Testing *CP* in $K_{\mu 3}$ Decays

Miriam Leurer

Randall Laboratory of Physics, University of Michigan, Ann Arbor, Michigan 48109-1120

(Received 2 December 1988)

K factories will open the way for high-precision *CP* tests in $K_{\mu 3}$ decays. The standard model does not predict *CP* breaking in this process. We consider here the effects of nonstandard interactions mediated by vector and scalar particles and by leptoquarks. We show that, for the only experimentally measurable quantity, vector particles alone never induce *CP* violation, and give a general expression for the *CP* breaking induced by scalars and leptoquarks.

PACS numbers: 11.30.Er, 12.15.Cc, 13.20.Eb

According to the standard model, there is a single source for *CP* violation—the Kobayashi-Maskawa (KM) matrix. *CP* is violated in processes which involve *several* KM matrix elements whose relative, convention-independent phase is nonzero. Some such typical processes are K-$\bar{K}$ mixing and nonleptonic *K* decays, where *CP* breaking has indeed been seen. In contrast, the decay of a charged kaon to $\pi l \nu$ (a pion, a lepton, and a neutrino) involves a *single* KM matrix element whose phase is convention dependent and cannot induce any *CP* breaking. A similar statement applies to the decay of neutral kaons, but in this case one must be careful to isolate *CP* breaking in the decay process itself from *CP* breaking induced by K-$\bar{K}$ mixing. This is done by identifying the decaying particle as a K^0 if the final state includes a positively charged lepton (l^+) and as a $\bar{K}^0$ if the final state includes an l^-. If the decaying particle is so identified then *CP* is again conserved for this process according to the standard model. (Here we assume the $\Delta S = \Delta Q$ rule in K^0 and $\bar{K}^0$ decays, that is, we neglect contributions which are second order in the weak interactions.) Therefore if, in the future, *CP* violation will be seen in any K_{l3} ($K \to \pi l \nu$) decay, the inevitable conclusion will be that new, beyond-standard physics is responsible.

Motivated by the prospects for high-accuracy *CP* tests in *K* factories, we analyze in this Letter the contributions to $K_{\mu 3}$ ($K \to \pi \mu \nu$) decays[1] of generic intermediate nonstandard particles: vectors, scalars, and leptoquarks. There is only one measurable *CP*-violating quantity in this case—the component of the muon spin orthogonal to the decay plane in the kaon rest frame. We will show that the contributions of intermediate vectors to the orthogonal spin *cancel* and will give a general expression for the contributions of intermediate scalars and leptoquarks.

For completeness we begin with the standard-model effective interaction relevant to our process:

$$\mathcal{L}^{\text{stand}} = \sqrt{2} G_F \sin\theta_C \bar{s} \gamma^\rho \mu \bar{\nu}_\mu \gamma_\rho \frac{1-\gamma_5}{2} \mu + \text{H.c.} \qquad (1)$$

G_F is the Fermi constant and θ_C is the Cabibbo angle. All the parameters in the standard-model interaction are real and *CP* is therefore conserved.

Note that $\mathcal{L}^{\text{stand}}$ is proportional to the hadronic vector current $\bar{s}\gamma^\rho u$. In general, the effective standard-model Lagrangian includes another term, which is proportional to the axial-vector current $\bar{s}\gamma^\rho \gamma_5 u$. However, this other term does not contribute to $K \to \pi \mu \nu$ decays as the matrix element $\langle \pi | \bar{s}\gamma^\rho \gamma_5 u | K \rangle$ vanishes, due to parity. For this reason, in the following we will ignore effective interactions that are proportional to the hadronic axial-vector current. Similarly, we may ignore interactions (induced by, e.g., nonstandard scalars) that are proportional to the pseudoscalar $\bar{s}\gamma_5 u$.

