported[13] from a numerical investigation of the partition-function zeros on L_tL^3 lattices with $L_t=2$.

In marked contrast to Eq. (9a), the SU(3) estimates of Eq. (9b) are completely inconsistent with one another. Assuming a Gaussian distribution, the likelihood that the difference between $\nu(8,4)$ and $\nu(10,6)$ is due to chance in 0.24 (=24%) for the SU(2) Eq. (9a), but less than 5×10^{-6} for the SU(3) results (9b). Relying on these data our statement is the following. We have *no* evidence for a second-order SU(3) transition. Consequently, we see no reason to doubt previous investigations,[1,2] which directly support the first-order nature of this transition. [We consider it an accident that $\nu(8,4)$ is in agreement with the value $1/d$ that the so-called "discontinuity fixed point"[14] of a 3D spin system is supposed to have. Such a fixed point is relevant for the L_tL^3 geometry, where for $L_t=2$; indeed $1/\nu=3.02\pm0.05$ was found by investigating the partition-function zeros numerically.[15]]

In summary, our finite-size scaling analysis of the 't Hooft electric flux yields an explicit critical exponent ν (10) for SU(2) lattice gauge theory and evidence that the SU(3) deconfining transition is of first order. In future work[16] an analysis of 3D spin systems is planned.

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