Polarization Correlation Analysis of the Radiation from a Two-Photon Deuterium Source Using Three Polarizers: A Test of Quantum Mechanics versus Local Realism

T. Haji-Hassan,[a] A. J. Duncan, W. Perrie,[b] and H. Kleinpoppen
Atomic Physics Laboratory, University of Stirling, Stirling, FK9 4LA, Scotland

E. Merzbacher
Department of Physics and Astronomy, University of North Carolina, Chapel Hill, North Carolina 27514
(Received 28 September 1988)

Measurements have been made of the polarization properties of the two photons emitted in the decay of metastable atomic deuterium in an experiment in which three linear polarizers, instead of the normal two, are used. The quantum-mechanical predictions for these imperfect polarizers have been calculated and shown to be in good agreement with the experimental results. The results have also been used to test local realistic models which are capable of explaining all existing two-polarizer-type experiments, and have provided conclusive evidence against these theories.

PACS numbers: 03.65.Bz, 42.50.Wm

Over the past two decades a series of experiments have been carried out[1-7] to observe the polarization properties of correlated photon pairs emitted in certain atomic decay processes. Most of the experiments have been performed with a view to testing Bell's inequality[8] and resolving the debate between the proponents of quantum mechanics and local realism. However, they may also be viewed as tests of the quantum formalism describing polarization and polarization analyzers in novel and hitherto unexplored situations.

The polarization state of the two collinear photons may be described, in the most common case of zero angular momentum transfer and no parity change, by the state vector

$$|\psi\rangle=(1/\sqrt{2})(|x\rangle_1|x\rangle_2+|y\rangle_1|y\rangle_2),$$

where $|x\rangle_1$ represents an x-polarized photon propagating to the right, and $|x\rangle_2$ represents an x-polarized photon propagating to the left, with similar definitions for $|y\rangle_1$ and $|y\rangle_2$. In the usual experimental arrangement, in which linear polarizers are placed diametrically on either side of the source, it follows that observation of, say, an x (y)-polarized photon on the left ensures that an $x(y)$-polarized photon will be detected on the right. The presence of the polarizer on the left destroys the rotational symmetry about the observation axis that would otherwise exist on the right and, in the spirit of the Einstein-Podolsky-Rosen argument, the "collapse of the state vector" brought about by the measurement on the left determines the polarization state of the correlated photon on the right. These experiments, that make use of two polarizers, can thus be considered to be the two-photon analog of the situation where light from an unpolarized single-photon source is analyzed by two polarizers in series. The above view can be validated in the quantum-mechanical formalism by our showing that the properties of the radiation on the right may be expressed in terms of an effective single-photon density matrix whose elements depend on the form of the two-photon state vector and the properties of the polarizer on the left.

For two-polarizer–type experiments, the analysis on both sides of the source is carried out by simple polarizers which can be represented quantum mechanically by *normal* operators in the sense that, if the effect of a polarizer is represented by the operator L then $[L,L^\dagger]=0$, where $L^\dagger$ is the Hermitian adjoint of L. There is then some interest in extending the measurements to cover the situation where compound analyzers, which cannot be represented by normal operators, are used on one side or both sides of the source. Since, in general, the product of two normal operators is not normal, such a situation can be achieved by, for example, the introduction of additional linear polarizers whose transmission axes may be oriented at arbitrary angles relative to the transmission axes of the original two polarizers. Such an experiment, illustrated in Fig. 1, in which an additional polarizer is introduced on one side of the source is described here. This experiment can, in fact, be considered the analog of one in which radiation from an unpolarized single-photon light source is analyzed by a train of *three* polar-

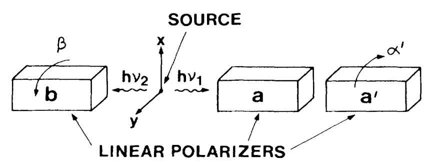

FIG. 1. Schematic diagram of the three-polarizer experiment. The orientation of polarizer a is fixed with its transmission axis parallel to the x axis, while the transmission axes of polarizers b and a′ are rotated, respectively, through angles β and α' relative to the x axis but in opposite senses.

