Comment on "How the Result of a Measurement of a Component of the Spin of a Spin-$\frac{1}{2}$ Particle Can Turn Out to be 100"

I shall argue that the above claim[1] is of little relevance to the theory of measurement as conventionally understood, because it relies on a highly nonstandard use of the concepts "value" and "measure," and in particular on the elevation of a particular form of interaction from a secondary and inessential ingredient of the measurement process to its defining characteristic.

Consider an ensemble of "systems" S which possess, *inter alia*, an observable A represented by a Hermitian operator $\hat{A}$ with a finite number of discrete eigenvalues a_i and corresponding eigenfunctions ϕ_i. Each system is allowed to interact with a different "device" D drawn from an ensemble of identical devices prepared in an initial pure state χ_0. The interaction between S and D is such that the initial state $\phi_i\chi_0$ of $S+D$ evolves into the final state $\phi_i'\chi_i$. It is emphatically *not* assumed at the present stage that the various χ_i are mutually orthogonal or that $\phi_i'=\phi_i$.

Use of D as a "measuring device" for S in the usual sense (i.e., so as to read off a unique value of A for *each individual system*) requires of course as a necessary condition that the χ_i be mutually orthogonal (see, e.g., AAV's Ref. 1, p. 440). AAV, however, content themselves[2] with a much less stringent notion of "measurement," according to which it is required only that inspection of the ensemble of devices D after the S-D interaction should yield the expectation value of A on the (initial) S ensemble. Since, as noted by AAV, by making the ensemble large enough we can determine the final-state density matrix of D to any desired accuracy, it is clear that for this purpose almost any choice of the χ_i will do; indeed, for the case of spin $\frac{1}{2}$ the only choice which would *not* serve is to make the two χ_i identical up to a phase! Despite this, AAV make a *very specific* choice: Assuming that D is characterized by a single variable q, they choose the S-D interaction Hamiltonian to be

$$\hat{H}_{S\text{-}D}=-g(t)\hat{A}q\,, \tag{1}$$

where $g(t)$ has the properties specified by them, and, moreover, demand (roughly speaking) that the quantity $\lambda\equiv\max(a_i)\Delta\pi\int g(t)dt$ be small compared to 1, where $\Delta\pi$ is the rms dispersion, in the state of χ_0, of the momentum conjugate to q.

If the initial state of the S ensemble is ψ_{in}, and we "postselect" a final state ψ_f as described by AAV, the (unnormalized) state of the subensemble of devices D so selected is

$$\chi=\sum_i(\psi_f,\phi_i')^*(\psi_{\text{in}},\phi_i)\chi_i\,. \tag{2}$$

In general this state bears no simple relation to that obtained in any experiment without postselection. However, in the special case described by Eq. (1) it is easily seen that up to order λ the state (2) is identical to that which would have been obtained, without postselection, by substituting for $\hat{A}$ in expression (1), the c-number quantity $A_w\equiv\langle\psi_f|\hat{A}|\psi_{\text{in}}\rangle/\langle\psi_f|\psi_{\text{in}}\rangle$. AAV state that "the standard interpretation" of this result is that A_w is "the measured value of A" and call A_w "the weak value of A for [the] preselected and postselected ensemble" (of S).

In what sense is A_w a "value" of RA for this ensemble? It is (trivially) neither the unique value nor the ensemble mean: The only thing it characterizes is the effect of the S ensemble on the state of an ensemble of devices essentially identical to D [i.e., coupled by Eq. (1) with $\lambda\ll1$]. (In a true measurement, by contrast, the measured value tells us much more than just the effect of the system on the measuring device.) Moreover, the equivalence of this effect to that of a nonpostselected ensemble with a unique value of A, which is apparently the sole basis for AAV's statement (above) is itself valid only to lowest nontrivial order in λ. (Cf. Comment by Peres.[3])

AAV's claims, then, rest crucially on their identification of the interaction (1) as essentially *defining* "measurement." In fact, however, when used as (one) component of a *true* measurement process of the Stern-Gerlach type, Eq. (1) (with $\lambda\gg1$, of course) has no fundamental significance in its own right, but is purely a *means to an end*, namely the orthogonalization of the different χ_i; in real-life experimental practice it is not even a component, let alone the essence, of all or even most important measurement processes. For a measurement in the less stringent sense considered by AAV it is, as pointed out above, even less essential. In other words, it is precisely the notion of "standard measuring procedure" which is at issue between us.

This work was supported through the John D. and Catherine T. MacArthur Foundation at the University of Illinois.

A. J. Leggett
Department of Physics
University of Illinois at Urbana-Champaign
Urbana, Illinois 61801

Received 6 May 1988
PACS numbers: 03.65.Bz

---

[1] Yakir Aharonov, David Z. Albert, and Lev Vaidman, Phys. Rev. Lett. **60**, 1351 (1988); hereafter AAV.

[2] Cf. also Y. Aharonov, D. Albert, A. Casher, and L. Vaidman, Phys. Lett. A **124**, 199 (1987), the bulk of which, however, refers to a significantly *different* experiment.

[3] Asher Peres, following Comment, Phys. Rev. Lett. **62**, 2326 (1989).