Three-Body Forces and the Description of Light Nuclei

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It is shown that a schematic three-body force in addition to a general two-body force leads to a strongly improved description of the energies of $A=4$–16 nuclei. The values of the two strength parameters of the proposed three-body force, which are obtained empirically from energies and static moments of p-shell nuclei, are found to be similar to those of a realistic three-nucleon force.

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The nature of three-body forces and their significance for the shell-model description of various properties of $A>3$ nuclei are still unclear. In practically all shell-model calculations performed so far the interaction is restricted to contain effective one- and two-body terms only. However, one should expect that many-body forces contribute as well. The main reason is that the corrections to a two-body force, which are required to compensate for configurations which are excluded from the model space, give rise to many-body terms in the resulting effective interaction. Moreover, it may not be excluded that nuclear forces do not contain a realistic three-body part.

The usual restriction to a two-body interaction is based on the assumption that effects of many-body forces can be absorbed largely in an effective two-body interaction. However, there are some problems associated with the restriction to effective two-body interactions only. For the $A=18$–40 nuclei it is found empirically that a uniform mass dependence, i.e., all matrix elements of the effective two-body interaction are multiplied by a factor $A^{-0.3}$, is needed in order to obtain a good description of sd-shell levels.[1] Such a mass dependence may well originate from excluded many-body effects, since any two-body force yielding matrix elements with a linear mass dependence is equivalent to a three-body force. On the other hand, not every three-body force can be interpreted as a two-body force with a linear mass dependence. In the p shell the large radii of ^{6}Li and ^{7}Li indicate that the single-particle orbits are mass dependent, which leads to a nontrivial mass dependence of the one- and two-body matrix elements. Changes in these orbits may therefore be reflected in an effective three-body force (when one uses a Hamiltonian with mass-independent strength parameters).

We will show below that a uniform mass dependence of the effective two-body interaction gives only a marginal improvement for the $A=4$–16 nuclei, whereas the inclusion of a rather simple three-body force largely reduces the deviations between experiment and theory. The problems encountered when a two-body force is used for the description of light nuclei are mainly concentrated in the $A=6$–8 nuclei. Some authors therefore even discarded levels in these nuclei from their fits.[2,3] On the one hand, one may thus argue that levels in these light nuclei are not very suitable for a simple shell-model description and should not be used to determine an effective two-body interaction. But on the other hand, it is interesting to investigate whether these levels are especially affected when (effective) many-body forces are included in the shell-model Hamiltonian.

In the past some attempts have been made to include three-body interactions. Nash[4] investigated Gaussian three-body forces and their effect on states with maximum spatial symmetry in ^{4}He, ^{8}Be, ^{12}C, and ^{16}O. However, he did not perform shell-model calculations. In the work of Goldhammer, Hill, and Nachamkin[3] the three-body interaction is based on the second-order effects of a pure two-body tensor force. They obtained large improvements compared to the results of Cohen and Kurath,[2] but could not reproduce the low-lying resonances in ^{8}Be, even if a schematic four-body force was included.[5] Dirim, Elliott, and Evans[6] obtained an effective three-body interaction from the Sussex interaction with perturbation theory, but did not try to optimize their interaction. They noticed that the three-body interaction energy for ^{16}O should be about +12 MeV. This follows directly from the binding energies of mass 4, 5, 15, and 16 nuclei and from the assumption that only nucleons in the p shell are active.

In this work we investigate the need for a three-body force, in order to improve the description of $A=4$–16 nuclei in a p-shell model space. The calculations have been performed with the Utrecht shell-model program RITSSCHIL.[7] This general shell-model code treats many-body operators essentially the same way as one- or two-body operators.

The Hamiltonian H used is split into four parts corresponding to the zero-, one-, two-, and three-body parts of the effective interaction, i.e.,

$$H=H^{(0)}+H^{(1)}+H^{(2)}+H^{(3)}. \quad (1)$$

The parameters specifying H are considered to be mass and state independent (except in the case where we included a term A^{λ} in $H^{(2)}$). The zero-body part represents the energy of the ^{4}He core. The one- and two-body parts are most generally expressed in the 2 single-particle energies and the 15 two-body matrix elements, respectively. The most general parametrization of the three-body part $H^{(3)}$ leads to 51 three-body matrix elements. Not all these parameters can be determined accurately by the available experimental data. Therefore we searched for a schematic three-body interaction with only a few parameters. Optimal values for all parameters are determined by means of an iterative least-squares-fitting procedure to experimental data. This selected set of data is essentially the same as used in our previous work.[8,9] This means that the binding energies of ground states and excited states as well as the (available) static moments of some 75 levels with normal parity, i.e., $(-1)^A$, are taken into account. The static moments are included in the fit since they considerably improve the accuracy of some parameters in the Hamiltonian, which are rather poorly determined by a fit to energy levels only.[9]