The effective interaction induced by the standard *W* may be significantly modified once we allow nonstandard physics. This is because neutrinos in a nonstandard model are usually massive and have a nontrivial KM-type matrix. The Lagrangian (1) is then modified to

$$\mathcal{L}^W = \sqrt{2} G_F \sin\theta_C \bar{s}\gamma^\rho u \sum_i \mathcal{U}_{\mu i} \bar{\nu}_i \gamma_\rho \frac{1-\gamma_5}{2} \mu + \text{H.c.}, \qquad (2)$$

where i is an index running over neutrino mass eigenstates and the $\mathcal{U}_{\mu i}$'s are the KM leptonic matrix elements, which satisfy the unitarity relation

$$\sum_i |\mathcal{U}_{\mu i}|^2 = 1 . \qquad (3)$$

If the neutrinos are of Dirac type, one could choose a phase convention in which all the $\mathcal{U}_{\mu i}$'s are real and $\mathcal{L}^W$ is then *CP* conserving. If the neutrinos are of Majorana type, only one overall phase could be removed. The relative phases are *CP* breaking. However, this *CP* violation will not have any effect on $K_{\mu 3}$ decays, as the various $\mathcal{U}_{\mu i}$'s contribute to *different* processes ($\mathcal{U}_{\mu i}$ contributes to $K \to \pi \mu \nu_i$) and never mix.

We will now introduce the effective interactions induced by various intermediate bosons in a generic nonstandard model. $\mathcal{L}^V$ describes the effect of intermediate vectors, including the standard *W*,

$$\mathcal{L}^V = \bar{s}\gamma^\rho u \sum_i \bar{\nu}_i \gamma_\rho \left(A_i^V \frac{1-\gamma_5}{2} + B_i^V \frac{1+\gamma_5}{2} \right) \mu + \text{H.c.} \qquad (4)$$

Here we do not include the contributions of vector leptoquarks, which will be discussed later. A_i^V, B_i^V are complex parameters. A_i^V is dominated by the contribution of the standard W boson, while B_i^V gives the effects of right-handed currents. Theoretical bounds on B_i^V in the framework of left-right symmetric models are very strong[2,3] and, in practice, imply that we could ignore its contribution to $K_{\mu 3}$ decays altogether. However, these bounds depend on some assumed relations between the left- and right-handed KM matrices and can be evaded (the stronger bounds[3] require further assumptions on neutrino mass matrices), so we shall retain B_i^V.

The effective interaction introduced by intermediate scalars is

$$\mathcal{L}^S = \bar{s}u\sum_i \bar{\nu}_i \left[A_i^S \frac{1+\gamma_5}{2} + B_i^S \frac{1-\gamma_5}{2} \right] \mu + \text{H.c.}, \qquad (5)$$

where, again, we do not yet include the effects of leptoquarks. The effective interactions of leptoquarks may be rewritten by applying Fierz transformations. The contributions of vector leptoquarks may then be included in $\mathcal{L}^V$ and $\mathcal{L}^S$. Those of scalar leptoquarks contribute to $\mathcal{L}^V$ and $\mathcal{L}^S$, as well as generating a new "tensor effective interaction:"

$$\mathcal{L}^T = \bar{s}\sigma^{\mu\nu}u\sum_i \bar{\nu}_i \sigma_{\mu\nu} \left[A_i^T \frac{1+\gamma_5}{2} + B_i^T \frac{1-\gamma_5}{2} \right] \mu + \text{H.c.} \quad (6)$$

We cast $\mathcal{L}^V$, $\mathcal{L}^S$, and $\mathcal{L}^T$ into the effective Lagrangian,

$$\mathcal{L}_{\text{eff}} = \mathcal{L}^V + \mathcal{L}^S + \mathcal{L}^T. \qquad (7)$$

Clearly, the CP-violating quantities of this Lagrangian are the relative phases of A_i^V, A_i^S, A_i^T, B_i^V, B_i^S, and B_i^T for every fixed neutrino flavor i. Parameters associated with different neutrino flavors contribute to different processes and their relative phase is irrelevant in our case.

