Two-Particle Interferometry

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An exposition is given of the fundamental ideas of the recently opened field of two-particle interferometry, which employs spatially separated, quantum mechanically entangled two-particle states. These ideas are illustrated by a realizable arrangement, in which four beams are selected from the output of a laser-pumped down-converting crystal, with two beams interferometrically combined at one locus and two at another. When phase shifters are placed in these beams, the coincident count rates at the two loci will oscillate as the phases are varied, but the single count rates will not.

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After more than a century of interference experiments with individual particles—photons, electrons, neutrons—a conceptually new field of interferometry has recently been opened by employing correlated two-particle systems.[1-3] Two-particle interferometry has already exhibited new nonclassical optical phenomena, new confirmations of quantum mechanics, and new violations of Bell's inequality (hence of the family of local realistic theories), and it promises a rich mine of further results.

The essence of two-particle interferometry is the application of techniques of beam recombination to two-particle quantum states of the general form

$$|\Psi\rangle = 2^{1/2}[|\alpha\rangle_1|\gamma\rangle_2 + |\delta\rangle_1|\beta\rangle_2]\,. \qquad (1)$$

Here $|\alpha\rangle_1$ and $|\delta\rangle_1$ are orthonormal vectors in the Hilbert space of particle 1, and $|\beta\rangle_2$ and $|\gamma\rangle_2$ likewise for particle 2. Schrödinger called states like $|\Psi\rangle$ "entangled," because they cannot be factored in any way into the form $|\chi\rangle_1|\xi\rangle_2$. Entanglement is an extraordinarily rich source of phenomena. When the two particles are spatially separated, as in the polarization correlation experiments of Wu and Shaknov[4] and of the followers of Bell,[5] there are dramatic nonlocal effects of entanglement. The new phenomena studied in this Letter are obtained when the production of entangled states of spatially separated particles is combined with interferometric techniques. The purpose of this Letter is to present the ideas of two-particle interferometry in some generality, to propose some general schemes for realizing these ideas, and to show how the experiments performed and proposed so far are special cases of these schemes.

The general ideas which we emphasize are the following. First, as stated in the preceding paragraph, two-particle interferometry depends upon the preparation of entangled two-particle quantum states. Second, there is a wide range of choices of operators of which the $|\alpha\rangle_1$, $|\gamma\rangle_2$, $|\delta\rangle_1$, $|\beta\rangle_2$ of Eq. (1) are eigenvectors. The proliferation of polarization experiments in the last few decades may have given rise to the notion that polarization eigenvectors are particularly appropriate for constructing entangled states. As a matter of fact, however, the eigenvectors of other operators can also be used, and in particular the most natural procedure in two-particle interferometry is to use (approximate) eigenvectors of linear momentum operators. Third, entangled states like $|\Psi\rangle$ of Eq. (1) can be prepared by capitalizing upon the fact that a conserved quantity, like linear momentum, can be partitioned in various ways between two particles. Fourth, standard interferometric techniques of directing, phase shifting, and recombining beams can be applied to entangled two-particle states, particularly when $|\alpha\rangle_1$, etc., are approximate linear-momentum eigenvectors.

The general arrangement which we propose for two-particle interferometry is shown in Fig. 1. An ensemble of particle pairs is emitted by a source into the beams A, B, C, D, with wave vectors $\mathbf{k}_A$, $\mathbf{k}_B$, $\mathbf{k}_C$, and $\mathbf{k}_D$, each pair in the ensemble being in the quantum state

$$|\Psi\rangle = 2^{-1/2}[|\mathbf{k}_A\rangle_1|\mathbf{k}_C\rangle_2 + |\mathbf{k}_D\rangle_1|\mathbf{k}_B\rangle_2]\,, \qquad (2)$$

where

$$|\mathbf{k}_A| = |\mathbf{k}_D|\,, \quad |\mathbf{k}_B| = |\mathbf{k}_C|\,. \qquad (3)$$

State $|\Psi\rangle$ describes a coherent superposition of two distinct pairs of correlated paths for particles 1 and 2. In one of these, particle 1 enters beam A and is reflected from mirror M_A to phase shifter ϕ_1 en route to beam splitter H_1, from which it proceeds either to detector U_1 or to detector L_1; while particle 2 enters beam C and is