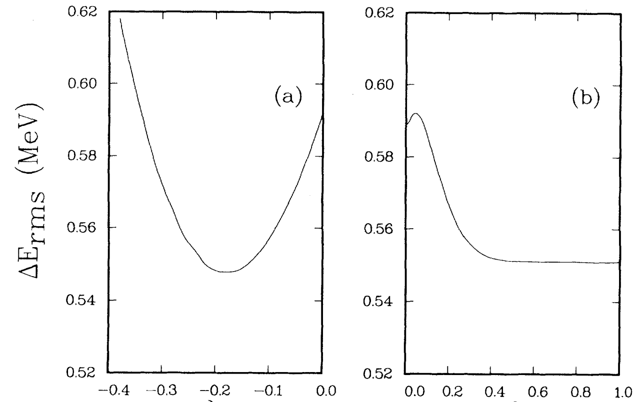

FIG. 1. (a) The rms deviation for energy levels vs the parameter λ in the mass-dependent term A^{λ} multiplying all two-body matrix elements. (b) The rms deviation for energy levels vs the range parameter β of a Gaussian three-body interaction. All other parameters are optimized.

First, we restrict ourselves to calculations without a three-body force. Similar to the work[1] of Wildenthal and Brown we investigated the effect of a mass-dependent multiplication factor A^{λ} for all two-body matrix elements. It is shown in Fig. 1(a) that optimal results are obtained for $\lambda \approx -0.18$ leading to an rms deviation for energy levels $\Delta E_{\rm rms} = 0.55$ MeV. However, the improvement is not significant compared to the result obtained with $\lambda = 0$, since the latter yields $\Delta E_{\rm rms} = 0.59$ MeV.

$$V^{(3)}_{\rm Gauss}(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3) = \frac{G}{\beta^3}\exp\left(-\frac{1-\beta}{\beta}[(\mathbf{r}_1-\mathbf{R})^2+(\mathbf{r}_2-\mathbf{R})^2+(\mathbf{r}_3-\mathbf{R})^2]\right). \tag{2}$$

The center of mass of the three nucleons is denoted by $\mathbf{R}$. The normalization is such that for $G=1$ the matrix element for the $|0s^3\rangle$ state is unity, independent of the value of the range parameter β. In the extreme short-range limit, i.e., with the range parameter $\beta \to 0$, this force becomes equal to a three-body δ force, whereas in the extreme long-range limit, i.e., $\beta = 1$, this yields a term proportional to $(A-4)^3$ in the Hamiltonian. In Table I we present analytical expressions for the matrix elements of $V^{(3)}_{\rm Gauss}$. These three-body matrix elements depend only on the spatial symmetry $[f]$ and the orbital angular momentum L, but not on the spin S, the total angular momentum J, or the isospin T. Matrix elements between states with different $[f]$, L, or S vanish. A variation between 0 and 1 of the parameter β does not lead to significant improvements; see Fig. 1(b). It is seen that a pure three-body δ force, i.e., $\beta=0$, gives almost no improvement at all over the results with the mass-independent two-body interaction.

Now we will show that much better results can be obtained with a three-body force of the following structure:

$$V^{(3)}_{A,C} = \sum_{\rm cycl}(A\{\sigma_1\cdot\sigma_2,\sigma_1\cdot\sigma_3\}\{\tau_1\cdot\tau_2,\tau_1\cdot\tau_3\} + C[\sigma_1\cdot\sigma_2,\sigma_1\cdot\sigma_3][\tau_1\cdot\tau_2,\tau_1\cdot\tau_3]) . \tag{3}$$

The summation runs over the three cyclic permutations of particles 1, 2, and 3. The $\{,\}$ and $[,]$ denote anticommutators and commutators with strength parameters A and C, respectively. The spin- and isospin-exchange terms correspond to those arising from the exchange of two (pseudoscalar and isovector) pions, so that the structure of this force is similar to that of the two-pion-exchange three-nucleon force first proposed by Fujita and Miyazawa.[10] The differences are the absence of a two-body tensor force in the spin (anti)commutators and the replacement of all radial functions in front of the

TABLE I. Three-body matrix elements for $V^{(3)}_{\rm Gauss}$ (with $G=1$).

$[f]$	L	$\langle p^3[f]LS\|V\|p^3[f]LS\rangle_{JT}$
[3]	1	$+\frac{7}{9}-\frac{10}{9}\beta+\frac{8}{9}\beta^2+\frac{4}{9}\beta^3$
[3]	3	$+\frac{2}{9}+\frac{1}{3}\beta^2+\frac{4}{9}\beta^3$
[21]	1	$+\frac{5}{9}\beta-\frac{1}{9}\beta^2+\frac{5}{9}\beta^3$
[21]	2	$+\frac{1}{3}\beta+\frac{1}{3}\beta^2+\frac{1}{3}\beta^3$
[111]	0	$+\beta^2$

TABLE II. Three-body matrix elements for $V_{A,C}^{(3)}$.

