Proton Decay in Three-Generation Matter-Parity-Invariant Superstring Models

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Proton decay in three-generation superstring models with intermediate scale is investigated. It is shown that matter-parity (M_2) invariance alone does not stabilize the proton. However, models with M_2 invariance and Calabi-Yau manifolds such that the exotic C-odd quarks and leptons are superheavy will generally lead to proton lifetimes consistent with existing data.

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Superstring models where compactification takes place on the three-generation Calabi-Yau manifold[1] allow for the breaking of the E_6 group symmetry to $[\mathrm{SU}(3)]^3 \equiv \mathrm{SU}(3)_C \times \mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$ by Wilson loops.[2] This compactification takes place at a mass scale M_c of order of the Planck mass ($M_{\mathrm{Pl}} = 2.4 \times 10^{18}$ GeV). Such models require an additional breaking of $[\mathrm{SU}(3)]^3$ to the standard-model $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ at an intermediate scale M_I. Previous discussions[3,4] have stressed the importance of the predictions of $\sin^2\theta_W$ and the proton-decay rate to delineate physically acceptable models. That valid intermediate-scale models with $\sin^2\theta_W$ in agreement with the experimentally determined value exists has been discussed elsewhere.[5] In this Letter we will show that these models also predict an experimentally acceptable rate of proton decay.

In the $[\mathrm{SU}(3)]^3$ phase, the massless multiplets consist[4] of nine nonets of leptons $L_i(\mathbf{1},\mathbf{3},\bar{\mathbf{3}})$ and six mirror generations $\bar{L}_i$, seven nonets of left-handed quarks $Q_i(\mathbf{3},\bar{\mathbf{3}},\mathbf{1})$ and four mirror generations $\bar{Q}_i$, and seven nonets of left-handed antiquarks Q_i^c with four mirror generations $\bar{Q}_i^c$. In conventional particle notation $L = L(l^\lambda, H^\lambda, H'_\lambda, e^c, \nu^c, N)$ and $Q = Q(q^\lambda, D)$, $Q^c = Q^c(u^c, d^c, D^c)$, where $l^\lambda, H^\lambda, H'_\lambda, q^\lambda$ are the lepton, Higgs-boson, and quark $\mathrm{SU}(2)_L$ doublets, D and D^c are Higgs color triplets, and N and ν^c are the SU(5) singlets which must grow vacuum expectation values (VEV's) if $[\mathrm{SU}(3)]^3$ is to break to the standard model at M_I. Recently, a detailed discussion of how the intermediate scale may form has been given.[6,7] The intermediate scale arises from three dynamical sources: (i) Supersymmetry breaking (assumed to occur at or near M_c), (ii) renormalizable $(\mathbf{27})^3$ and $(\overline{\mathbf{27}})^3$ contributions W_{R} to the superpotential W, and (iii) nonrenormalizable $(\mathbf{27} \times \overline{\mathbf{27}})^n / M_c^{2n-3}$ contributions W_{NR} to W arising from the integration out of the tower of Planck-mass superstring states. Supersymmetry breaking is assumed to give rise to soft-breaking running masses for each generation, and the interactions of W_{R} are assumed to turn one or more of these negative at scale M_I. (That this actually happens for the known Yukawa couplings of a simple symmetric Calabi-Yau manifold at a scale $M_I \gtrsim 10^{15}$ GeV has recently been demonstrated.[8]) This, combined with the nonrenormalizable couplings W_{NR}, then leads to the necessary VEV growths with $\langle N \rangle, \langle \nu^c \rangle \sim 10^{15}$ GeV breaking $[\mathrm{SU}(3)]^3$ to $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$.

The analysis of Refs. 6 and 7 assumed that the Calabi-Yau manifold at M_c preserves matter parity $M_2 \equiv CU_z$, where U_z is the element of $\mathrm{SU}(3)_C \times \mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$ which reverses the sign of $\mathrm{SU}(2)_{L,R}$ doublets (leaving other states unchanged) and C is a permutation transformation on the Calabi-Yau polynomial coordinates x_i, y_i, $i = 0,1,2,3$, such that $x_2 \leftrightarrow x_3$ and $y_2 \leftrightarrow y_3$ (leaving other coordinates unchanged). Matter parity is an essential ingredient in superstring models as it prevents[4,9] the disastrously rapid dimension-4 proton decay.[10] A remarkable feature of the results of Ref. 6 is that the lowest-lying extremum of the spontaneous breaking of $[\mathrm{SU}(3)]^3$ at M_I *preserves* M_2 (the extrema which break M_2 lying higher). In addition it has the physically necessary property that the M_2-preserving extremum also preserves $\mathrm{SU}(2) \times \mathrm{U}(1)$ at M_I (i.e., again extrema where VEV's of H^λ and H'_λ form also lie higher). It was thus possible in Refs. 6 and 7 to analyze which particles became superheavy and which remained light within the framework of matter-parity invariance, and we use these results here to analyze the problem of proton decay.

