Fragmentation of Stretched Spin Strength in ^{28}Si

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Calculations have been made to explore the effect of configuration mixing in a large basis on the fragmentation of the "stretched" $M6$ strength in ^{28}Si. Our work extends a previous calculation, which allowed a single particle in the $f_{7/2}$ orbit and the remainder in the (unrestricted) $1d_{5/2}$ and $2s_{1/2}$ orbits, by also allowing up to four particles in the $1d_{3/2}$ orbit. It is found that configuration mixing within this expanded basis gives an improved description of the spectrum and several other properties of the observed states.

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The past decade has seen the accumulation of an extensive collection of experimental data[1-3] concerning "stretched" particle-hole states in nuclei. These states, for the even-even nuclei typically studied, have negative parity and total angular momentum $J=j_h+j_p$ which is the fully aligned sum of the maximum possible hole and particle angular momenta, $j_h=l_h+\frac{1}{2}$ and $j_p=l_p+\frac{1}{2}$, in the valence shell and the shell lying immediately above it, respectively. In ^{28}Si, the nucleus of interest here, these are 6^- states. What makes such states most interesting is that, of the many thousands of configurations with this spin and parity in a $1\hbar\omega$ basis, only the stretched configuration $1f_{7/2}1d_{5/2}^{-1}$ can contribute to a one-step $M6$ transition from the ground state. As a result, inelastic scattering measures the distribution of this unique configuration among all the 6^- states. Interpretation of the inelastic-scattering data is simplified because this $M6$ excitation is mediated by a single spin density which provides a common source (albeit multiplied by a probe-dependent interaction strength) for the cross section measured in electron, nucleon, and pion inelastic-scattering reactions.[1-5]

Experiments with the above probes give consistent results for the spin strength attributable to the stretched configuration.[1-3,5] The observed strength is typically a small fraction of the strength expected for pure particle-hole excitation—generally less than $\frac{1}{3}$ for isoscalar excitations and less than $\frac{1}{2}$ for isovector excitations. It is this depletion of strength we wish to understand. For magnetic transition of lower multipolarity, particularly $M1$ and Gamov-Teller transitions, calculations[6-8] indicate that non-nucleonic degrees of freedom such as the Δ give important contributions to the transition matrix elements and convection currents enter to complicate the analysis of scattering data. When realistic interactions are employed, these same models[6-10] indicate only small corrections to stretched transitions due to Δ degrees of freedom and core polarization. Furthermore, the core-polarization corrections tend to be nearly identical for $T=0$ and 1 excitations,[8,10] as emphasized in Ref. 5. Thus it is expected that valence-configuration-mixing effects are primarily responsible for the observed depletion of stretched spin strength. Because of this, stretched transitions provide an ideal laboratory for an acute test of the shell model. It will be shown that the shell model passes this test.

Initial studies of this problem using the shell model[11-13] were only able to explain about half of the depletion of strength, with the exception of studies of stretched 4^- states in p-shell nuclei where the small basis size allowed full $(0+1)\hbar\omega$ calculations. The challenge is to extend these results to the s-d shell (and beyond) so that systematic comparison can better test the model. In the only prior shell-model study in the s-d shell, Amusa and Lawson[13] (hereinafter to be called AL) examined the effect of limited configuration mixing on several aspects pertaining to the single-particle transition $1d_{5/2}\rightarrow 1f_{7/2}$ in ^{28}Si. Liu and Zamick[14] have made related studies using the rotational model with Coriolis mixing. In the work of AL, the zero-order (pure particle-hole) prediction is based on the $(1d_{5/2})^{12}\rightarrow(1d_{5/2})^{11}(1f_{7/2})$ transition. By adding the $1s_{1/2}$ level to this zero-order model, AL showed significant improvement in the inelastic-scattering cross sections and spectroscopic factors for the lowest $T=0$ and 1 6^- state and the $B(M1)$ transition rate between them. However, AL noted a discrepancy between their findings and the experimental limits on the full spectrum of 6^- strength determined from pion and electron scattering. With respect to all of this, AL conjectured that additional improvements could be effected by further enlarging the model space to include the $1d_{3/2}$ level to some degree.