izers.[9] It is also of interest, however, because of its relevance to a suggestion by Garuccio and Selleri,[10] based on a particular class of local realistic models, that a finite difference might exist, in this situation, between the predictions of quantum-mechanics and local realistic theories involving enhanced photon detection made possible by the low detection efficiency of the photomultipliers. In this class of theories, capable of explaining all existing two-polarizer experiments, Garrucio and Selleri postulate that a detection vector $\boldsymbol{\lambda}$ in addition to a polarization vector $\mathbf{l}$ is to be attributed to each photon of the pair emitted by a two-photon source. The detection vector $\boldsymbol{\lambda}$ is assumed to be unaffected by passage through a linear polarizer and the probability of detection of a photon is assumed to depend on the angle between $\mathbf{l}$ and $\boldsymbol{\lambda}$. A previous experiment[7] involving the insertion of a half-wave plate rather than a third linear polarizer found no evidence for such an enhancement effect, but could not be regarded as conclusive because of the possibility that the $\boldsymbol{\lambda}$ vector was affected by passage through the half-wave plate.

The basic experimental arrangement and procedure has been described in detail elsewhere.[6] Briefly, the source consists of a beam of metastable atomic deuterium, the two-photon radiation from which is detected and analyzed by a conventional electronic coincidence technique. As indicated in Fig. 1, the orientation of polarizer a is held fixed while polarizer b is rotated through an angle β in a clockwise sense and polarizer a′ through an angle α' in the opposite sense. Then the ratio $R(\beta,\alpha')/R(\beta,\infty)$ is measured as a function of angle α' for various angles β, where $R(\beta,\alpha')$ is the coincidence rate with all the polarizer plates in place while $R(\beta,\infty)$ is the rate with the plates of polarizer a′ removed.

The quantum-mechanical theory for this situation is considered in detail elsewhere.[11] In brief, if A, A', and B are the 2×2 matrices which represent the action of polarizers a, a′, and b on the state vectors for the photons which are incident upon them, the quantum-mechanical expression for the ratio $R(\beta,\alpha')/R(\beta,\infty)$ is obtained from the 4×4 density matrix ρ, representing the two-photon state $|\psi\rangle$ by the formula

$$\frac{R(\beta,\alpha')}{R(\beta,\infty)}=\frac{\mathrm{Tr}_1(A'A\rho_{\mathrm{eff}}A^{\dagger}A'^{\dagger})}{\mathrm{Tr}_1(A\rho_{\mathrm{eff}}A^{\dagger})}.$$

Here the traces are taken only over the polarization variables of photon 1 (on the right in Fig. 1), and ρ_{eff} is the effective density matrix obtained from ρ by our taking the trace over the polarization variables of photon 2 (on the left in Fig. 1):

$$\rho_{\mathrm{eff}}=\mathrm{Tr}_2(B\rho B^{\dagger}).$$

The result is

$$\frac{R(\beta,\alpha')}{R(\beta,\infty)}=\tfrac{1}{2}(M_{A'}+m_{A'})+\tfrac{1}{2}(M_{A'}-m_{A'})\left(\frac{M_AP-m_AQ}{M_AP+m_AQ}\right)\cos2\alpha'-\Delta(\beta,\alpha'),\qquad(1)$$

where

$$P=\tfrac{1}{2}[(M_B+m_B)+(M_B-m_B)\cos2\beta],\quad Q=\tfrac{1}{2}[(M_B+m_B)-(M_B-m_B)\cos2\beta].$$