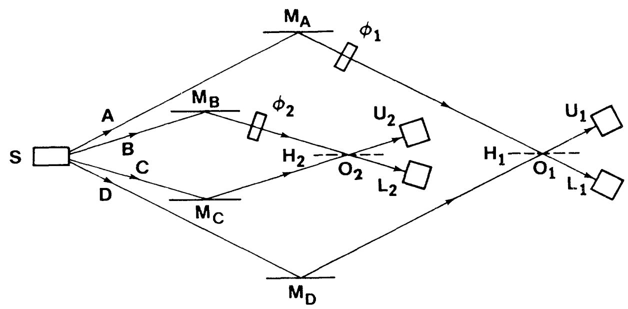

FIG. 1. An arrangement for two-particle interferometry with variable phase shifters. The source S emits two particles, 1 and 2, into four beams A, B, C, D. Index i ($i=1,2$) labels the particle that is registered in detectors U_i or L_i. The state of the pair is assumed to be given by Eq. (2), which is a superposition of two amplitudes: (I) particle 1 in beam A and particle 2 in beam C, and (II) particle 1 in beam D and particle 2 in beam B. The two beams A and D of particle 1 are given a variable relative phase shift ϕ_1 before recombination near the point O_1 on the half-silvered mirror H_1 (Mach-Zehnder interferometry). Likewise, the two beams B and C of particle 2 are given a variable phase shift ϕ_2 before recombination near O_2 on the half-silvered mirror H_2. The observed quantities of interest are the two-particle coincident count rates as functions of ϕ_1 and ϕ_2 quantum mechanically predicted by Eq. (5).

reflected from mirror M_C to beam splitter H_2, from which it proceeds to detectors U_2 or L_2. In the other pair of paths particle 1 enters beam D and proceeds to U_1 or L_1 via M_D and H_1, while particle 2 enters beam B and proceeds to U_2 or L_2 via M_B, ϕ_2, and H_2. If the detectors are assumed to have quantum efficiency η then the quantum-mechanical probability for joint detection of particles 1 and 2 by detectors U_1 and U_2, when phase shifts ϕ_1 and ϕ_2 have been chosen, is η^2 times the absolute square of the total amplitude $A(U_1,U_2|\phi_1,\phi_2)$. This total amplitude is the superposition of the amplitudes associated with each of the two pairs of correlated paths:

$$A(U_1,U_2|\phi_1,\phi_2)=2^{-1/2}[(2^{-1/2}ie^{i\phi_1})(2^{-1/2}) + e^{i\theta}(2^{-1/2})(2^{-1/2}ie^{i\phi_2})] , \quad (4)$$

where the factors $e^{i\phi_1}$ and $e^{i\phi_2}$ arise from the phase shifters encountered along the respective paths, and the factors $2^{-1/2}i$ and $2^{-1/2}$ arise[6] respectively from reflection and transmission at the beam splitters. In each term of Eq. (4) there are two factors in parentheses, the first factor referring to a path of particle 1 and the second to the correlated path of particle 2. The phase factor $e^{i\theta}$ depends upon the detailed placement of the mirrors and beam splitters and is independent of ϕ_1 and ϕ_2. Expressions analogous to Eq. (4) can be given for the amplitudes $A(L_1,L_2|\phi_1,\phi_2)$, $A(U_1,L_2|\phi_1,\phi_2)$, and $A(L_1,U_2|\phi_1,\phi_2)$, and η^2 times the absolute squares of these amplitudes are the quantum-mechanical probabilities for joint detection by the detector pairs (L_1,L_2), (U_1,L_2), and (L_1,U_2), respectively. The results are

$$p(U_1,U_2|\phi_1,\phi_2)=p(L_1,L_2|\phi_1,\phi_2) = \eta^2[\tfrac{1}{4}+\tfrac{1}{4}\cos(\phi_2-\phi_1+\theta)] \quad (5a)$$

and

$$p(U_1,L_2|\phi_1,\phi_2)=p(L_1,U_2|\phi_1,\phi_2) = \eta^2[\tfrac{1}{4}-\tfrac{1}{4}\cos(\phi_2-\phi_1+\theta)] . \quad (5b)$$

The sinusoidal dependence of the joint detection probabilities in Eqs. (5a) and (5b) on the phase shifts is characteristic of quantum-mechanical interference phenomena. By monitoring the coincident count rates while varying the phase shifts ϕ_1 and ϕ_2, interference fringes will be exhibited. However, if one focuses attention upon only one of the particles, it is easy to see that no interference fringes will be exhibited. Specifically, the probabilities of single detections by the various detectors are

$$p(U_1|\phi_1,\phi_2)=p(L_1|\phi_1,\phi_2) = p(U_2|\phi_1,\phi_2)=p(L_2|\phi_1,\phi_2)=\eta/2 . \quad (6)$$

In short, Eq. (6) shows that the count rate of each of the four detectors singly is constant, independent of ϕ_1 and ϕ_2. Only the coincident count rate exhibits interference fringes. Consequently, we have an essentially two-particle interferometric phenomenon. We emphasize that this phenomenon is a consequence of the entangled character of the state $|\Psi\rangle$.