In this Letter we report the results of carrying out just such a project, having in fact performed calculations in the basis $(d_{5/2},s_{1/2})^{11-n}d_{3/2}^{n}f_{7/2}$ with up to $n=4$ allowed. This contains 75% of the 6^- states in a full $(sd)^{11}f_{7/2}$ basis, and thus gives a good measure of the results to be expected from full $(0+1)\hbar\omega$ configuration mixing in s-d shell nuclei. Since our goal was mainly to observe the effect of changing the basis size in the s-d-shell part of the problem, we use the Shiffer-True[15] spin-dependent central interactions as AL to connect the s-d and f configurations, and compare the results that AL obtained with $n=0$ and the Wildenthal, McGrory, Halbnert, and Glaudemans[16] effective Hamiltonian appropriate to the $d_{5/2},s_{1/2}$ space to those with $n\le 4$ and the Wildenthal[17] effective Hamiltonian for the full s-d shell. (Initial checks indicate that the relatively small truncation in the $d_{3/2}$ orbit does not have large effects on the problem; we will publish results for the full s-d-shell calculation in a future paper.) Like AL, we adjust the $f_{7/2}$ single-particle energy to give the correct excitation energy for the yrast 6^- $T=0$ state.

Since our central concern is with the depletion of strength seen in inelastic scattering, we are interested in matrix elements

$$Z_T=\langle\Psi_{6^-,T}||A^{6}_{T,0}(f_{7/2},d_{5/2})||\Psi_{\rm g.s.}\rangle \quad (1)$$

(reduced in spin space only) of the operator[18]

$$A^{6,M}_{T,T_z}(j_f,j_i)=[a^{\dagger}_{j_fm_f,t_{z_f}}a_{j_im_i,t_{z_i}}]^{6,M}_{T,T_z} \quad (2)$$

for the full spectrum of 6^- states in ^{28}Si. In this expression, $a^{\dagger}$ creates a particle, a annihilates one, and the square brackets indicate coupling to the total angular momentum 6 and isospin T with the usual factors to ensure the proper rotational properties.

The simplicity of the stretched states guarantees that the inelastic-scattering cross section is proportional to Z_T^2, but the large number of possible states (about 60 000 in our basis) makes it impractical to solve for all of the eigenstates of the model system. The Lanczos algorithm provides a convenient alternative, since it obviates the need to pursue a full diagonalization in this space but, as Whitehead[19] has shown, still gives a description of the distribution of strength as a function of energy accurate to the 2Nth moment after N Lanczos iterations. This algorithm also converges fastest for the lowest-lying states, so we get "sharp" states for the yrast $T=0$ and 1 levels observed experimentally.

The specific procedure is to first get a good ground state for ^{28}Si within the (relatively small) s-d-shell basis. Then we form the "collective vector"[20] for the 6^- state of isospin T defined by

$$|\chi_T\rangle=A^{6,6}_{T,0}|\Psi_{\rm g.s}\rangle\,, \quad (3)$$

which contains all of the $M6$ strength in this basis and thus determines $\Sigma=\langle\chi_T|\chi_T\rangle$, a "sum rule" which measures the reduction in strength due to depletion of the $d_{5/2}$ orbit in the ground state. (This method is simplest to use for an $I=T=0$ ground state such as we consider here, where we also have that Σ is independent of T.) By choosing the first Lanczos vector to be the normalized collective vector, we can directly and cheaply compute Z^2 from Σ and the coefficient of this vector (a clear measure of its fragmentation) in each pseudoeigenvector at any stage of the calculation. The eigenvectors for the observed states are formed at the end of the sequence of iterations so that we can also calculate such things as the

TABLE I. Measured properties of the observed 6^- states in ^{28}Si compared to results from the simple model $(f_{7/2}d_{5/2}^{-1})$, the AL model $(f_{7/2}[d_{5/2}s_{1/2}]^{11})$ from Ref. 13, and this work. Note that the full sd space would give 29 908 $T=0$ states and 53 637 $T=1$ states.

Observable	Simple model	AL	This work	Expt.
11.71 MeV ($T=0$)				
Z_T^2	1.0	0.258	0.203	0.14 ± 0.04[a]
C^2S	0.5	0.206	0.156	0.21 ± 0.03[b]
14.36 MeV ($T=1$)				
Z_T^2	1.0	0.522	0.374	0.33 ± 0.04[a]
C^2S	0.5	0.368	0.234	0.20 ± 0.02[b]
$T=1\rightarrow T=0$ transition				
$B(M1)$ (e^2fm^2)	0.162	0.073	0.052	0.031 ± 0.004[c]
ΔE (MeV)	0.70	1.4	2.65	2.78
Basis size				
6^- $T=0$	1	95	21 653	
6^- $T=1$	1	144	40 386	
Sum rule Σ	1.0	0.854	0.785	

[a]Average of Refs. 1–3, 5, and 22 with appropriate error estimate.
[b]Average of Refs. 23 and 24 with appropriate error estimate.
[c]Reference 25.

spectroscopic factor for stripping to these states and the $B(M1)$ transition rate between them. All calculations were done with the VLADIMIR system of codes[21] adapted to the CYBER-205 and ETA10 computers at Florida State.

