Relativistic Pion-Ring Series for Nuclear Matter

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The binding energy, density, and incompressibility (saturation properties) of normal nuclear matter are reproduced using a Lagrangian with approximate chiral symmetry. The pion-ring series including nucleons and Δ's provides the necessary attraction and its density dependence, while the effects of the scalar field are not significant, and vacuum-polarization effects are expected to be small.

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Understanding the saturation properties of normal nuclear matter has been one of the most challenging problems in theoretical physics. Recently, successful treatments of nuclear matter have been obtained from relativistic quantum field theories[1] of nuclear matter. However, the pionic effects are not important in the commonly used mean-field or Hartree-Fock approximation to such theories.[1,2] This is surprising because pions mediate the nucleon-nucleon interaction at all but the shortest ranges. Other reasons for thinking pions to be relevant are given in Refs. 3–5. Furthermore, chiral symmetry is an important approximate symmetry of strongly interacting systems which requires pionic degrees of freedom.

In the present work, we aim to obtain a relativistic chiral treatment of nuclear matter studying the pion-ring series, because the attractive nature of the π-N interaction can lead to pionic enhancement. The pion-ring series is the proper partial summation for this collectivity.

The dynamics is obtained with a Lagrangian employing hadronic degrees of freedom. We use Weinberg's[6] pseudovector (PV) representation of the linear σ model, with vector ω mesons and Δ isobars also included:

$$\mathcal{L}=\mathcal{L}_0(N)+\mathcal{L}_0(\Delta)+\mathcal{L}_0(\omega)+\mathcal{L}_0(\pi)+\mathcal{L}_0(\phi)-U(\phi)+\mathcal{L}_{\pi\phi}+\mathcal{L}_{\omega NN}+\mathcal{L}_{\pi NN}+\mathcal{L}_{\pi\pi NN}+\mathcal{L}_{\pi N\Delta}\,,$$

where $\mathcal{L}_0$'s are the free-field Lagrangians for the hadronic degrees of freedom and

$$U(\phi)=-(m_\phi^2-m_\pi^2)\left[\frac{g_\pi}{2M}\phi^3-\frac{1}{2}\left(\frac{g_\pi}{2M}\right)^2\phi^4\right], \tag{1a}$$

$$\begin{aligned}
\mathcal{L}_{\pi\phi}&=-\frac{g_\pi\phi}{M}\left(1-\frac{g_\pi\phi}{2M}\right)\partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi}+\frac{g_\pi\phi}{2M}m_\pi^2\boldsymbol{\pi}^2+O(\boldsymbol{\pi}^3)\,,\\
\mathcal{L}_{\phi NN}&=g_\pi\phi\bar{N}N\,,\\
\mathcal{L}_{\omega NN}&=-g_\omega\omega_\mu\bar{N}\gamma^\mu N\,,\\
\mathcal{L}_{\pi NN}&=-\frac{g_\pi}{2M}\bar{N}\gamma^5\gamma^\mu\boldsymbol{\tau}\cdot\partial_\mu\boldsymbol{\pi}N+O(\boldsymbol{\pi}^3)\,,\\
\mathcal{L}_{\pi\pi NN}&=-\left(\frac{g_\pi}{2M}\right)^2\bar{N}\gamma^\mu\boldsymbol{\tau}\cdot\boldsymbol{\pi}\times\partial_\mu\boldsymbol{\pi}N+O(\boldsymbol{\pi}^3)\,,\\
\mathcal{L}_{\pi N\Delta}&=\left(\frac{f_\Delta}{m_\pi}\right)\bar{\Delta}_\mu(g^{\mu\nu}-\gamma^\mu\gamma^\nu\xi)\mathbf{T}\cdot\partial_\nu\boldsymbol{\pi}N+O(\boldsymbol{\pi}^3)+\text{H.c.}\,.
\end{aligned} \tag{1b}$$

We include isobars (Δ) in addition to nucleons (N) because two-pion exchanges with intermediate Δ's supply important mid-range attraction in nucleon-nucleon scattering. We use the simplest form of Rarita-Schwinger propagator for the Δ, and off-mass-shell effects are considered in terms of ξ which modifies the $\pi N\Delta$ vertex.[7] This Lagrangian has been studied extensively by Matsui and Serot[8] for the case $\Delta_\mu=0$. The nucleon, Δ, pion, and ω masses (M, M_Δ, m_π, and m_ω) and pion-baryon cou-

TABLE I. Isospin-even π-N scattering parameters.