We have divided our parameters into "A type" and "B type." In the $m(\nu_i) \to 0$ limit, A_i and B_i contribute to different processes and their relative phase does not induce CP breaking. In this limit, $(1-\gamma_5)/2$ and $(1+\gamma_5)/2$ are projection operators onto left- and right-handed neutrino states, respectively. A's then contribute to $K \to \pi\mu(\nu_i)_L$ decay and B's to $K \to \pi\mu(\nu_i)_R$ decay. When $m(\nu_i) \neq 0$, the CP breaking induced by the relative phase of an A_i and a B_i is suppressed by powers of $m(\nu_i)/E$, where E is some energy scale characteristic to the process. Assuming $m(\nu_i) \leq 0.25$ MeV[4] and $E \sim 100$ MeV we find suppression by 2.5×10^{-3} or by a still smaller factor. This observation has important consequences. In particular, it implies that CP violation induced by intermediate vectors alone is strongly suppressed. One suppression factor is $m(\nu_i)/E$, which is due to the fact that such CP breaking arises from the relative phase A_i^V and B_i^V. The other suppression is by a factor of $(M_W/M_R)^2$, typical to effects that depend on right-handed currents (M_W is the standard W mass, and M_R is the mass scale of these currents). If CP breaking is seen in $K_{\mu 3}$ decays at the level of $\sim 10^{-3}$, it will be a signature of a nonstandard scalar or leptoquark. This fact was pointed out by Cheng[5] a few years ago. Here we will carry his result further and show that the contributions of the vectors alone to the orthogonal spin of the muon completely cancel. Therefore, if CP breaking is seen at any level, it must be the effect of a scalar or a leptoquark.

For every fixed helicity state of every neutrino flavor, the orthogonal spin of the muon, $s_\perp$, is a CP-violating quantity. However, we do not observe the neutrinos directly and so cannot distinguish between their various states. The only CP-violating quantity we can measure is the average of $s_\perp$ over neutrino flavors and helicities. It is the averaging over helicity states that wipes out the effects of the relative phase of A_i^V and B_i^V. To prove the last statement, we introduce some notation. Following Lee and Wu[6] we work in the center-of-mass frame of the two leptons. In this frame the three-momenta of the muon and the neutrino are opposite and the three-momenta of the kaon and the pion are equal. All momenta lie in one plane. The transformation to the kaon rest frame is accomplished by a boost along its direction of motion. Such a boost does not change the plane of the momenta, so that the "orthogonal direction" is identical in both frames. The kinematics is completely specified by q^2, the mass squared of the two-lepton system, and α, the angle between the pion and the muon.

For fixed q^2 and α we define $\mathcal{A}_i(h,h')$ to be the amplitude for a muon with helicity $h/2$ and a neutrino of flavor i and helicity $h'/2$. The amplitudes $\mathcal{A}_i(h,h')$ obey

$$\mathcal{A}_i(h,h') = hh'\mathcal{A}_i(-h,-h')\,|_{A\leftrightarrow B}\,, \qquad (8)$$

where by $A \leftrightarrow B$ we mean the interchange of $A_i^V \leftrightarrow B_i^V$, $A_i^S \leftrightarrow B_i^S$, and $A_i^T \leftrightarrow B_i^T$. Equation (8) is due to the fact that $\mathcal{A}_i(h,h')$ and $\mathcal{A}_i(-h,-h')$ are related to each other by a parity transformation followed by 180° rotation around the orthogonal direction. (Under this operation the momenta are unchanged, the helicities are flipped, and the A and B parameters are exchanged in the Lagrangian.) The factor hh' on the right-hand side of (8) reflects the phase (-1) that a fermion gains under a 360° rotation. Strictly speaking, one should be careful when applying parity to Majorana fermions. We may do that by endowing them with imaginary internal parity: $\eta_P(\nu) = i$. When we apply parity, the factors of i (one from the Majorana particle in the final state and one from the Majorana field in the Lagrangian) cancel each other and do not modify Eq. (8).

The CP-violating quantity $s_\perp$ averaged over ν_i helicity states is given by (see Ref. 6)

$$\langle s_\perp^i \rangle = \frac{\text{Im}\sum_{h'}\mathcal{A}_i(+,h')\mathcal{A}_i^*(-,h')}{\sum_{h,h'}|\mathcal{A}_i(h,h')|^2}\,. \qquad (9)$$

Let us study the numerator in the right-hand side of Eq.

