

**Aharonov and Vaidman Reply:** In our recent Letter<sup>1</sup> we defined a *new* concept: a *weak value* of a quantum variable. We showed that a standard measuring procedure with weakened coupling, performed on an ensemble of both preselected and postselected systems, yields the weak value. The intuitive picture can be seen from our general approach<sup>2</sup> in which we consider two wave functions for a single system at a given time: the usual one evolving toward the future, and another evolving backward in time toward the past. Weak enough measurements do not disturb the above two wave functions and thus, the outcomes of such measurements should reflect properties of both states. The weakness of the interaction, therefore, is the essential requirement for the above measuring process. We claim that *for any measuring procedure of a physical variable the coupling can be made weak enough such that the effective value of the variable for a preselected and postselected ensemble will be its weak value.*

Leggett<sup>3</sup> argues that our result has “little relevance to the theory of measurements as conventionally understood” because it relies on a “very specific choice” of the interaction: our (and his) Eq. (1). This equation represents the measuring interaction of the von Neumann formalism, the conventional theory of measurements, and we used it only for the proof; the result itself does not rest on the specific form of the interaction. The only requirement is the weakness of the interaction.

Another point of Leggett is that our result is valid only up to first order in  $\lambda$ . Our requirement of weakness [Eq. (4)] ensures that all contributions beyond the first order can be neglected; therefore, the first order is all that we need. Our “weakness” requirement [Eq. (4)] is, however, too strong. We have since refined it,<sup>2</sup> and it turns out that there are situations (as the one in Ref. 4) in which our result is valid for  $\Delta \gg 1/\lambda$ , i.e., for higher orders in  $\lambda$ .

The weakness condition implies that the requirement of the orthogonality of  $\chi_i$  (see Leggett), which is the property of ideal quantum measurements, cannot be fulfilled. Although any realistic experiment has some overlap between  $\chi_i$ , the situation considered in our Letter is very different: The overlap is almost complete. It corresponds to a very large uncertainty of a single measurement. This is what prevented Peres<sup>5</sup> from considering  $\langle p \rangle$  (see his Comment) as an outcome of the measurement. We, however, believe that “the competent experimenter” will not be concerned with wide spread for final  $p$ , when he knows that the initial spread was large too. He will not consider it as “conflicting data for  $p$ ” when the average of the outcomes converges to a definite value as  $1/\sqrt{N}$ . Even if the experimenter, for whatever reason, computes the entire histogram of  $p$ , he will not be able to “unmistakably recognize the two peaks at  $p = \pm 1$ .” Although  $f(p)$  is, indeed, a superposition of two Gaussians

shifted by  $\pm 1$ , it is, approximately, equal (in the limit  $\Delta \rightarrow 0$ ) to the Gaussian shifted by the value 100. The two peaks can be found only if it is known in advance that  $f(p)$  is a superposition of two Gaussians.

In a previous paper<sup>4</sup> we described an example in which the measurement yielded a weak value with a small uncertainty. The disadvantage of that example was that it could be obtained only very rarely. (It explains why nobody, so far, has reported an unusual weak value as an outcome of a real experiment.) But even in the situation considered in our Letter, which is not a rare event, there is a physical variable whose measurement yields a weak value with an arbitrarily small uncertainty. This is, in fact, the variable which is measured in the experiment proposed in our Letter. The outcome of this experiment is the position of the center of the spot on the screen. The shift of the spot is proportional to the total momentum in the  $z$  direction of the  $N$  postselected particles  $\sum_{i=1}^N p^{(i)}$  which is, in turn, proportional to the  $z$  component of the total spin  $\sum_{i=1}^N \sigma_z^{(i)}$ . Consequently, the shift of the spot yields the (weak) value:  $(\sum_{i=1}^N \sigma_z^{(i)})_w = 100N$ . Its uncertainty, however, is proportional to  $\sqrt{N}$  and, therefore, much smaller than the observed shift. Thus, from the fact that the sum of the  $z$  components of the  $N$  spin- $\frac{1}{2}$  particles is equal to  $100N$  we *infer* that for each particle  $(\sigma_z)_w = 100$ .

Finally, there is a confusion that may arise from the last sentence of Peres’s Comment. We state again that our result *does not* contradict “the rules of elementary quantum mechanics.” Our result is hidden in the standard formalism behind the unexpected mathematical identity: our Eq. (7).

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<sup>1</sup>Y. Aharonov, D. Z. Albert, and L. Vaidman, Phys. Rev. Lett. **60**, 1351 (1988).

<sup>2</sup>Y. Aharonov and L. Vaidman (to be published).

<sup>3</sup>A. J. Leggett, preceding Comment, Phys. Rev. Lett. **62**, 2325 (1989).

<sup>4</sup>Y. Aharonov, D. Albert, A. Casher, and L. Vaidman, Phys. Lett. A **124**, 199 (1987).

<sup>5</sup>Asher Peres, preceding Comment, Phys. Rev. Lett. **62**, 2326 (1989).