Baryon-Number Violation in a Quantum Gas of W and Higgs Bosons

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Use of a coherent-state representation of the sphaleron allows a direct calculation of its production rate in a thermal gas of W and Higgs bosons. Technical considerations permit a straightforward calculation only in the case $\lambda/g^2 \simeq 1$ ($M_H \simeq 3M_W$), where λ is the Higgs-boson quartic coupling, and g is the SU(2) gauge coupling. For this case it is found that the rate is unsuppressed for temperatures $T \gtrsim 2.4 M_W(0)$, where $M_W(0)$ is the zero-temperature W mass. Thus anomalous $B+L$ violation is also unsuppressed above this temperature.

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Baryon-number violation in the standard model is induced by a U(1) anomaly[1] for the axial-vector baryon current. At zero temperature, the $\Delta B \neq 0$ processes are instanton dominated, and are suppressed[2] by factors $\sim \exp(-4\pi/\alpha_w)$. However, it has been argued[3] that at high temperatures $\Delta B \neq 0$ processes are unsuppressed because classical thermal fluctuations can allow ambient field configurations to cross the potential barrier between distinct topological vacua. The resulting violation of baryon number, which could wash out any pre-existing $B+L$ excess, may then be understood as resulting from fermion level crossings[4] in the presence of the unstable classical field configurations interpolating the vacua. The configuration of minimum energy,[5] called the "sphaleron,"[6] plays a central role in these semiclassical discussions: The transition rate is controlled by the Boltzmann factor $\exp(-E_{\rm sph}/T)$, and by the Gaussian and zero-mode fluctuations of the fields about the sphaleron.[7]

Several recent studies have focused on the nature of the transition in real time. A computer study[8] of the $1+1$ Abelian Higgs model (which contains an anomaly, sphalerons, and topologically distinct vacua) confirms that a random field configuration selected from a heat-bath environment will evolve in real time, at an unsuppressed rate, to configurations with nontrivial winding number, and hence will precipitate fermion level crossings. In doing so, the field passes through the "kink" of the $1+1$ theory, which is the sphaleron of this model. These real-time transitions have also been discussed for the pendulum problem,[9] and, in a variational approximation, for the gauge theory itself.[10]

In all of these studies, the sphaleron evolves from initial configurations chosen in a basis of classical fields. The question then arises: Can one calculate the rate for sphaleron creation in the more canonical early universe environment, an incoherent quantum gas of free W's, Higgs bosons, and other particles? Such a calculation is the subject of this work. In what follows, it is shown explicitly that for the (calculable) case $\lambda/g^2 \simeq 1$, the rate for the process W's,H's$\rightarrow$ sphaleron is unsuppressed for temperatures $T \gtrsim 2.4 M_W(0)$, where $M_W(0)$ is the zero-temperature W mass. Here, as usual, λ is the Higgs-boson quartic coupling, and g is the SU(2) gauge coupling. There is no reason to suspect that the rate is suppressed for other values of λ/g^2—it is just that, for reasons which will become clear, the calculation becomes grossly unreliable unless $\lambda/g^2 \simeq 1$. Accompanying the unsuppressed sphaleron production is, of course, the violation of $B+L$ described above.

The starting point for the analysis is the scattering formula

$$S_{i\rightarrow \rm sph} = -2\pi i\, \delta(E_i - E_{\rm sph}) T_{i\rightarrow \rm sph}\,, \tag{1}$$

where

$$T_{i\rightarrow \rm sph} = \langle \psi_{\rm sph}(\mathbf{R}) \,|\, H_{\rm int} \,|\, k_1, \ldots, k_{n_W}; q_1, \ldots, q_{n_H}\rangle \tag{2}$$

describes the formation of a sphaleron, centered at $\mathbf{R}$, from an initial state consisting of n_W W's and n_H Higgs scalars. For typographic economy, both the spin and isospin components of the ith W are included in the momentum label k_i.

For the purpose of this work, I ignore the finite lifetime ($\sim M_W^{-1}$) of the sphaleron, and take the state $|\psi_{\rm sph}(\mathbf{R})\rangle$ to be a normalized eigenstate of the complete Hamiltonian: $\langle \psi_{\rm sph}(\mathbf{R}') \,|\, \psi_{\rm sph}(\mathbf{R})\rangle = \delta_{\mathbf{R},\mathbf{R}'}$. It is convenient to work on a finite lattice constructed in configuration space, with lattice spacing $\sim 2r_{\rm sph} \sim M_W^{-1}$. This description reflects the fact that sphalerons are not eigenstates of the total three-momentum, but are appropriately labeled by their centers $\mathbf{R}$. The state $|k_1, \ldots, k_{n_W}; q_1, \ldots, q_{n_H}\rangle$ is a Fock-space ket describing the initial state, constructed from single-particle states normalized in a box of volume V: $\langle \mathbf{k}_i \,|\, \mathbf{k}_j\rangle = \delta_{\mathbf{k}_i,\mathbf{k}_j}$. Finally, $H_{\rm int}$ is the SU(2)+Higgs-boson interaction Hamiltonian.