(9). We may apply to it the following chain of identities:

$$\begin{aligned}\mathrm{Im}\sum_{h'}\mathcal{A}_i(+,h')\mathcal{A}_i^*(-,h') &= -\mathrm{Im}\sum_{h'}\mathcal{A}_i(-,h')\mathcal{A}_i^*(+,h')\\ &= -\mathrm{Im}\sum_{h'}\mathcal{A}_i(-,-h')\mathcal{A}_i^*(+,-h')\\ &= \mathrm{Im}\sum_{h'}[\mathcal{A}_i(+,h')\mathcal{A}_i^*(-,h')]\,|_{A\leftrightarrow B}\,. \end{aligned}\tag{10}$$

The first two steps are trivial mathematical manipulations. Note that in the second step we used the fact that h' is a dummy index, namely, we used the averaging over neutrino helicities. The last step follows from (8). Equation (10) implies that the numerator in the expression for $\langle s^i_\perp\rangle$ is *symmetric* under $A\leftrightarrow B$ interchange.

Consider now the contributions of intermediate vectors only. The amplitudes $\mathcal{A}_i(h,h')$ are linear in the parameters A_i^V,B_i^V. The numerator expression (9) for $s^i_\perp$ must therefore be proportional to $\mathrm{Im}[A_i^V(B_i^V)^*]$, which is clearly *antisymmetric* under $A\leftrightarrow B$ interchange. The obvious conclusion is that $\langle s^i_\perp\rangle$ vanishes in this case. *CP* violation induced by vectors alone disappears when we average over neutrino helicities and is therefore not detectable.

We now proceed to give the general expression for the orthogonal spin of the muon including the effects of the scalars and leptoquarks. To this end we present the amplitudes $\mathcal{A}_i(h,h')$ for K^+ (or K^0) decay. Up to an overall constant they are given by

$$\mathcal{A}_i(+,-) = -f_+\sin\alpha\left[A_i^V + B_i^V\frac{m_\nu}{E_\nu+|\mathbf{p}|}\,\frac{m_\mu}{E_\mu+|\mathbf{p}|}\right] + f_T\sin\alpha\left[A_i^T\frac{m_\mu}{E_\mu+|\mathbf{p}|} + B_i^T\frac{m_\nu}{E_\nu+|\mathbf{p}|}\right],$$

$$\begin{aligned}\mathcal{A}_i(-,-) = -A_i^V\frac{m_\mu}{E_\mu+|\mathbf{p}|}(f_0+f_+\cos\alpha) + B_i^V\frac{m_\nu}{E_\nu+|\mathbf{p}|}(f_0-f_+\cos\alpha)f_S\left[A_i^S - B_i^S\frac{m_\nu}{E_\nu+|\mathbf{p}|}\,\frac{m_\mu}{E_\mu+|\mathbf{p}|}\right]\\ + f_T\cos\alpha\left[A_i^T + B_i^T\frac{m_\nu}{E_\nu+|\mathbf{p}|}\,\frac{m_\mu}{E_\mu+|\mathbf{p}|}\right].\end{aligned}\tag{11}$$

The other two amplitudes, $\mathcal{A}_i(-,+)$ and $\mathcal{A}_i(-,-)$, are determined by the identity (8). Here m_ν and m_μ are the masses of the neutrino and the muon, E_ν and E_u are their energies, and $\mathbf{p}=\mathbf{k}_\mu=-\mathbf{k}_\nu$. f_+, f_0, f_S, and f_T are functions of q^2 and are related to the hadronic matrix elements in the center-of-mass frame of the two leptons through

$$\begin{aligned}&\langle\pi|\bar s\gamma^0 u|K\rangle = 2|\mathbf{k}_\pi|f_0\,,\\ &\langle\pi|\bar s\boldsymbol{\gamma} u|K\rangle = 2\mathbf{k}_\pi f_+\,,\\ &\langle\pi|\bar s u|K\rangle = 2|\mathbf{k}_\pi|f_S\,,\\ &\langle\pi|\bar s\sigma^{0i}u|K\rangle = 2i\mathbf{k}_\pi f_T\,,\\ &\langle\pi|\bar s\sigma^{ij}u|K\rangle = 0\,.\end{aligned}\tag{12}$$