The term $\Delta(\beta,\alpha')$ results from interference between the wanted light (light polarized in a direction parallel to the transmission axis of the polarizer) and the unwanted light (light polarized in a direction perpendicular to the transmission axis) passing through polarizer a, and is given by

$$\Delta(\beta,\alpha')=\frac{(M_Am_A)^{1/2}(M_B-m_B)(M_{A'}-m_{A'})}{2(M_AP+m_AQ)}\sin2\beta\sin2\alpha'\cos\phi,\qquad(2)$$

where M_A and m_A are the transmission efficiencies (the moduli squared of the transmission amplitudes) for light polarized parallel and perpendicular to the transmission axis of polarizer a, respectively, with similar definitions for M_B, m_B, $M_{A'}$, and $m_{A'}$. The angle ϕ represents the relative phase between the complex transmission amplitudes for wanted and unwanted light through polarizer a, and, for light passing directly through the polarizer, it would be expected that $\phi=0°$, $\cos\phi=1$. Of course, if the polarizer were perfect, the interference term would not occur since in that case $m_A=0$.

A complication arises when we use imperfect pile-of-plates polarizers in that a portion of the transmitted light results from internal reflections from the plates of the polarizers. It is assumed that the contribution of internal reflections to the wanted component is negligibly small since they occur near to Brewster's angle, but a significant part of the unwanted component does arise from these reflections. However, because of the small deviations of the plate alignment from Brewster's angle, the lack of parallelism of the surfaces of the individual plates and the imperfect polish of the plate surfaces, it is unlikely that the component of the unwanted light resulting from these internal reflections will interfere with the light passing straight through the polarizer. If this assumption is made then, in the expression for $\Delta(\beta,\alpha')$, the factor $(M_Am_A)^{1/2}$ must be modified to $(M_Ah_A)^{1/2}$ where h_A represents the transmission efficiency of the unwanted component not resulting from internal reflections. The quantity h_A is wavelength dependent and cannot be readily measured so, in practice, a weighted mean value was calculated, taking account of the optical properties of the polarizer plates and the spectral distribution of the radiation from the two-photon source. This procedure resulted in the value $h_A=0.0182$. The other transmission efficiencies were measured in a subsidiary experiment described previously.[6] For polarizers a and b, it was found that $M_A=M_B=0.908\pm0.013$, $m_A=m_B=0.0299\pm0.0020$, whereas for polarizer a′, made from plates supplied by a different manufacturer, $M_{A'}=0.938\pm0.010$, $m_{A'}=0.040\pm0.002$.

It would be possible to use the above values for the transmission efficiencies directly in expression (1) to obtain the quantum-mechanical predictions. However, strictly, the quantity $R(\beta,\alpha')/R(\beta,\infty)$ is wavelength dependent and depends nonlinearly on wavelength-dependent transmission efficiencies. To evaluate the importance of this fact, a wavelength-averaged value for $R(\beta,\alpha')/R(\beta,\infty)$ was computed taking into account the spectral distribution of the two-photon source, the quantum efficiency of the photomultipliers and the wavelength dependence of the transmission efficiencies of the polarizers. The result of these calculations are shown as the quantum-mechanical predictions in Figs. 2 and 3 but, as can easily be verified, the curves shown do not differ significantly from those obtained by direct substitution of the above quoted values of the transmission efficiencies in expression (1).

The experimental and theoretical results for $\beta=0°$, 33°, and 67.5° are shown in Fig. 2. Clearly, within the limits of experimental error, the results are in good agreement with the predictions, although there is a suggestion that the results for $\beta=33°$ are systematically slightly too high. It is also worth noting that the quantum-mechanical prediction for $\beta=0°$ is extremely close to the form $M_A\cos^2\alpha'+m_{A'}\sin^2\alpha'$, and the experimental results for this case can be considered a test of Malus' law for the transmission of single photons through polarizer a′.