Central to this work is the use of a coherent- (Fock-) state representation of the classical field configuration. Such representations have been previously given.[11] It is a property of a coherent state (prepared at $t=0$) that if the interaction energy is small and the occupation number is large, then the field eigenvalue changes only by a phase as time passes (the minimal wave packet does not spread).[12] If the interaction energy is large, the field eigenvalue is unstable with time (the wave packet will show a large spread). Hence, in order to sensibly use the formula (2), we must have a configuration which is almost an eigenstate of the free Hamiltonian. From Ref. 11, this occurs for the case $\lambda/g^2 \simeq 1$, in which case the expectation value of the free Hamiltonian in the sphaleron state is approximately equal to the mass of the sphaleron, $E_{\rm sph}$. For this case, it is also true that the contribution of Higgs bosons to the energy and occupation number is small, and can be ignored. Thus, I concentrate on this case.

We may then expand $T_{i\to \rm sph}$ in the free-particle basis,

$$T_{i\to \rm sph}=\sum_{n_W'}\sum_{\{k_i'\}}\frac{1}{n_W'!}\langle\psi_{\rm sph}(\mathbf{R})\,|\,k_1',\ldots,k_{n_W'}'\rangle \times\langle k_1',\ldots,k_{n_W'}'\,|\,H_{\rm int}\,|\,k_1,\ldots,k_{n_W}\rangle. \quad (3)$$

The interaction Hamiltonian $H_{\rm int}$ mediates various $2\to 2$ scattering processes, containing all possibilities of $(W/H)+(W/H)\to(W/H)+(W/H)$. To avoid notational confusion, I will proceed including only $WW \to WW$. It will be obvious that this will suffice to obtain the results of this paper. Thus, I take

$$\langle k_1',\ldots,k_{n_W'}'\,|\,H_{\rm int}\,|\,k_1,\ldots,k_{n_W}\rangle = \delta_{n_W,n_W'}\sum_{lmrs}\langle k_r'k_s'\,|\,H_{\rm int}\,|\,k_lk_m\rangle\sum_{P\{a_i\}}\prod_{\substack{i\neq lm\\ a_i\neq rs}}\delta_{\mathbf{k}_i,\mathbf{k}_{a_i}'}\,. \quad (4)$$

The first matrix element in Eq. (3) was obtained in the previously referred to analysis.[11] With a slight change of notation, and a shift of the origin to $\mathbf{R}$, the results of Ref. 11 give

$$\langle\psi_{\rm sph}(\mathbf{R})\,|\,k_1',\ldots,k_{n_W'}'\rangle=(n_W'!)^{-1/2}e^{-N_W/2}e^{i\mathbf{P}\cdot\mathbf{R}} \times\left(\frac{1}{\sqrt{V}}\right)^{n_W'}\prod_{i=1}^{n_W'}w(k_i'\lambda_i'a_i')\,, \quad (5)$$

where

$$\mathbf{P}=\sum_i\mathbf{k}_i\,,\quad w(k\lambda a)\equiv(\tfrac{1}{2}\omega_k)^{1/2}\boldsymbol{\epsilon}(\mathbf{k}\lambda)\cdot\int d^3r\,e^{i\mathbf{k}\cdot\mathbf{r}}\tilde{\mathbf{W}}^a(\mathbf{r})\,,$$

and

$$\omega_k\equiv(k^2+M_W^2)^{1/2}\,.$$

$\tilde{\mathbf{W}}^a$ is the regulated, unitary-gauge vector wave function in configuration space,[11] and

$$N_W=\int\frac{d^3k}{(2\pi)^3}\sum_{\lambda,a}|\,w(k\lambda a)\,|^2. \quad (6)$$