Contributions from N, Δ, and ϕ pole terms	$a_s^{(+)}(m_\pi^{-1})$	$r_s^{(+)}(m_\pi^{-3})$	$a_{p_{3/2}}^{(+)}(m_\pi^{-3})$	$a_{p_{1/2}}^{(+)}(m_\pi^{-3})$
N	−0.010	+0.012	+0.053	−0.104
Δ ($\xi=0$)	−0.049	−0.039	+0.062	+0.032
Δ ($\xi=0.75$)	−0.019	−0.009	+0.055	+0.025
Δ ($\xi=1.0$)	0	+0.007	+0.046	+0.015
ϕ ($m_\phi=5m_\pi$)	−0.074	−0.137	+0.047	+0.048
ϕ ($m_\phi=15m_\pi$)	−0.008	−0.016	+0.005	+0.005
Expt.[a]	−0.010	−0.044	+0.133	−0.057
	±0.003	±0.007	±0.002	±0.002

[a]Reference 9.

plings $(g_\pi, f_\Delta) = (13.45, 2.0)$ are well determined physical parameters. The resulting low-energy π-N scattering lengths and effective ranges are shown as a function of ξ and m_π in Table I. For example, $\xi = 0$ and $m_\pi = 15 m_\phi$ give $r_s^{(+)} = -0.043 m_\pi^{-3}$. The scalar-meson mass m_ϕ and vector-meson–nucleon coupling g_ω are treated as adjustable parameters. The experimentally determined values of g_ω^2 are in the range $60 < g_\omega^2 < 300$.[10] The scalar meson is not a well-known "particle" and is expected to have a large width. The above Lagrangian represents an effective theory applicable for not very high momentum transfers. We make no attempt to calculate vacuum properties. We also include form-factor effects as discussed below. We may set m_ϕ to be infinite without violating chiral symmetry; however, we keep it finite to investigate the effects of the scalar meson. This allows the possibility of obtaining a solution such as in Ref. 8. [Possible additions of higher-order terms to $U(\phi)$ lead to small effects; see below.]

Typical calculations of nuclear-matter properties use the well-known mean-field approximation (MFA). For uniform, static, symmetric ($N = Z$) nuclear matter one neglects the Δ's and replaces the fields ϕ, ω_μ, and $\boldsymbol{\pi}$ appearing in $\mathcal{L}$ by the nuclear-matter expectation values:

$$\phi(x) = \phi_0, \quad \omega_\mu(x) = \omega_0 \delta_{\mu 0}, \quad \boldsymbol{\pi}(x) = 0 .$$

If one neglects the term $U(\phi_0)$ and computes the energy density $\epsilon^{\rm MFA}$, the result is that of the nonchiral Walecka model.[11] The procedure is to minimize $\epsilon^{\rm MFA}$ with respect to $M^* \equiv M - g_\pi \phi_0$ and adjust g_ω^2 and m_ϕ^2 to the saturation point ($k_F = 1.3\ {\rm fm}^{-1}$, $E/A - M = -15.75$ MeV).

To obtain a result that maintains chiral symmetry and achieves nuclear-matter saturation it is necessary to include terms beyond the MFA. We add the pion-ring series in Fig. 1, which consists of the second-order one-pion exchange with π propagation modified by the nuclear medium. The motivation is that the pions appearing in the two-particle–two-hole term can be scattered by other nucleons. The importance of the resulting diagrams (ring series, or random-phase approximation) has been stressed elsewhere.[4,12] The π-ϕ interaction is included by rescaling the pion field $\boldsymbol{\pi} \to \boldsymbol{\pi}/(1 - g_\pi \phi_0/M)$ so that $m_\pi^2 \to m_\pi^2 (1 - g_\pi \phi_0/M)$. (This is a small effect.)

The calculation is simplified by neglecting terms of $O(\boldsymbol{\pi}^3)$ to avoid including purely mesonic loops.[13] The composite nature of the baryons and pion should cause such effects to be small.[14] The resulting $\mathcal{L}$ incorporates the physics of the original Walecka model plus PCAC (partial conservation of axial-vector current), the nonlinear scalar-meson interaction [Eqs. (1a) and (1b)], and Δ contributions to the π-N interaction. We calculate the nuclear medium effect on π propagation through the baryon one-loop approximation to the pion proper self-energy $\Pi(q)$ [Fig. 1(a)]. Then $\Pi(q)$ is used to compute the ring series of Fig. 1(b).