Referring to Eq. (1), it is clear that, according to quantum mechanics, the ratio $R(\beta,\alpha')/R(\beta,\infty)$ should not be symmetrical with respect to a change of sign of angle α', because of the presence of interference between the coherent wanted and unwanted components of radiation transmitted through polarizer a. To investigate this prediction, an additional measurement for $\beta=67.5°$, $\alpha'=-45°$ was carried out and the result is shown, in comparison with the results for positive α', in Fig. 3 along with the appropriate quantum-mechanical prediction. Again the results clearly indicate the quantum-mechanical prediction, particularly with regard to the asymmetry with respect to angle α'. It is, perhaps, interesting to note how the use of imperfect polarizers reveals these interesting features and provides even more convincing verification of the quantum-mechanical formalism than might otherwise be obtained.

Finally, returning to the predictions of the class of local realistic theories proposed by Garuccio and Selleri,[10] for the three-polarizer experiment they showed that for

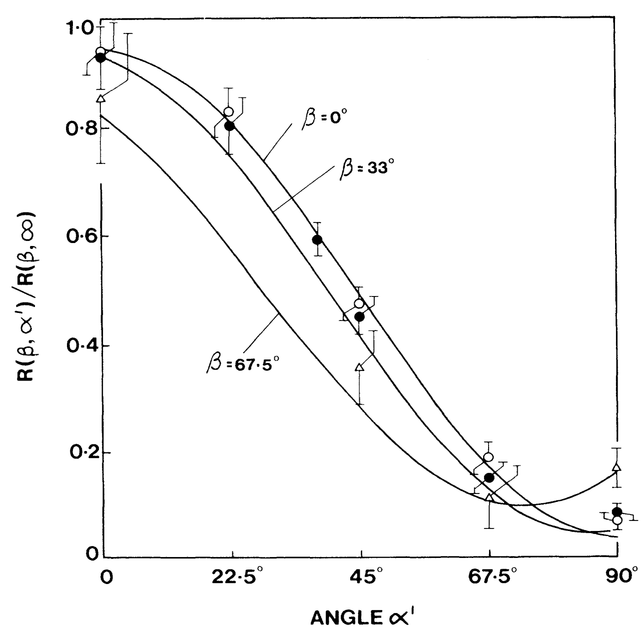

FIG. 2. Variation of the ratio $R(\beta,\alpha')/R(\beta,\infty)$ as a function of α' for $\beta=0°$ (○), 33° (●), and 67.5° (△). The solid curves represent the quantum-mechanical predictions.

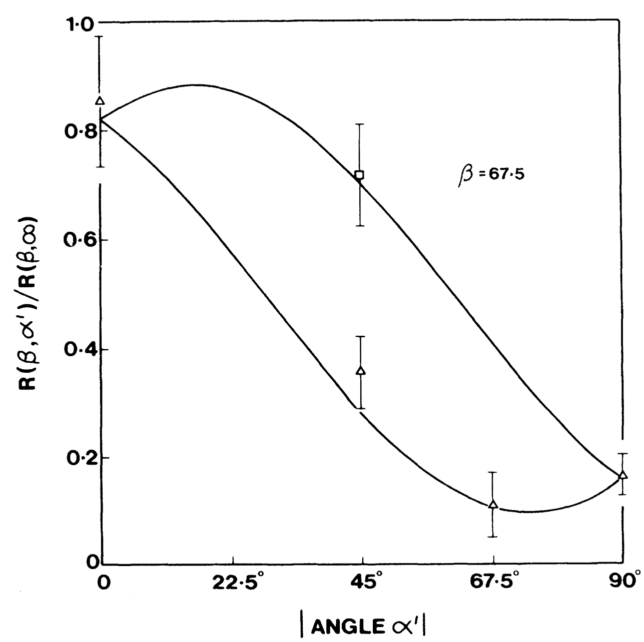

FIG. 3. The ratio $R(\beta,\alpha')/R(\beta,\infty)$ as a function of α' for $\beta=67.5°$. The upper (lower) curve is the theoretical result for $\alpha'<0$ ($\alpha'>0$). The experimental points are marked □ for $\alpha'<0$, △ for $\alpha'>0$.