While matter parity is a necessary element for a phenomenologically acceptable model, as will be seen below, it is not sufficient to prevent too rapid proton decay as it does not inhibit decay via exotic quark channels. When combined, however, with models that have acceptable mass spectra for these exotic generations,[7] these models then do yield proton lifetimes consistent with experiment.

The renormalizable $[\mathrm{SU}(3)]^3$-invariant superpotential reads[6]

$$W_R = \lambda^1_{ijk}(\epsilon_{aa'a''} d_i^a u_j^{a'} D_k^{a''}) + \lambda^2_{ijk}(\epsilon^{aa'a''} u^c_{ai} d^c_{a'j} D^c_{a''k}) + \lambda^3_{ijk}(-H_i^\lambda H'_{\lambda j} N_k - H_i^\lambda \nu_j^c l_{\lambda k} + H'_{\lambda i} e_j^c l_k^\lambda) - \lambda^4_{ijk}(D_i^a N_j D^c_{ak} - D_i^a e_j^c u^c_{ak} + D_i^a \nu_j^c d^c_{ak} + q_i^{a\lambda} l_{\lambda j} D^c_{ak} - q^a_{\lambda i} H_j^\lambda u^c_{ak} - q_i^{a\lambda} H'_{\lambda j} d^c_{ak}), \quad (1)$$

where λ^1_{ijk}, etc., are coupling constants, i,j,k are generation indices, a,a',a'' are $SU(3)_C$ indices, and c means conjugate. When N and ν^c grow VEV's, proton decay can arise from four distinct sources which we examine now in detail.

(i) The first term in the third set of parentheses of Eq. (1) produces a mass growth for D^a fields: $M^d_{ik} D_i^a D^c_{ak}$, where $M^D_{ik} \equiv -\lambda^4_{ijk}\langle N_j\rangle$. Proton decay then proceeds through the superheavy D^a and D^c_a fields as in SU(5) supergravity theory. This case has been previously discussed,[11] and we do not consider it further here other than to note that agreement with experimental rates requires that M^D have no zero eigenvalues, and that the eigenvalues of M^D be $\gtrsim 10^{14}$ GeV. The former condition is true for the simple symmetric Calabi-Yau manifolds where the Yukawa coupling constants are known, while the latter condition, i.e., $\lambda^4\langle N\rangle \gtrsim 10^{14}$ GeV, is shown to be expected in Ref. 6 for most mechanisms leading to N VEV growth.

(ii) When N and ν^c *simultaneously* grow VEV's, there is mixing between the D^a and d^a fields[12] leading to proton decay via the *light* d^a quarks. Including in the VEV the growth of H' at the electroweak scale, the mass terms read

$$W_3^{D\text{-}d} = (M_{mn} D_m^a D^c_{an} + M'_{mr} D_m^a d^c_{ar} + \mu_{rs} d_r^a d^c_{as}) + (M_{rs} D_r^a D^c_{as} + M'_{rn} D_r^a d^c_{an} + \mu_{mn} d_m^a d^c_{an}), \quad (2)$$

where $M_{mn} = -\lambda^4_{mjn}\langle N_j\rangle$, $M'_{mr} = -\lambda^4_{mjr}\langle \nu_j^c\rangle$, $\mu_{rs} = \lambda^4_{rjs} \times \langle H_j'\rangle$, etc. In Eq. (2), m,n run over the C-even generations and r,s over the C-odd generations. The matter parity of each generation is given in Table I, showing that Eq. (2) is the most general matter-parity-preserving mass term. Equation (2) can be diagonalized by a biunitary transformation leading to eigenstates $\hat{D}^a_m$, $\hat{d}^a_m$, etc. It is clear that D_m and d_r mix and D^c_m and d^c_r mix in this diagonlization, i.e.,

$$D^c_m = C^1_{mn}\hat{D}^c_n + S^1_{ms}\hat{d}^c_s, \quad d^c_r = C^1_{rs}\hat{d}^c_s - S^1_{rn}\hat{D}^c_n, \qquad D_m = C^2_{mn}\hat{D}_n + S^2_{ms}\hat{d}_s, \quad d_r = C^2_{rs}\hat{d}_s - S^2_{rn}\hat{D}_n, \quad (3)$$

and similar equations holding for the D_r, d_m system. Just below M_I, the three massless generations of d quarks of the standard model are linear combinations of the C-even d_m states,[6] while all other particles get superheavy masses from the nonrenormalizable W_{NR} interactions.[7] Thus after SU(2)×U(1) breaking, Eq. (2) implies that the $\hat{D}_r$ states have mass $\sim M_{rs} \sim 10^{15}$ GeV, while the three standard-model linear combinations of $\hat{d}_m$ have electroweak masses $\sim\mu_{mn}$ (and are the d, s, and b quarks). Similarly the light leptons are three linear combinations of[6] l_m, e^c_m, other leptons being superheavy.[7]

TABLE I. Matter parity (M_2) of particle states. r denotes C-even and n denotes C-odd states.