$[f]$	S	T	$\langle p^3[f]LS\,|\,V\,|\,p^3[f]LS\rangle_{JT}$
[3]	$\frac{1}{2}$	$\frac{1}{2}$	$-36A+144C$
[21]	$\frac{1}{2}$	$\frac{1}{2}$	$+12A-144C$
[21]	$\frac{3}{2}$	$\frac{1}{2}$	$-12A$
[21]	$\frac{1}{2}$	$\frac{3}{2}$	$-12A$
[111]	$\frac{1}{2}$	$\frac{1}{2}$	$+60A+144C$
[111]	$\frac{3}{2}$	$\frac{3}{2}$	$+12A$

$\sigma_i \cdot \sigma_j$ terms by unity. Results are found to be rather insensitive to such a radial dependence since all active particles occupy p-shell orbits. The anticommutator term in (3) can be written as a linear mass-dependent two-body force.[11] However, this does not hold for the commutator term.

In Table II the analytical expressions for the matrix elements of $V_{A,C}^{(3)}$ are presented. Note that in contrast to the three-body Gaussian force the matrix elements do not depend on L, but on S and T. Like the three-body Gaussian force, this one also contains only diagonal matrix elements and it is a central force, i.e., its matrix elements do not depend on J. A fit of the two parameters A and C, together with the eighteen parameters of the most general zero-, one-, and two-body interactions, leads to the three-body matrix elements presented in the fifth column of Table III.

The most general diagonal central three-body interaction in our model space is characterized by ten parameters, since in the p shell ten three-particle states with different $[f]$, L, S, and T exist. A fit of these ten parameters, instead of the two parameters A and C, leads to the matrix elements presented in the last column of Table III. Note that the two sets of matrix elements in this table are remarkably similar.

The energy levels have deviations $\Delta E_{\rm rms}=0.38$ MeV and $\Delta E_{\rm rms}=0.36$ MeV for the two- and ten-parameter cases, respectively. Both numbers are considerably smaller than those obtained without a three-body force; i.e., $\Delta E_{\rm rms}=0.59$ and 0.55 MeV for the optimum mass-independent and mass-dependent two-body interactions, respectively [see also Fig. 1(a)]. Most impressive therefore is the result obtained with the two-parameter three-body force $V_{A,C}^{(3)}$ for which we present some results.

The contribution of the three-body interaction to the binding energy of the ground states for a number of p-shell nuclei is presented in Fig. 2. In our model space the three-body interaction can contribute only to $A \geq 7$ nuclei, since it requires at least three particles in the p shell. One observes from Fig. 2 that the effect of the three-body force is attractive for $A<11$ and repulsive for $A \geq 11$.

The negative three-body energy for the ^{7}Li and ^{8}Be ground states can easily be understood. Their calculated

TABLE III. Empirical values of the three-body matrix elements (MeV); 2P stands for two parameters, etc.

$[f]$	L	S	T	$V_{A,C}^{(3)}$ (2P)	Central force (10P)
[3]	1	$\frac{1}{2}$	$\frac{1}{2}$	-0.83	-0.93
[3]	3	$\frac{1}{2}$	$\frac{1}{2}$	-0.83	-0.79
[21]	1	$\frac{1}{2}$	$\frac{1}{2}$	$+1.16$	$+0.71$
[21]	2	$\frac{1}{2}$	$\frac{1}{2}$	$+1.16$	$+1.08$
[21]	1	$\frac{1}{2}$	$\frac{3}{2}$	$+0.17$	$+0.38$
[21]	2	$\frac{1}{2}$	$\frac{3}{2}$	$+0.17$	$+0.14$
[21]	1	$\frac{3}{2}$	$\frac{1}{2}$	$+0.17$	$+0.13$
[21]	2	$\frac{3}{2}$	$\frac{1}{2}$	$+0.17$	$+0.24$
[111]	0	$\frac{1}{2}$	$\frac{1}{2}$	-2.15	-1.62
[111]	0	$\frac{3}{2}$	$\frac{3}{2}$	-0.17	-0.37

wave functions are almost completely symmetric in the spatial coordinates of the p-shell nucleons. Hence in ^{7}Li the three-body energy is about equal to the value of the $[f]=[3]$ three-particle matrix element, i.e., -0.83 MeV. In ^{8}Be the three-body energy for the spatial symmetric state is even 4 times as large. For the heavier nuclei the wave function necessarily is of mixed symmetry and the positive $[f]=[21]$ three-body matrix elements contribute as well. The result is that the three-body energy for the ground states gradually increases from $A=8$ to $A=16$.