The last matrix element in (12) vanishes by straightforward tensor analysis. It is impossible to construct an antisymmetric tensor with the single three vector ($\mathbf{k}_\pi$) that we have.

We will make a few approximations. First, terms of second order in $m(\nu_i)/E$ will be neglected. In particular, this means that we neglect the effect of the various neutrino masses on the kinematics. The average of the orthogonal μ spin over neutrino flavors and helicities is then,

$$\langle s_\perp\rangle = \frac{\mathrm{Im}[\sum_{i,h'}\mathcal{A}_i(+,h')\mathcal{A}_i^*(-,h')]}{\sum_{i,h,h'}|\mathcal{A}_i(h,h')|^2}\,.\tag{13}$$

Next, we will neglect terms that are of second order in B_i^V, A_i^S, B_i^S, A_i^T, and B_i^T, namely, we will ignore second-order effects induced by nonstandard intermediate bosons. In this approximation we may consistently replace A_i^V by the contribution of the standard W:

$$A_i^V \simeq \sqrt{2}G_F\sin\theta_C\,\mathcal{U}_{\mu i}\,.\tag{14}$$

Substituting the amplitudes (11) into expression (13), we get

$$\begin{aligned}\langle s_\perp\rangle = \sin\alpha\sum_i\Bigg[\mathrm{Im}\left(\frac{A_i^S\mathcal{U}_{\mu i}^*}{\sqrt{2}G_F\sin\theta_C}\right)\frac{f_+f_S}{(f_+\sin\alpha)^2+R_\mu(f_0+f_+\cos\alpha)^2}\\ +\mathrm{Im}\left(\frac{A_i^T\mathcal{U}_{\mu i}^*}{\sqrt{2}G_F\sin\theta_C}\right)\frac{f_T[f_+\cos\alpha-(f_+\cos\alpha+f_0)R_\mu]}{(f_+\sin\alpha)^2+R_\mu(f_0+f_+\cos\alpha)^2}\Bigg],\end{aligned}\tag{15}$$

where $R_\mu = m_\mu^2/q^2$. The unitarity relation (3) was used in (15). We should remark that by using unitarity we introduced still another approximation: The sums on the i index in expressions for $\langle s_\perp\rangle$ extend only over these neutrinos which are light enough, so that the decay $K \to \pi\mu\nu_i$ is kinematically allowed, while the sum in the unitarity relation extends over all neutrinos. Use of unitarity in (15) is justified only under the assumption that the mixing parameter $\mathcal{U}_{\mu i}$ is negligible when the ith neutrino is very heavy.

Before we describe the present experimental situation and speculate about future results we should note the following: The orthogonal spin of the muon is actually a measure of T breaking, rather than a direct test of CP, and T symmetry only implies that $s_\perp$ vanishes *at tree level.*[7] Standard-model electromagnetic loop effects introduce $s_\perp \sim \alpha$ in $K^0_{\mu 3}$ decays[8] and $s_\perp \lesssim 10^{-6}$ in $K^+_{\mu 3}$ decays.[9] Experiments designed to test CP in $K_{\mu 3}$ decays in the region $s_\perp \lesssim 10^{-3}$ will need to concentrate on K^+ decays or to measure $\Delta s_\perp = s_\perp - \bar{s}_\perp$, where $s_\perp$ is the orthogonal spin of the μ^+ in K^0 decay and $\bar{s}_\perp$ is the orthogonal spin of the μ^- in K^0 decay. $\Delta s_\perp$ has the advantage of being a direct CP (and not T) test and it is a truly CP-violating quantity which gets no contributions from electromagnetic loops. To include the loop effects in the formulas given in this paper one should replace $s_\perp$ by $\frac{1}{2}\Delta s_\perp$ everywhere.