The two-particle interferometric experiments referred to in Refs. 1 and 2 used photon pairs produced by parametric down-conversion, a process in which a single photon incident upon a crystal gives rise to a pair of correlated photons. In all of these experiments a pinhole arrangement selected only two beam directions, *symmetrically* placed about the incident beam. If parametric down-conversion is used as the source in the arrangement of Fig. 1, with the incident beam into the crystal coming from the left, then the correlated directions of Eq. (2) (A with C, D with B) are *asymmetrical* with respect to the incident beam. Fortunately, both experimental[7] and theoretical[8] studies reveal the existence of asymmetrically directed down-converted pairs of photons. Consequently, the state $|\Psi\rangle$ of Eq. (2) can be prepared by piercing four linearly arranged pinholes in a diaphragm downstream from the down-converting crystal. The beams A, B, C, D of Fig. 1 will emerge from the pinholes with wave vectors $\mathbf{k}_A$, $\mathbf{k}_B$, $\mathbf{k}_C$, $\mathbf{k}_D$ satisfying

$$\mathbf{k}_A+\mathbf{k}_C=\mathbf{k}_D+\mathbf{k}_B=\mathbf{k} , \quad (7)$$

where $\mathbf{k}$ is the wave vector of the incident beam. Because of the placement of the pinholes,

$$|\mathbf{k}_A|=|\mathbf{k}_D|, \ |\mathbf{k}_B|=|\mathbf{k}_C| , \quad (8)$$

but

$$|\mathbf{k}_A|<|\mathbf{k}_B| . \quad (9)$$

From Eq. (8) it follows that the frequencies of the two interfering beams at H_1 are equal, and the frequencies of the two interfering beams at H_2 are equal, even though the frequencies at H_1 and H_2 are unequal.

We stress that the arrangement of Fig. 1 is only one example of the resources of two-particle interferometry. To indicate the richness of this field we shall briefly sketch three other arrangements.

(1) In general, the beams A, B, C, and D need not be coplanar in order to exhibit two-particle interference. For example, if the source of the pairs is parametric down-conversion, Eq. (7) is still satisfied in the arrangement of four pinholes shown in Fig. 2. The pinholes from which A and D emerge are pierced at arbitrary points on a circle drawn on the diaphragm centered about the incident beam direction I, and beams B and C emerge from pinholes placed on another circle centered about I, with the constraint that the AC and BD planes intersect along I. If the beams A and D are brought together at a vertex lying in the AD plane, and the beams B and C are brought together at a vertex in the BC plane, then Eq. (5) still holds. In the special case in which the two circles coincide, the four pinholes are located at the corners of a rectangle centered about I, and $|\mathbf{k}_A|, |\mathbf{k}_B|, |\mathbf{k}_C|$, and $|\mathbf{k}_D|$ are all equal, so that the frequencies at H_1 and H_2 are equal.[9] Suppose, furthermore, that the four pinholes are reduced to two and the following identifications are made: A with B, C with D, M_A with M_B, M_C with M_D, and H_1 with H_2. The resulting arrangement is that used in all the experiments of Refs. 1 and 2 except that of Ghosh and Mandel.

(2) The arrangement of Fig. 1 may be modified by blocking two pinholes, leaving only beams A and D, and by replacing M_B and M_C by half-silvered mirrors M_B' and M_C', which are moved upward and downward, respectively, into beams A and D. The transmitted beams from M_B' and M_C' will then proceed respectively to M_A and M_D to join at H_1, exactly as in Fig. 1; and the reflected beams from M_B' and M_C' will join at H_2 at the same location as in Fig. 1. The resulting arrangement is essentially that proposed by Reid and Walls.[10] We disagree, however (as we shall argue in a later paper), with their interpretation that "one does not have a quantum superposition state as an original source. Quantum interference effects are provided by the beam splitters."[11]

(3) The arrangement of Fig. 1 can be modified by removing the beam splitters H_1 and H_2, the phase shifters ϕ_1 and ϕ_2, and the detectors U_1, L_1, U_2, L_2, and by placing miniature detectors at variable positions in the two diamond-shaped regions R_1 and R_2 where beams intersect. (Detailed views of R_1 and R_2 are presented in Fig. 3.) O_1 and O_2 are points in R_1 and R_2, and $\mathbf{r}_1$ and $\mathbf{r}_2$ are position vectors from O_1 and O_2, respectively. Note that in Fig. 3 the beam with wave vector $\mathbf{k}_C$ is the reflection of beam B from the mirror M_B, the beam with wave vector $\mathbf{k}_D$ is the reflection of beam A from the mirror M_A, etc. If $\mathbf{r}_1$ and $\mathbf{r}_2$ fall in the respective regions R_1 and R_2 (where all the intersecting beams are described to good approximation by correlated plane waves), the two-photon quantum state evolving from the state $|\Psi\rangle$ of Eq. (2) has the Schrödinger form

