Universal Critical Normal Sheet Resistance in Ultrathin Films

Tao Pang
School of Physics and Astronomy, University of Minnesota, 116 Chruch Street S.E., Minneapolis, Minnesota 55455
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A simple analysis is given to ultrathin-film systems. Based on the physical picture proposed that the superconducting-insulating phase transition at $T=0$ is a combined effect of the pairing and localization in two-dimensional systems, the normal-state sheet resistance is evaluated in the vicinity of the transition. It is found to be universal and equal to $h/(2e)^2$, independent of the details of different samples as observed in experiments. A similar formulation can also be used to explain the quantized conductance in narrow channels.

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Recent experiments have found many rich phenomena in lower-dimensional systems. The most noticeable phenomena, such as the quantum Hall effect[1] and fractional quantum Hall effect[1] in two-dimensional electron systems under strong magnetic fields, the quantized conductance[2,3] of narrow conducting channels, and the weak localization effect in disordered two-dimensional systems,[4] have been fairly well understood. The experimental studies of the superconducting transitions and the temperature dependence of the sheet resistance in all kinds of ultrathin films[5,6] have revealed many interesting aspects of quantum fluctuations and dissipation, Coulomb interaction effects, and various phase transitions.[5,6] Some aspects mentioned above have been extensively studied theoretically.[7-9] All these phenomena have to be described in the language of quantum transport, fluctuation, dissipation, or many-body correlation. Most of the time, the phenomena are the combined results of several quantum mechanisms and the key to understanding each phenomenon is to isolate each contribution and to analyze it carefully. The study of weak localization in disordered two-dimensional systems is a typical example.[4]

A set of extremely interesting experiments[10] carried out most recently on ultrathin bismuth films reveal some fundamental aspects of ultrathin films which could not be seen before on other systems because of the complexity of the data. The essential difference between Bi and the materials studied experimentally before is that Bi crystal samples are not superconductors. This excludes the formation of coupled superconducting grains in the Bi thin films with an average thickness around one monolayer and indicates the difficulty of applying the coupled-superconducting-grain model to Bi films. The basic difference between the data of the temperature dependence of the sheet resistance of the Bi films and other well studied materials is that Bi data seem to indicate that the onset of the superconductivity happens at $T=0$, and with a normal-state sheet resistance $R(T=0)=R_c$ with $R_c \simeq h/(2e)^2$ and a clear bifurcation behavior between the normal states in insulating and superconducting films. The data also suggest that the possible sheet resistance at $T=0$ can only have three values: 0, ∞, or R_c. The R_c observed in Bi films is further evidence that the onset of superconductivity in ultrathin films is universally governed by the normal-state sheet resistance.[5] Since the superconducting grains are strongly suppressed in Bi samples and R-T curves do not show the effects of strongly coupled superconducting grains, namely, local minima and flat tails at $T=0$ with all possible $R(T=0)$, a more fundamental picture seems to be necessary to account for the universal threshold in ultrathin films. The idealized experimental curves for Bi samples around the threshold are shown in Fig. 1 for illustration.

The purpose of this Letter is to provide a more fundamental picture for the quantum phase transition between insulating and superconducting states for the systems like Bi ultrathin films and to evaluate the normal sheet resistance for these systems in the vicinity of the quan-

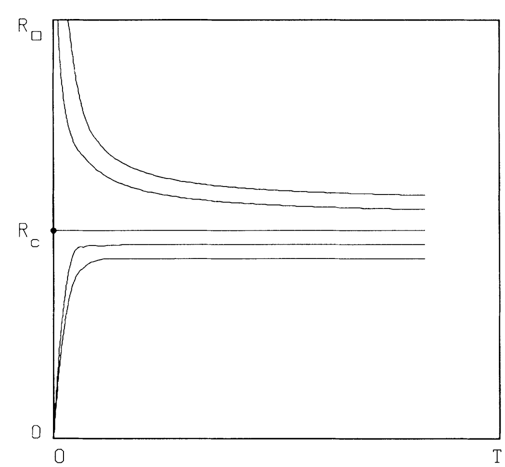

FIG. 1. Idealized sheet-resistance–temperature curves in ultrathin films such as Bi samples. R_c is the universal critical sheet resistance with $R_c = h/(2e)^2$. The units are arbitrary. The critical point discussed in the text is indicated by a heavy dot.

tum phase transition from a quantum transport theory.[11,12] The analysis given here realizes that the observed R_c is another kind of quantized transport coefficient observed in a completely different kind of system than the quantum Hall effect, fractional quantum Hall effect, and the quantized conductance in inversion layers or heterojunctions. More importantly, this critical point separates the superconducting and insulating phases at $T=0$ in the ultrathin films, and the transported charge is exactly $2e$, which is fundamentally different from the quantum Hall effect and quantized conductance effect where the transported charge is e, or the fractional quantum Hall effect where the transported charges are fractional-charge excitations. The R_c is found to be equal to $h/4e^2$ exactly and independent of any detail of a specific sample, such as, the size, the degree of disorder, the electron density, and the coherence length of the sample.