At present, experimental results[10] are consistent with CP conservation, as predicted by the standard model: $\langle s_\perp \rangle = (-1.85 \pm 3.60) \times 10^{-3}$. The error is controlled by statistics and the same experimental setting in a K factory could observe $s_\perp$ values as small as 10^{-3}.

If nonvanishing $s_\perp$ is observed, we will be able to determine if its origin is A_i^S or A_i^T interaction. This will be done by analysis of the dependence of $s_\perp$ on the kinematical variables q^2 and α and comparison to Eq. (15).

In most theoretical beyond-standard models the predicted contribution of leptoquarks to $\langle s_\perp \rangle$ is too small to be detected. This is because leptoquarks are required to be very heavy in order to avoid their mediating various rare decays. As for nonstandard intermediate scalars, their coupling to a fermion pair is usually suppressed by m_f/M_W, where m_f is the mass of one of the two fermions. A_i^S is then suppressed by $m_S m_\mu / M_W^2 \approx 2 \times 10^{-6}$ relative to G_F. Such suppression will make $\langle s_\perp \rangle$ hopelessly small and render the effect undetectable. However, in some Weinberg-type models the Higgs scalar is much lighter then the W and consequently A_i^S is enhanced by $(M_W/M_S)^2$ relative to G_F. It was argued[9,11] that such light scalars could induce $\langle s_\perp \rangle$ of the order of 10^{-3}.

In summary, CP violation in $K_{\mu 3}$ decays may only be induced by nonstandard physics and here we considered the effects of intermediate nonstandard vectors, scalars, and leptoquarks. We showed that the CP-breaking effects of the vectors completely cancel. If nonvanishing orthogonal spin of the muon is seen, comparison of (15) with the observed dependence of $s_\perp$ on q^2 and α could give us some clue about the nature of the underlying physics responsible for this CP-violating phenomena.

I thank Gordy Kane for pointing out to me that one could search for nonstandard physics by looking for CP breaking in semileptonic K decays, and for useful comments during the course of this work. For related issues discussed by him and collaborators see Ref. 12.

[1] B. Kayser, in Proceedings of the Conference on Elementary Particle Physics, Aspen, 1988 (to be published).

[2] G. Beall, M. Bander, and A. Soni, Phys. Rev. Lett. **48**, 848 (1982); G. Ecker, W. Grimus, and H. Neufeld, Phys. Lett. **127B**, 365 (1983); **132B**, 467 (1983); H. Harari and M. Leurer, Nucl. Phys. **B233**, 221 (1984).

[3] H. Harari and Y. Nir, Phys. Lett. B **188**, 163 (1987); Nucl. Phys. **B292**, 251 (1987).

[4] Particle Data Group, M. Aguilar-Benitez *et al.*, Phys. Lett. B **204**, 13 (1988).

[5] H. Y. Cheng, Phys. Rev. D **28**, 150 (1983).

[6] T. D. Lee and C. S. Wu, Annu. Rev. Nucl. Sci. **16**, 471 (1966).

[7] N. Byers, S. W. MacDowell, and C. N. Yang, in *Proceedings of the International Seminar in High Energy Physics and Elementary Particles, Trieste, 1965* (International Atomic Energy Agency, Vienna, 1965), p. 953.

[8] L. B. Okun and I. B. Khriplovich, Yad. Fiz. **6**, 821 (1967) [Sov. J. Nucl. Phys. **6**, 598 (1968)].

[9] A. R. Zhitnitskii, Yad. Fiz. **31**, 1024 (1980) [Sov. J. Nucl. Phys. **31**, 529 (1980)].

[10] S. R. Blatt *et al.*, Phys. Rev. D **27**, 1056 (1983).

[11] H. Y. Cheng, Phys. Rev. D **26**, 143 (1982).

[12] P. Castoldi, J. M. Frere, and G. L. Kane, Phys. Rev. D (to be published).