With only the two-body interaction we calculate the four lowest states in ^{7}Li, and find that all of them have predominantly symmetry $[f]=[3]$, on the average 0.8 MeV too strongly bound, whereas the three lowest levels in ^{8}Be are calculated to be on the average 0.8 MeV too weakly bound. With the inclusion of the three-body force these systematic discrepanceis are both reduced by

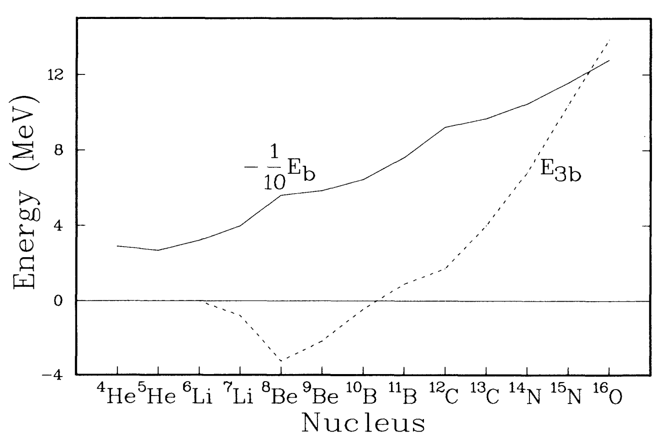

FIG. 2. Calculated three-body energy E_{3b} and total binding energy E_b for the ground states of p-shell nuclei with $T_z=0$ or $T_z=+\frac{1}{2}$.

a factor of 2. Also for $A > 8$ similar improvements occur. However, the discrepancies seem to be much less systematic than they are for the $A=7$ and $A=8$ nuclei. Therefore the evidence for the three-body force comes from nuclei throughout the p shell.

It follows from Table I that any Gaussian interaction with $0 \le \beta \le 1$ leads to three-body matrix elements which all have the same sign. Hence such an interaction can never produce a set of matrix elements similar to those given in Table III. Apparently exchange terms, leading to the negative signs of the $[f]=[3]$ and $[f]=[111]$ matrix elements and the positive signs of the $[f]=[21]$ matrix elements, are crucial ingredients for an optimized effective three-body interaction.

The two strength parameters of $V^{(3)}_{A,C}$ as determined from our fit to energies and static moments in p-shell nuclei are given by $A=-0.14$ MeV and $C=-0.009$ MeV. It is interesting to point out the similarity in ratio and sign between the present empirical values and those obtained from a realistic three-nucleon potential such as the Tuscon-Melbourne potential.[12] If one drops the tensor force and the radial dependence in this potential, one obtains the same structure as (3), with strength parameters $A=-0.035$ MeV and $C=-0.018$ MeV.

In conclusion, a schematic central three-body force leads to a strongly improved description of energies in the p-shell nuclei. The choice for the radial dependence of this schematic three-body force appears to be rather unimportant within the present model space. This may not hold anymore for calculations in a larger model space, where orbits with different radial dependence are active. For other observables, e.g., magnetic dipole and electric quadrupole moments, one does not obtain large improvements. However, in analogy with the present addition of three-body forces to the Hamiltonian, one might expect significant improvements for electromagnetic observables, if two-body corrections are added to the one-body operators.

[1]B. H. Wildenthal, Prog. Part. Nucl. Phys. **11**, 5 (1985).

[2]S. Cohen and D. Kurath, Nucl. Phys. **73**, 1 (1965).

[3]P. Goldhammer, J. R. Hill, and J. Nachamkin, Nucl. Phys. A**106**, 62 (1968).

[4]G. F. Nash, Nuovo Cimento **42B**, 8 (1966); **61B**, 261 (1969).

[5]J. L. Norton and P. Goldhammer, Nucl. Phys. **A165**, 33 (1971).

[6]H. Dirim, J. P. Elliot, and J. A. Evans, Nucl. Phys. **A244**, 301 (1975).

[7]D. Zwarts, Comp. Phys. Commun. **38**, 365 (1985).

[8]A. G. M. van Hees and P. W. M. Glaudemans, Z. Phys. A **314**, 323 (1983); **315**, 223 (1983).

[9]A. G. M. van Hees, A. A. Wolters, and P. W. M. Glaudemans, Phys. Lett. B **196**, 19 (1987); Nucl. Phys. **A476**, 61 (1988).

[10]L. Fujita and H. Miyazawa, Prog. Theor. Phys. **17**, 360 (1957).

[11]J. Carlson, V. R. Pandharipande, and R. B. Wiringa, Nucl. Phys. **A401**, 59 (1983).

[12]S. A. Coon, M. D. Scadron, P. C. McNamee, B. R. Barett, D. W. E. Blatt, and B. H. J. McKellar, Nucl. Phys. **A317**, 242 (1979).