For the interacting pairs, we have

$$\langle k_r'k_s'\,|\,H_{\rm int}\,|\,k_lk_m\rangle=(2\pi)^3\delta^3(\mathbf{k}_l+\mathbf{k}_m-\mathbf{k}_r'+\mathbf{k}_s')(2\omega_l2\omega_m2\omega_r'2\omega_s')^{-1/2}(1/V^2)\mathcal{T}(lm\to rs)\,, \quad (7)$$

so that, on replacing the closure sum on k_r',k_s' with $V^2(2\pi)^{-6}\int d^3k_r'd^3k_s'$, one obtains [from Eqs. (3)-(7)]

$$T_{i\to\rm sph}=\frac{1}{(n_W!)^{1/2}}\frac{1}{n_W(n_W-1)}e^{-N_W/2}e^{i\mathbf{P}\cdot\mathbf{R}}\sum_{lm}\prod_{i\neq l,m}^{n_W}w(k_i\lambda_ia_i)A_{lm}\,,$$

$$A_{lm}=\sum_{\lambda_r'\lambda_s'a_r'a_s'}\frac{1}{(2\pi)^3}\int d^3k_r'd^3k_s'\delta^3(\mathbf{k}_l+\mathbf{k}_m-\mathbf{k}_r'+\mathbf{k}_s')w(k_r'\lambda_r'a_r')w(k_s'\lambda_s'a_s')(2\omega_l2\omega_m2\omega_r'2\omega_s')^{-1/2}\mathcal{T}(lm\to rs)\,. \quad (8)$$

On dimensional grounds, I set

$$A_{lm}=(\alpha_w/\pi)TC_{lm}w(k_l\lambda_la_l)w(k_m\lambda_ma_m)\,, \quad (9)$$

where T denotes temperature, $\alpha_w=g_{\rm SU(2)}^2/4\pi\simeq 1/28$, and C_{lm} is a number ~ 1. It will be sufficient for the purposes of this paper to set $C_{lm}\sim C\sim{\rm const}\sim 10^{0\pm 2}$. The sum on $\{lm\}$ in Eq. (8) will approximately cancel the combinatoric factor $n_W(n_W-1)$ in the denominator, and I obtain as my approximation to the amplitude

$$T_{i\to\rm sph}\simeq C\left(\frac{\alpha_w}{\pi}\right)T\frac{1}{(n_W!)^{1/2}}e^{-N_W/2}e^{-i\mathbf{P}\cdot\mathbf{R}}\prod_{i=1}^{n_W}w(k_i\lambda_ia_i)\,. \quad (10)$$

From (1) and (10), the rate of sphaleron formations in the plasma is given by

$$\Gamma_{i\to\rm sph}=2\pi\,|\,C\,|^2\left(\frac{\alpha_w}{\pi}\right)^2T^2e^{-N_W}\sum_{\mathbf{R}}\sum_{n_W=0}^{\infty}\frac{1}{n_W!}\int\prod_{i=1}^{n_W}\frac{d^3k_i}{(2\pi)^3}\bar{n}_W(\omega_{k_i})\sum_{\lambda_i,a_i}|\,w(k_i\lambda_ia_i)\,|^2\delta\left(E_{\rm sph}-\sum_{i=1}^{n_W}\omega(k_i)\right), \quad (11)$$

where $\bar{n}_W(\omega_k)=[\exp(\omega_k/T)-1]^{-1}$. According to the previous discussion, we make the replacement $\sum_{\mathbf{R}}\to VM_W^3$. The energy-conservation condition is implemented by the replacement

$$2\pi\delta\left(E_{\rm sph}-\sum_{i=1}^{n_W}\omega(k_i)\right)=\int_{-\infty}^{\infty}dt\,\exp\left[-i\left(E_{\rm sph}-\sum_{i=1}^{n_W}\omega(k_i)\right)\right].$$

The sum can then be carried out, and the equation for the rate per unit volume becomes

$$(\Gamma/V)_{i\to \text{sph}} = |C|^2 \left(\frac{\alpha_w}{\pi}\right)^2 T^2 M_W^3 e^{-N_W} \int_{-\infty}^{\infty} dt\, e^{-iE_{\text{sph}}t + f(t)}, \tag{12}$$

where

$$f(t) \equiv \int \frac{d^3k}{(2\pi)^3} \bar{n}_W(\omega_k) \sum_{\lambda,a} |w(k\lambda a)|^2 e^{i\omega_k t}. \tag{13}$$

The t integral in Eq. (12) can be evaluated using the saddle-point method. The saddle point is located at a point $t_0 = i\tau$ on the imaginary t axis, where τ is given by

$$E_{\text{sph}} = \int \frac{d^3k}{(2\pi)^3} \bar{n}_W(\omega_k)\omega_k \sum_{\lambda,a} |w(k\lambda a)|^2 e^{-\omega_k \tau}. \tag{14}$$

If the second derivative of $f(t)$ at the saddle point is approximated by M_W^2, the rate becomes

$$(\Gamma/V)_{i\to \text{sph}} = |C|^2 \left(\frac{\alpha_w}{\pi}\right)^2 T^2 M_W^2 e^{\Delta N}, \tag{15}$$

where

$$\Delta N = E_{\text{sph}}\tau - N_W + f(i\tau). \tag{16}$$

It should be remembered that one-loop corrections will induce a temperature dependence in the vacuum expectation value of the Higgs field, and hence a temperature dependence in the quantities M_H, M_W, and E_{sph}.[7] This dependence must be implemented whenever one of these quantities appears.