M_2-even states	M_2-odd states
l_r, e^c_r, ν^c_r	l_n, e^c_n, ν^c_n
q_r, u^c_r, d^c_r	q_n, u^c_n, d^c_n
D_n, D^c_n, N_n	D_r, D^c_r, N_r
H_n, H'_n	H_r, H'_r

The dimension-5 proton-decay interactions discussed in Ref. 11 and (i) above arise from the λ^1, λ^2, and λ^4 D and D^c interactions of Eq. (1). Eliminating these fields in terms of the mass eigenstates by Eq. (3) leads to the dangerous possibility of very rapid proton decay through the light $\hat{d}_m$ quarks. We now show that matter-parity invariance forbids this. Consider, for example, the λ^1, λ^4 proton-decay couplings in Eq. (1). From Table I, the M_2-invariant possibility involving light quarks is

$$\lambda^1_{nn'm}\epsilon_{aa'a''} d_n^a u_{n'}^{a'} D_m^{a''} - \lambda^4_{nn'm} q_n^{a\lambda} l_{\lambda n'} D^c_{am}. \quad (4)$$

Upon eliminating D_m, d_n, etc., in terms of the mass eigenstates by Eq. (3), all terms involve at least one particle from the exotic d_s sector, i.e., $\lambda^1 S^2 \hat{d}_n u_{n'} \hat{d}_s$, and $\lambda^4 S^1 \hat{q}_n l_{n'} \hat{d}^c_s$. This will lead to dimension-5 proton-decay amplitudes analogous to those discussed in (i) above but mediated by $\hat{d}_s$ rather than $\hat{D}_m$. Further, the vertices now contain the factor $S^1 S^2$. One may estimate the size of S^1 and S^2 from Eq. (2). Since $\mu_{rs} \ll M_{mn}, M'_{mr}$, in the limit $\mu_{rs} \to 0$ one sees that S^2 vanishes. The D^c_m-d^c_r mixing is governed by M and M', i.e., $S^1 \sim M'/(M^2+M'^2)^{1/2}$ and hence can be ~ 1. For nonzero μ_{rs}, S^2 must be proportional to both μ_{rs} and M'_{rn} to get mixing, i.e.,[13] $S^2 \sim \mu M'/(M^2+M'^2)$. Thus S^2 is very small: $S^2 \sim 10^{-13}$. The $\hat{d}_s$-mediated decay then effectively replaces $1/M_{\hat{D}}$ in (i) above by $S_1 S_2/m_{\hat{d}}$. Thus even if the $\hat{d}_s$ were of electroweak mass, i.e., ~ 100 GeV, this decay amplitude would be no larger than that of (i) and hence compatible with experiment. Actually, however, as discussed above, the $\hat{d}_s$ states all grow superheavy masses (i.e., $\sim 10^{15}$ GeV) from the nonrenormalizable interactions,[7] and hence this decay mode is negligibly smaller than those discussed in (i).

Baryon- and lepton-number-violating terms such as $\lambda' \hat{d}_n u_{n'} \hat{d}_m$ or $\lambda^4 \hat{q}_n l_{n'} \hat{d}_m$ where all particles are light would indeed have produced rapid proton decay, and the fact that they did not appear in the above discussion is indeed just a consequence of M_2 invariance, as can be seen from Table I.

(iii) Baryon- and lepton-number violating interactions of the type discussed in (ii) above lead to an alternate

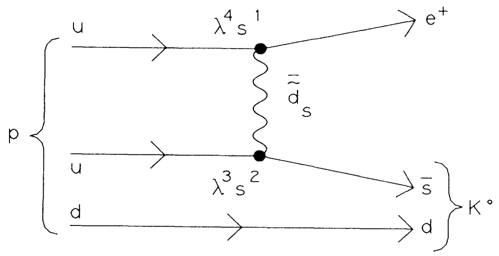

FIG. 1. Proton decay via the heavy squark $\tilde{d}_s$.