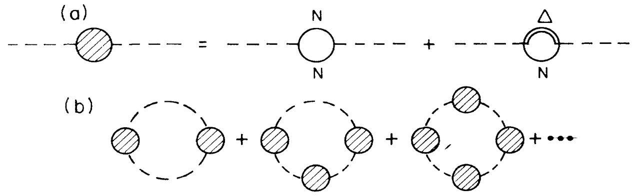

FIG. 1. (a) Second-order pion proper self-energy $\Pi^0(q)$. (b) Pion-ring diagrams.

The formalism of Chin[15] may be generalized to include the pions interacting with the baryons via derivative coupling. The resulting pion-ring energy density $\epsilon^{\rm ring}$ is given by

$$\epsilon^{\rm ring} = -\frac{i}{2} \int \frac{d^4 q}{(2\pi)^4} \{\ln[1 - \Pi(q) D_\pi^0(q)] + \Pi(q) D_\pi^0(q)\} . \tag{2}$$

In addition to $\epsilon^{\rm ring}$, we also include the familiar exchange energy (Fock term) in our calculations.

Equation (2) is a compact expression enabling ready computation. However, high momentum effects are included via the appearance of loops and the integration over all values of q. Furthermore, the effects of short-range repulsion are not yet incorporated. Therefore, first, we include a form factor $F(q^2)$ at the πNN and $\pi N \Delta$ vertices:

$$F(q^2) = \exp(q^2/\Lambda^2) . \tag{3}$$

This provides suppression in the spacelike region of q^2 of a form similar to both bag and nonrelativistic models of confined quarks and provides a stronger suppression (at large values of $-q^2$)[16] than monopole or dipole form factor. The second step is to neglect the medium modification of the terms of Fig. 1(a) with antibaryons. Then, the baryon-pair terms which are independent of the nuclear-matter density are taken into account by using the physical pion mass m_π as our input. The third step is to include the effects of baryon-baryon short-range correlations in the p-wave part of the pion self-energy via the familiar Landau-Migdal parameter g'. The use of g' in nonrelativistic calculations is needed to avoid the appearance of a pion condensate for experimentally ruled out regions of the nuclear-matter density.[4] The total pion proper self-energy with short-range N-N, N-Δ correlation effects may be written as

$$\Pi(q) = \Pi_s(q) + \frac{\Pi_p(q)}{1 - (g'/\mathbf{q}^2)\Pi_p(q)} ,$$

$$\Pi_s(q) = \Pi_N^0(\mathbf{q}=0) + \Pi_\Delta^0(\mathbf{q}=0) ,$$

$$\Pi_p(q) = \Pi_N^0(q) + \Pi_\Delta^0(q) - \Pi_N^0(\mathbf{q}=0) - \Pi_\Delta^0(\mathbf{q}=0) ,$$

where Π_s and Π_p are the s-wave and p-wave parts, respectively, in the nonrelativistic limit, and any q or k_F

dependence of g' is neglected. Here $g'=0.7$ is sufficient to prevent a pion condensation for values of $k_F \lesssim 2.2$ fm^{-1}.

Our approximations respect chiral symmetry in the limit $m_\pi^2 \to 0$. The pseudovector nature of our pion-baryon coupling preserves the property $\Pi(q=0)=0$ so that the pion remains massless even in the presence of our approximations.

The calculation is now specified. The mean-field expression for the energy density is supplemented by the pionic-ring series and the pionic Fock term. We minimize the energy density with respect to M^* and adjust m_ϕ^2 and g_ω^2 so that saturation properties may be reproduced. The results for average binding energy $(E/A-M)$ as a function of the Fermi momentum k_F are shown in Fig. 2(a) in the cases of $\xi=0.0$, 0.75, and 1.0. We took different Λ's for each case so that the resulting binding energies are the same in the case without the scalar mesons [see Fig. 2(b)], because we want to