$$\Psi(\mathbf{r}_1,\mathbf{r}_2) \sim [\exp(i\mathbf{k}_C\cdot\mathbf{r}_2)\exp(i\mathbf{k}_A\cdot\mathbf{r}_1) + \exp(i\theta)\exp(i\mathbf{k}_B\cdot\mathbf{r}_2)\exp(i\mathbf{k}_D\cdot\mathbf{r}_1)], \quad (10)$$

where an overall time-dependent phase factor has been suppressed, and the relative phase factor $\exp(i\theta)$ depends upon the detailed placement of the optical elements independently of $\mathbf{r}_1$ and $\mathbf{r}_2$. By Eq. (10) the coincident count rate of the miniature detectors located at $\mathbf{r}_1,\mathbf{r}_2$ is

$$n(\mathbf{r}_1,\mathbf{r}_2) = \text{const} \times \{1+\cos[\mathbf{K}\cdot(\mathbf{r}_2-\mathbf{r}_1)+\theta]\}, \quad (11)$$

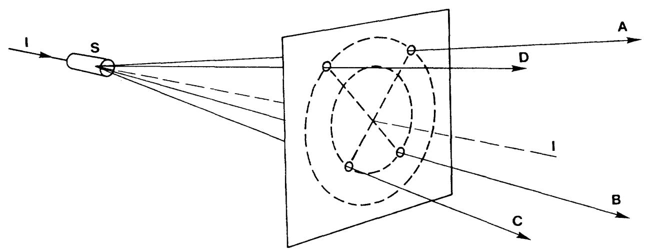

FIG. 2. A three-dimensional arrangement of four beams selected from the output of a down-converting crystal S. The diaphragm downstream from S is normal to the incident beam direction I. The pinholes from which the beams A and D emerge lie on a circle centered about I, and the pinholes from which B and C emerge lie on another circle centered about I. The plane AC intersects the plane BD along I. Beams A and D and beams B and C are to be recombined.

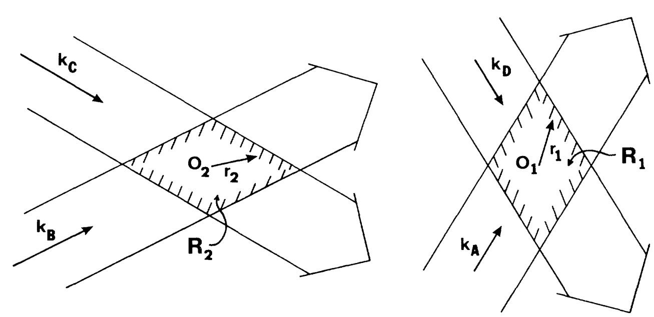

FIG. 3. A detailed view of the regions R_1 and R_2 where the beams of Fig. 1 intersect. O_1 and O_2 are points in R_1 and R_2 which serve as origins for the position vectors $\mathbf{r}_1$ and $\mathbf{r}_2$. The beam-splitters, phase-shifters, and large-aperture detectors of Fig. 1 have been removed, but miniature detectors (not shown) are placed at $\mathbf{r}_1$ and $\mathbf{r}_2$. The coincident count rate of the miniature detectors as a function of $\mathbf{r}_1$ and $\mathbf{r}_2$ is predicted by Eq. (11).

where

$$\mathbf{K} = \mathbf{k}_B - \mathbf{k}_C = \mathbf{k}_A - \mathbf{k}_D .$$

If one focuses upon the miniature detector in R_1, disregarding the detector in R_2, one finds its count rate to be independent of $\mathbf{r}_1$—no fringes; and likewise for the detector in R_2 by itself. Consequently, the fringe pattern predicted by Eq. (11) is unequivocally a two-photon interference phenomenon. Finally, we consider the result of the following simplifications: eliminating two of the pinholes (thereby making beams A and B coincident and beams C and D coincident), and identifying M_B with M_A, M_C with M_D, and O_1 with O_2. The resulting arrangement, in which the regions R_1 and R_2 of beam intersection coincide, is that of the experiment performed by Ghosh and Mandel.[1]

In a separate paper we shall examine in more detail the role of entanglement in two-particle interferometry and shall make some proposals for new nonpolarization tests of Bell's inequality.

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[9]The rectangular arrangement of pinholes may be indispensable if high-frequency down-conversion photon pairs [as observed by P. Eisenberger and S. L. McCall, Phys. Rev. Lett. **26**, 684 (1971)] are employed in two-particle interferometric experiments. Then a *single* Laue-case crystal can be used (instead of the four mirrors M_A, M_B, M_C, M_D of Fig. 1) to recombine both the pair of beams A and D and the pair of beams B and C, and one other Laue-case crystal will function as both of the beam splitters H_1 and H_2. (See Ref. 3 for use of the Laue-case crystals for interferometry on high-frequency photons.)

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