any angle $\theta < 90°$, if it is arranged that the angles of the transmission axes of the three polarizers satisfy the relation $\beta = 3\theta$, $\alpha' + \beta = \theta$, then the ratio γ of the quantum-mechanical prediction to their local realistic prediction must always be greater than some minimum value, γ_L say, which depends on θ in a specified way. For the parameters of this experiment the important range of angles occurs for $58° < \theta < 80°$, since for these values of θ the lower limit γ_L is greater than unity and a definitive test between quantum mechanics and their class of local realistic models becomes possible. The maximum value of $\gamma_L = 1.447$ occurs here for $\theta = 71°$, corresponding to the experimental angles $\beta = 33°$, $\alpha' = 38°$, so that the approach of Garuccio and Selleri sets an upper limit[12] on $R(33°,38°)/R(33°,\infty)$ of 0.413, whereas the actual experimental point has the value 0.585 ± 0.029, violating the prediction of the Garuccio-Selleri model by over 6 standard deviations. More recently Selleri,[13] taking other factors into account, modified the prediction of the local realistic model to give $\gamma_L = 1.162$ with a corresponding upper limit on $R(33°,38°)/R(33°,\infty)$ of 0.514 which, however, is still violated by the experimental result by almost 3 standard deviations. The three-polarizer experiment, therefore, provides further strong evidence against the possibility of enhancement in the detection process and appears to rule out the class of local realistic theories proposed by Garuccio and Selleri.

The authors wish to thank Dr. H. J. Beyer and Professor F. Selleri for useful discussions and acknowledge the financial support of the Science and Engineering Research Council. One of us (T.H.) also wishes to thank the International Islamic University, Malaysia, for financial support. The U.S. Department of Energy provided partial support to one of the authors (E.M.).

(a)Present address: International Islamic University, P.O. Box 70, Jalan Sultan, 46700 Petaling Jaya, Selangor, Malaysia.

(b)Present address: British Nuclear Fuels plc., Capenhurst Works, Chester CH1 6ER, England.

[1]J. F. Clauser and A. Shimony, Rep. Prog. Phys. **41**, 1881 (1978).

[2]F. M. Pipkin, in *Advances in Atomic and Molecular Physics,* edited by D. R. Bates and B. Bederson (Academic, New York, 1978), Vol. 14, pp. 281–340.

[3]A. Aspect, P. Grangier, and G. Roger, Phys. Rev. Lett. **47**, 460 (1981).

[4]A. Aspect, P. Grangier, and G. Roger, Phys. Rev. Lett. **49**, 91 (1982).

[5]A. Aspect, J. Dalibard, and G. Roger, Phys. Rev. Lett. **49**, 1804 (1982).

[6]W. Perrie, A. J. Duncan, H. J. Beyer, and H. Kleinpoppen, Phys. Rev. Lett. **54**, 1790, 2647(E) (1985).

[7]T. Haji-Hassan, A. J. Duncan, W. Perrie, H. J. Beyer, and H. Kleinpoppen, Phys. Lett. A **123**, 110 (1987).

[8]J. S. Bell, Physics (N.Y.) **1**, 195 (1964).

[9]R. Clark Jones, J. Opt. Soc. Am. **46**, 528 (1956).

[10]A. Garuccio and F. Selleri, Phys. Lett. **103A**, 99 (1984).

[11]T. Haji-Hassan, Ph.D. thesis, Stirling University, 1988 (unpublished).

[12]The number 0.413 is obtained by our dividing the quantum-mechanical prediction for $\beta = 0°$, $\alpha' = 38°$ by 1.447, thus strictly overestimating the upper limit. If the quantum-mechanical prediction for $\beta = 33°$, $\alpha' = 38°$ were used, the disagreement between the Garuccio-Selleri–type model and quantum mechanics would be even greater.

[13]F. Selleri, private communication.