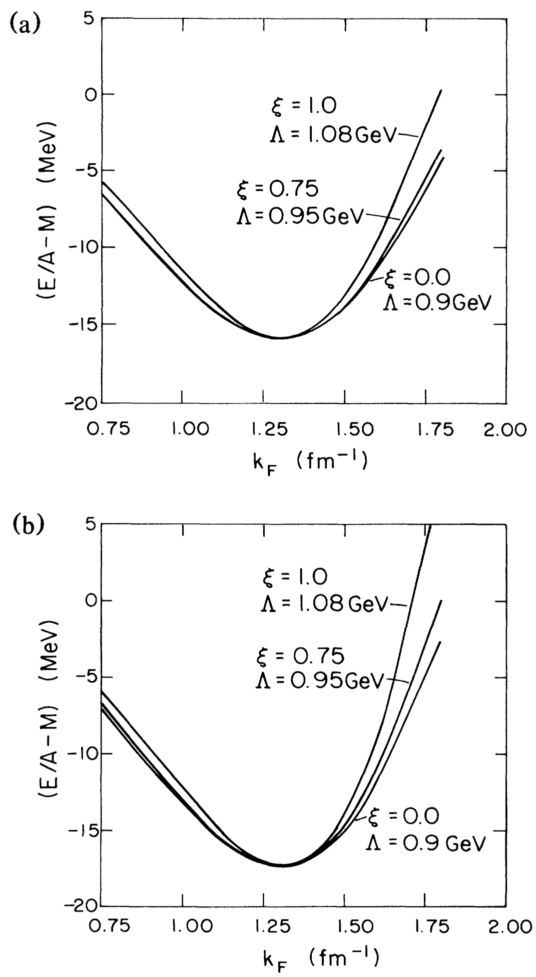

FIG. 2. (a) Binding energy per nucleon vs Fermi momentum with scalar mesons. (b) Binding energy per nucleon vs Fermi momentum without scalar mesons.

isolate the effects of the scalar mesons. Our calculations also have a Lee-Wick solution at $M^*=0$, and this is not shown.

The scalar-meson mass turns out to be quite large, $\gtrsim 2000$ MeV, which is consistent with the low-energy π-N scattering (see Table I). The ω-nucleon coupling is given by $g_\omega^2 \approx 35$–50, corresponding to a repulsive vector potential $g_\omega\omega_0 \approx 65$–$95$ MeV. This is smaller than the MFA result and is essential in allowing our values of the incompressibility κ^{-1} to be small enough to agree with the experimental value (≈ 200 MeV). The resulting value of M^* is $\approx (0.95$–$0.97)M$, so that the attractive scalar potential $g_\pi\phi_0 \approx 25$–45 MeV is not a significant effect. Note that our M^* is not the same m^* associated with single nucleon energies. If we neglect the scalar-meson effects and adjust the only parameter g_ω^2 so that $k_F=1.3$ fm^{-1} may be the saturation point, the corresponding binding energy is increased by only about 20% and the incompressibility is closer to the experimental value [Fig. 2(b)]. Thus, in our treatment, the nuclear-matter binding and its density dependence is mostly provided by the pionic terms. The small value of M^*-M reduces the influence of medium modification effects on vacuum-polarization terms. Hence, the second step of our approximation seems consistent.

Our dynamics is qualitatively similar to that required by the NN potential. The influence of the Δ in the two-pion-exchange potential [first term of Fig. 1(b)] provides substantial intermediate-range attraction, and ω exchange supplies the short-range repulsion. We have not attempted a detailed comparison with NN phase shifts. Our work is similar to that of Refs. 8 and 11 in this respect.

The importance of the relativistic version of the pionic-ring series is the salient feature of our results. Δ's are important. Setting Π^Δ to zero in the calculation of Fig. 2 would lead to an unbound nuclear system. We also describe the sensitivity to the parameters: ξ, g', and Λ. Changing ξ from 1.0 to 0.75–0.0 causes the incompressibility to decrease from 190 to 145–135 MeV. This is because keeping E/A fixed in the presence of s-wave repulsion from the Δ is achieved by reducing the g_ω^2. Next consider an increase of g' from our value of 0.7. This, important at high density, can be compensated by decreasing g_ω^2. The most crucial sensitivity comes from the choice of form factors. Using a monopole form factor with the same Λ instead of the form of Eq. (3) increases the pionic attraction by at least a factor of 2. Furthermore, increasing Λ by about 10% significantly enhances the pionic attraction which is important at high density. More repulsion from the ω meson cancels this high-density attraction, leaving E/A fixed. These effects cause saturation to occur at a low value of the nuclear density unless there is a decrease in M^*. The net result of increasing Λ is that E/A varies slowly with k_F and κ^{-1} is too small.

The sensitivity to the parameters is unfortunate, but not unexpected. Further relativistic studies of the microscopic nature of g', Λ, and ξ are needed to improve our understanding and justify the values we use.

The result that nuclear binding is dominated by the pionic-ring series, with Δ isobars playing a significant role and scalar mesons relatively unimportant, is somewhat different than the findings of other relativistic treatments. It is therefore interesting to seek experiments to distinguish the approaches. One consequence of our treatment is the existence of a pion condensate for $k_F \gtrsim 2.3$–$2.6\ \mathrm{fm}^{-1}$. Another challenge is to understand the spin effects observed in proton-nucleus scattering.

The successful reproduction of the experimental values of E/A, ρ, and κ^{-1} indicates that studies of relativistic pion dynamics are worthwhile.

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