Quantum Measurements with Postselection

In a recent Letter,[1] Aharonov, Albert, and Vaidman (AAV) claim that, with a suitable preselection and postselection of quantum systems, the result of a measurement of a quantum variable A can be larger than the largest eigenvalue of A. This surprising result is due to a faulty approximation in Eq. (3) of AAV. To see the error, consider explicitly the simple case $A=\sigma_z$. The final wave function of the measuring device—the left-hand side of Eq. (3)—is

$$\langle\psi_f|\exp(-iq\sigma_z-q^2/4\Delta^2)|\psi_i\rangle = (\mu e^{-iq}+\nu e^{iq})\exp(-q^2/4\Delta^2)\,,$$

where $\mu=\frac{1}{2}\langle\psi_f|(1+\sigma_z)|\psi_i\rangle$ and $\nu=\frac{1}{2}\langle\psi_f|(1-\sigma_z)\times|\psi_i\rangle$. The Fourier transform of this wave function (its p representation) is proportional to

$$\mu\exp[-\Delta^2(p-1)^2]+\nu\exp[-\Delta^2(p+1)^2]\,,$$

with two peaks at $p=\pm 1$, as expected. On the other hand, Eq. (5) of AAV gives a single peak at $p=(\mu-\nu)/(\mu+\nu)$.

A normally designed measurement must have $\Delta\gg 1$, so that the peaks are narrow.[2] On the other hand, a "weak" measurement as defined by AAV has $\Delta\ll 1$ and then the two broad peaks overlap and may even interfere destructively, for suitable μ and ν. In that case, the average value of p,

$$\langle p\rangle=\frac{|\mu|^2-|\nu|^2}{|\mu|^2+|\nu|^2+(\mu\bar{\nu}+\bar{\mu}\nu)\exp(-2\Delta^2)}\,,$$

may be larger than 1. This should not be a matter of concern because the standard deviation $[\langle p^2\rangle-\langle p\rangle^2]^{1/2}$ is even larger than that (this is due to the destructive interference mentioned above, which causes probabilities not to add in the classical way). As AAV point out, the large dispersion does not prevent us from determining $\langle p\rangle$ with arbitrary accuracy, if the experiment may be repeated as many times as we wish.

However, achieving this perfect accuracy is no license to proclaim that σ_z was actually observed to have an average value equal to the measured $\langle p\rangle$. Any competent experimenter, faced with a large number of conflicting data for p, would compute not only $\langle p\rangle$ and $\langle p^2\rangle$, but the entire histogram of p, and then would unmistakably recognize the two peaks at $p=\pm 1$, and ascribe their widths to the large initial uncertainty in p. The experimental results, if correctly interpreted, obey the rules of elementary quantum mechanics.

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[2] A. J. Leggett, preceding Comment, Phys. Rev. Lett. **62**, 2325 (1989).