The basic idea is that for the samples like Bi thin films, there are no superconducting grains (more precisely the number of superconducting grains is minimized). So the effect of the coupling between superconducting grains can be ignored or taken as a secondary effect. However, this effect may cover up other effects when the coupled superconducting grains are largely present as apparently happens in other materials.[5,6] The coupled-superconducting-grain model has accounted well for many aspects when coupled superconducting grains are present.[8] If there is a way to suppress the superconducting grains in other materials, the conclusions from the present work should be observed in those systems too. One might like to consider changing the external conditions or internal parameters of other materials in order to achieve the suppression.

The physical picture can be viewed as follows. When the localization length $l(\epsilon)$ for a single electron state with energy ϵ is much smaller than the average Cooper-pair size or the coherence length of the superconducting state, Cooper pairs cannot form because electrons see each other as localized particles on the scale of the coherence length. Therefore, the systems are in the insulating state at $T=0$ based on the scaling theory of localization[4] which shows that all single-particle states in disordered two-dimensional systems are localized. However, when $l(\epsilon)$ is much larger than the average size of a Cooper pair, the electrons see each other as extended particles on the scale of the coherence length, and therefore, the films are in the superconducting state at $T=0$. This is the exact result of Ma and Lee[13] on the criterion for disordered superconductors with the transition at $l_c \simeq \xi$,[4] where ξ is the superconducting coherence length. The single-particle excitations of the superconducting ground state in two-dimensional systems are localized too, since the single-particle states are always localized for disordered samples with infinite size except that the excitations are created within the localization length of the boundary of the samples.

Let us take the samples as squares with side length L for convenience and imagine the circuit is closed by leads outside the sample. Consider that all electrons are in localized states and all states below the Fermi level are occupied. The qualitative picture of the single-particle energy levels is that the energy levels are discrete and the separation between two levels decreases with increasing energy. The localization length of each energy level increases with increasing energy. The localization length at the Fermi level is the largest one of the occupied states if we consider the case at $T=0$. Now assume that the situation is such that the localization length at the Fermi level $l(\epsilon_F)$ is equal to the critical localization length l_c which allows the Cooper instability and the two electrons on the first level form a Cooper pair or vice versa. Another way to think of it is to assume that the energy-level structure and the localization length $l(\epsilon)$ do not change when the electron density is changing and so is the Fermi level. Let us increase the electron density from the insulating side until ϵ_F reaches ϵ_F^c with $l(\epsilon_F^c) = l_c$. Then when $\epsilon_F > \epsilon_F^c$, the system is superconducting and when $\epsilon_F < \epsilon_F^c$, the system is insulating. This is in some sense similar to the quantum phase transition in disordered boson systems.[14] The difference is that there is a marginal metallic state in the vicinity of the transition in the ultrathin-film systems. The experimental data of Bi samples seem to suggest that three states are possible at $\epsilon_F = \epsilon_F^c$ and $T=0$. If one had a sample with $\epsilon_F \equiv \epsilon_F^c$ to measure the sheet resistance at $T=0$, one would obtain three values: $R(T=0)=0$, ∞, or R_c. Let us assume that at time t_1, two electrons at the Fermi level ϵ_F^c are in the pairing state and at a later time t_2 they are in the localized single-particle state. If the sheet resistance were measured at t_1, one would obtain $R=0$ and if it were measured at t_2, one would obtain $R=\infty$. A third state, the marginal normal state, corresponds to the pair developing into a single-particle state at the edge of the sample (two edges are the same with the periodic boundary condition). The probability to observe $R=R_c$ is about l_c/L for $\epsilon_F = \epsilon_F^c$ and $T=0$. This situation is similar to the ordinary superconducting–normal-metal transition. When $T \equiv T_c$, both superconducting and normal states are possible. If one did measurements on the resistance, the resistance would fluctuate between zero and the normal resistance at T_c. Now let us try to evaluate R_c by assuming the following conditions: An external voltage is applied to the sample and at the time t_0 the normal state is realized from the pairing state when the only pair developing into the single-particle state is at the boundary. The two electrons from that marginally stable pair will keep moving *together* ballistically afterwards because the distance to cross the external voltage is practically zero now.

Based on the above argument, we can define a normal current density at t_0 under the assumed condition that only the two electrons from the marginally stable pair are transported together through the sample. The normal current density of these two special electrons is given by

$$J_x = -2e\sum f_n V_{xn}/L^2, \tag{1}$$

where e is the electron charge strength, f_n is the occupation number at state n participating in the conduction, and V_{xn} is the group velocity of the state in the x direction and is given by

$$V_{xn} = \frac{1}{\hbar}\frac{\partial\epsilon(n)}{\partial k_x(n)}, \tag{2}$$

where $\epsilon(n)$ is the energy level of state n and $k_x(n)$ is the quasi-wave-vector of the state n. Using periodic boundary conditions in the x direction and a ballistic assumption for the last-moment motion of two electrons in the marginally stable pair, we have