From Eq. (15) we can see that the formation of sphalerons will be (exponentially) unsuppressed at temperature T if $\Delta N \geq 0$, with τ at any temperature given by Eq. (14).

The result is very simply stated, and is shown with a bit of detail in Table I: The rate of formation of sphalerons (with $\lambda/g^2 \simeq 1$) is exponentially suppressed at temperatures $T < 2.4 M_W(0)$, and is unsuppressed at temperatures $T \geq 2.4 M_W(0)$. [$M_W(T) = \frac{1}{2} g v(T)$ denotes the value of W mass at temperature T.] The relation between sphaleron creation and the net rate of generation of $B+L$ in a radiation-dominated universe is given by[13]

$$\frac{1}{n_B}\frac{dn_B}{dT} \approx \frac{m_{\text{Pl}}}{T^2} \frac{(\Gamma/V)_{i\to \text{sph}}}{T^4}, \tag{17}$$

so that unsuppressed sphaleron creation in a finite-temperature interval will imply the washout of preexisting $B+L$.

In conclusion, several comments are in order.

(1) The calculation presented here shows that in at least one nontrivial case ($\lambda/g^2 \neq 1$), the classical sphaleron field configuration is formed at an unsuppressed rate in a high-temperature environment. The more general cases $\lambda/g^2 \simeq 1$ cannot be consistently examined using the simple approach in this paper. As a check on the consistency of the approach used here for the case $\lambda/g^2 \simeq 1$, one may calculate the decay rate of the sphaleron in vacuum. This is given by the expression (11), with the summation over **R** removed, and with all the $\bar{n}_W(\omega_{k_i})$ set equal to 1. The result is of $O(M_W)$ (i.e., $\Delta N \simeq 0$), which is the result expected from the classical analysis.

(2) Scaling arguments applied to the finite-temperature action[7] make plausible the idea that static configurations (like the sphaleron) will dominate the transitions at high temperatures [$T \gg 2M_W(T)$]. I find that the sphaleron is actually formed in the plasma at an unsuppressed rate at temperatures only slightly above $2M_W(T)$. [More quantitatively, for $\lambda/g^2 = 1$ and in the temperature range shown, the ratio $M_W(T)/M_W(0) = 0.75$. Hence the rate is unsuppressed for $T \geq 3.2 \times M_W(T)$.]

(3) At these temperatures, a simple calculation shows that the radius of the sphaleron, $R_{\text{sph}} \simeq [2M_W(T)]^{-1}$, is approximately equal to the Debye screening length $R_D \simeq (gT)^{-1}$. Thus screening corrections (discussed extensively in Ref. 7) should not affect the results of this work in any qualitative manner.

(4) During the era [$T > 2.4 M_W(0)$] in which sphalerons are produced at an unsuppressed rate, the electroweak plasma should be considered as strongly interacting. Perhaps sphalerons and other nearby classical field configurations are the correct quasiparticles of the system. This then may provide justification for the quasiclassical approach which has thus far been used in the analysis of the problem.

(5) It is important to state that fermion level crossings will take place for all field configurations with nonzero Chern-Simons density. The work done here samples only the (finite width) sphaleron.

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TABLE I. Numerical profile of the transition to unsuppressed sphaleron creation, for $\lambda/g^2 = 1$, at a temperature $T = 2.4 M_W(0)$. $e^{\Delta N}$ is the suppression (or enhancement) of the rate, as defined in Eqs. (15) and (16). The quantity $i\tau$ is the position of the saddle point in the energy-conservation integral [Eq. (12)]. For $\lambda/g^2 = 1$, the mass of the sphaleron $E_{\text{sph}} = 2.10 \times [2M_W(T)/\alpha_w]$, and in the temperature range shown, the ratio $M_W(T)/M_W(0) = 0.75$.

$T/[2M_W(0)]$	$\tau/[2M_W(T)]^{-1}$	ΔN
1.1	−0.17	−11
1.2	0	0
1.3	0.19	12

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