Table I and Fig. 1 show the result of our calculation after 24 Lanczos iterations for each isospin compared to the AL results.[13] Following the Livermore approach,[20] the curves in Fig. 1 are based on the strength of Z^2 for the individual eigenstates spread by a Gaussian of width $\langle (H - \langle H \rangle)^2 \rangle^{1/2}$ or 100 keV, whichever is larger. This curve is proportional to the inelastic-scattering cross section at the peak of the angular distribution. These curves are to be compared with the "data" points derived from the experimental strengths and energies of the observed states as described in Table I. The energy of the yrast 6^- $T=0$ state is a free parameter in both calculations, so one should focus on the improvement in the splitting between the $T=0$ and 1 states and the essential elimination of inelastic excitation of several low-lying $T=0$ states, which are strong enough in the AL calculation to have been observed by existing experiments, but were not. The effect of configuration mixing in the larger basis is to push the missing strength farther up in energy and spread it over many states. This is the mechanism which leads to a single yrast state with a small fraction of the total strength while the bulk of the single-particle strength remains unobserved. It is also significant that these effects are greater on the $T=0$ states. This is the key result, emphasized in Ref. 5, that cannot be obtained in core-polarization models with realistic forces.[8,10]

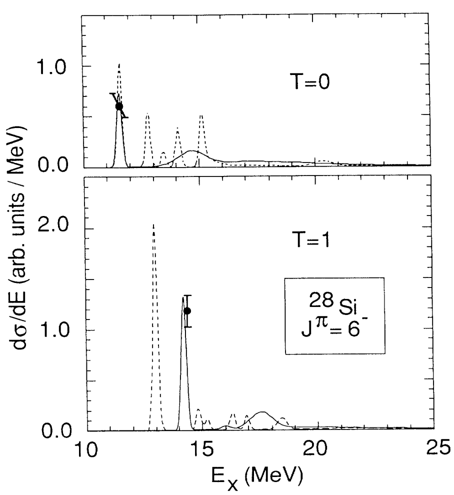

FIG. 1. The curves show the strength function for inelastic scattering calculated by AL (dashed curve) in a $d_{5/2},s_{1/2}$ basis (Ref. 13) and in this work (solid curve) after allowing up to four particles in the $d_{3/2}$ orbit compared to "data points" representative of the observed scattering strength. The error bar on the data point in the $T=0$ part of the figure is skewed so as not to obscure the curves behind it.

Following Amusa and Lawson, we also look at other features of these states to further judge the model. In Table I a number of measured properties of the 6^- states in ^{28}Si are compared with the prediction of the simple valence picture, the AL calculation, and the present results. One observes that there is general improvement as the basis space is increased. Clearly a large basis with configuration mixing is the essential element required to explain the details of the reduced strength observed for stretched magnetic transitions in the middle of the *s-d* shell. Diminishing returns from the effort to expand the basis are also evident. The full *s-d*-shell calculation mentioned above should lead to some additional improvement. Schmid's work[26] for the positive-parity states in ^{28}Si, with much larger model spaces and a different interaction, gives $\Sigma \cong 0.57$ which is consistent with this assertion. Finally, the core-polarization corrections,[6-10] which enter at the 10%-20% level, provide a small additional source of strength depletion.

There are a couple of limitations on our results that should be noted. The spuriosities of the lowest $T=0$ and 1 6^- states are 0.7% and 0.8%, respectively; however, we cannot remove spurious components in the rest of the spectrum without significantly expanding the basis. Additionally, the experimentally observed states are slightly unbound, and the resulting change in the wave functions would affect both the $M6$ strength calculation[27] and the matrix elements that enter our Hamiltonian. Proper inclusion of continuum effects is an important open problem. Nonetheless, our approach has the advantage of giving us the spectrum of $f_{7/2}d_{5/2}^{-1}$ strength which is the principal experimental observable.

In summary, fragmentation of the "stretched" state via conventional configuration mixing provides, within the slight limitations of our model space, a clear explanation of the observed properties of 6^- states seen in inelastic scattering from ^{28}Si. We plan to extend our work to the other nuclei in the *s-d* shell. Early results appear promising. Eventually we intend to explore the effects produced when we expand the basis to include the full, unrestricted *sd* basis and add the $f_{5/2}$ orbit to the space. The experimental challenge is to devise means of using spin observables to map out the full strength distribution even when it is otherwise obscured by background states.

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