$$k_x(n) = 2\pi n/L. \tag{3}$$

Using the condition given above, one easily obtains

$$J_x = -\frac{2e}{hL}\sum\frac{\partial\epsilon(n)}{\partial n}f_n. \tag{4}$$

In the case of the quantum Hall effect[11,12] $\sum f_n\,\partial\epsilon(n)/\partial n$ was averaged over one flux quantum, with $\sum f_n\,\partial\epsilon(n)/\partial n = -eV$ under the external voltage V. Here we are dealing with two electrons developed from a marginally stable pair and the $\sum f_n\,\partial\epsilon(n)/\partial n$ should be averaged over this pair. Therefore, $\sum f_n\partial\epsilon(n)/\partial n = -2eV$ and the current density is

$$J_x = (2e/hL)2eV. \tag{5}$$

One special feature which needs to be clarified immediately here is that the factor 2 associated with the charge strength is actually the residue effect of the marginally stable pair. The summation on n is actually converted into an integral of an energy dispersion under the applied voltage and one may wish to view the process as a process to transport a $-2e$ charge ballistically through a voltage V.

The quantized conductance observed in the ballistic narrow channels in inversion layers[2] can also be simply explained by the above formulation, if we take the spin index into account, sum up all ballistic states, and take the energy shift as eV. However, the underlying physics here is different and the corrections due to the quantum interference effect may need to be taken into account for some special geometries.

Now we can easily obtain the critical normal-state sheet resistance of the ultrathin films in the vicinity of the superconducting-insulating transition at $T=0$,

$$R_c = 1/\sigma = E_x/J_x = h/(2e)^2, \tag{6}$$

which is universal for all ultrathin films as long as the two-dimensional assumption is still valid.

If a system has a normal resistance R larger than R_c when T approaches zero, the localized single-particle state is more stable, and the system approaches the insulating phase at $T=0$. If a system has a normal resistance R smaller than R_c when T approaches zero, the pairing state is more stable, and the system approaches the superconducting phase. If a system has a normal resistance that goes to R_c exactly when T approaches zero, the system can be superconducting, insulating, or normal at $T=0$.

The critical normal sheet resistance can also be argued from a macroscopic point of view[15] analogous to Laughlin's argument[16] for the quantum Hall effect if one accepts the physical picture proposed above that the normal state at the critical point corresponds to the system with $\epsilon_F = \epsilon_F^c$ when the marginally stable pair develops into the single-particle state at the edge of the sample. Imagine that the sample is connected into a loop. The current in the loop is given by $I = c\Delta U/\Delta\phi$. ΔU is the change of the total electric energy in the system and $\Delta\phi$ is the change of the magnetic flux through the loop. ΔU is $2eV$ as argued above and $\Delta\phi$ is $hc/2e$ because only one marginally stable pair existed and has developed into the normal state. Then one can obtain $R_c = h/(2e)^2$ immediately.

Now let us simply analyze the influence of finite temperature, the Coulomb interaction, and the quantum fluctuation from one superconducting grain to another when coupled superconducting grains are presented.

Generally speaking, a finite-temperature analysis using the renormalization group should show flows to two stable fixed points at $T=0$, $R=0$ and ∞, and flows away from the unstable fixed point at $R=R_c$ and $T=0$.

The fluctuation effect due to the formation of local superconducting islands will introduce some rich phenomena. However, since the discussion of the universal critical normal sheet resistance given above does not depend on the quantum fluctuation from one superconducting grain to another, the conclusion is still true even when superconducting grains are formed in the systems. Now the question is how one can suppress the superconducting-grain effects in the systems where superconducting grains can form easily. Ultrathin Bi films are very good examples where the superconducting-grain effects are minimized because Bi crystal samples are not superconductors.

The Coulomb interaction will introduce some other interesting features to the systems. An issue such as how the disorder, crystallization in the grains, and the Coulomb interaction influence the behavior of the Cooper pairs is still not very clear. However, for the normal state, the Coulomb interaction may only renormalize the effective mass of the quasiparticles. Therefore, it is likely that the Coulomb interaction in such a sense will not influence the universal critical normal sheet resistance discussed above.

In summary, the normal sheet resistance of ultrathin films in the vicinity of the superconducting-insulating phase transition has been analyzed and evaluated. This critical sheet resistance is found to be universal and equal to $h/(2e)^2$, independent of any detail of a specific sample. Based on the analysis, it has been realized that the universal critical sheet resistance indicated in the experiments is another kind of quantized transport coefficient. This quantized transport coefficient happens in a completely different kind of system, ultrathin films. Other quantized transport coefficients like the quantum Hall effect and quantized conductance are all observed in inversion layers or heterojunctions of semiconductors. More importantly, this critical point separates the insulating and superconducting phases for all ultrathin superconducting films. The analysis provided here also brings up a challenge to experimentalists, namely, to what degree the experiments can suppress the coupled-superconducting-grain effects and how close the experiments can approach the universal critical normal sheet resistance in ultrathin-film systems.

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