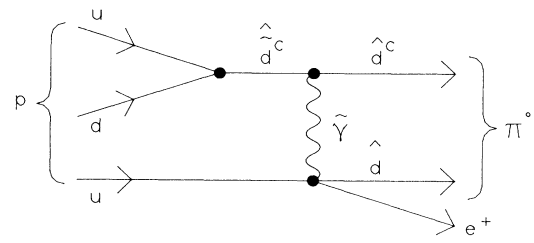

FIG. 2. Proton decay from R-parity violation if matter parity is *not* conserved.

mode of proton decay arising from the λ^2 and λ^4 interactions, e.g.,

$$\lambda^2_{nn'm}\epsilon^{aa'a''}u^c_{an}d^c_{a'n'}D^c_{a''m}+\lambda^4_{mnn'}D^a_m e^c_n u^c_{an'} . \tag{5}$$

From Eq. (3) we thus generate interactions of the type $\lambda^2 S^1 u^c_n \hat{d}^c_{n'} \hat{d}^c_s$ and $\lambda^4 S^2 \hat{d}_s e^c_n u^c_{n'}$ which lead to the proton decay of Fig. 1. One may estimate this decay rate as $\Gamma_P = A(\lambda^2\lambda^4 S^1 S^2)^2(\tilde{m}_{\hat{d}})^{-4} m_p^5$, where m_p is the proton mass, $\tilde{m}_{\hat{d}}$ is the $\hat{d}_s$ squark mass, and $A\sim 1$. This leads to a proton lifetime of

$$\tau_p = A^{-1}\left(\frac{\tilde{m}_{\hat{d}}}{10^8\ \mathrm{GeV}}\right)^4\left(\frac{\alpha_{\mathrm{em}}}{\lambda^2\lambda^4}\right)^2\left(\frac{10^{-13}}{S_1 S_2}\right)^2(3\times 10^{30}\ \mathrm{yr}) . \tag{6}$$

This decay mode was indeed the initial problem in supersymmetric theories.[10] Matter parity forbids it only through the three light standard-model generations d_m but not via the exotic quarks $\hat{d}_s$. We see from Eq. (6) that one needs $\tilde{m}_{\hat{d}} \gtrsim 10^8$ GeV to sufficiently suppress this mode. For the superstring models being considered here, however, one has[7] $m_{\hat{d}} \sim 10^{15}$ GeV, making this decay mode negligible.

(iv) The effect of the D-d mixing also produces a violation of R parity, allowing gauginos to convert into ordinary quarks and leptons. Thus the usual photino interaction $\tilde{\gamma}$-e^c-$\tilde{e}$ combined with the lepton-number-violating $\lambda^4 q^\lambda_n l_{\lambda n'} D^3_m$ interaction of Eq. (1) produces an effective Lagrangian interaction $\lambda^4 S^1 \tilde{\gamma} e^c u \hat{d}^c$ upon eliminating the selectron and using Eq. (3). If matter parity were *not* conserved and a light $\hat{d}^c_m$ could occur in this interaction, then the photino would become unstable (e.g., decay $\tilde{\gamma} \to e^+ + u + \bar{d}$) and the additional proton decay of Fig. 2 could occur. However, as we have seen, matter-parity invariance makes all the $\hat{d}$ states of Fig. 2 superheavy, i.e., they must lie in the $\hat{d}_s$ generations. Thus the "decay" of Fig. 2 is energetically forbidden and cannot destabilize the proton.

Matter-parity invariance is a powerful constraint in eliminating unwanted mechanisms for proton decay in Calabi-Yau superstring models. By itself, however, M_2 invariance is not sufficient to make the proton lifetime consistent with existing data. M_2 invariance does forbid proton decay to run through the light states of the three standard-model generations, but does not exclude decays via the other exotic generations that occur in Calabi-Yau models. One thus needs additional dynamical assumptions that make the exotic quarks and leptons superheavy. This, *combined* with M_2 invariance, then leads to superstring models with a proton lifetime consistent with experiment. In Ref. 7 it was shown that dynamical assumptions (on the nonrenormalizable interactions W_{NR}) can indeed lead to superheavy exotic particles. Thus it is important to see if these assumptions can be deduced directly from the properties of the Calabi-Yau manifold. Finally, we note that a phenomenologically acceptable Calabi-Yau compactification requires the existence of at least one light pair of Higgs doublets. As discussed in Ref. 7, this will arise if the Higgs-boson mass matrix from the $(\mathbf{27})^3$ interactions has a zero eigenvalue, which does indeed occur in the symmetric Calabi-Yau compactification.[14] If the zero-mode Higgs boson then lies in the generations producing intermediate scale breaking, the higher-dimension operators produce only electroweak-size mass contributions to this light Higgs boson.

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