Electron Quantum Interference and $1/f$ Noise in Bismuth

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The $1/f$ resistance noise of thin Bi films of lateral dimensions 1–10 μm increases at low temperatures approximately as T^{-1}, and the noise at 1 K is larger than at room temperature. The noise magnitude is reduced by a factor of 2 above a temperature-dependent characteristic magnetic field. These phenomena demonstrate that below liquid-nitrogen temperature the $1/f$ noise in a weakly disordered metal arises from defect-mediated quantum interference of conduction electrons.

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The last several years have seen substantial progress in the understanding of the role of quantum interference in transport properties of metals in the presence of disorder.[1] Several new phenomena have shown the inadequacy of the traditional Boltzmann approach to the problem, even in the limit of weak disorder. Two of the most striking observations are the weak localization corrections[2] to the electrical conductance, and the "universal conductance fluctuations"[3] (or UCF) that occur in small metallic samples. Both of these phenomena arise from the phase coherence of the electronic wave function over a distance L_{IN}, the inelastic diffusion length, which can be much longer than the elastic mean free path, l_e, at low temperatures. In addition, the UCF reflect the detailed interference pattern of the electron paths in a specific sample and not the ensemble average. The UCF have been observed in very small samples, either by varying the magnetic field[4] or by changing the Fermi energy.[5] However, it has been predicted that the sensitivity of the resistance to the motion of defects[6,7] implied by the theory of UCF should be observable as dynamic $1/f$ noise in disordered metals, even in samples with dimensions much larger than L_{IN}.

To test the relevance of quantum interference to $1/f$ noise in metals, we have measured the $1/f$ noise spectrum in several Bi films over the temperature range 0.3–300 K. The samples are not particularly small—they have lateral dimensions of 1–10 μm, the same dimensions as found in very-large-scale integration electronics. We find that the magnitude of the noise increases below about 70 K, and continues to increase on cooling to the lowest temperatures measured. The magnitude of the noise is strongly influenced by a weak magnetic field—it decreases by a factor of 2 at very low temperatures. The static magnetic-field-dependent resistance fluctuations (the "magnetofingerprint") undergo slow temporal changes even below 1 K. These measurements demonstrate that the $1/f$ noise below 70 K originates in quantum interference phenomena.

Polycrystalline Bi films of thickness 11 or 90 nm were thermally evaporated at room temperature onto oxidized Si substrates at a pressure of about 5×10^{-7} Torr. The low-temperature sheet resistance $R_\square$ of the 90-nm samples is about 110 Ω, while that of the 11-nm-thick samples is about 600 Ω. The resistance varies by only a few percent between 4 K and liquid-nitrogen temperature, while it decreases by (35–65)% on warming to room temperature.[8] The samples were patterned into five-terminal devices with photolithography and liftoff processing. Sample areas ranged from 1.3×5 μm^2 to 10×100 μm^2. The power spectrum of resistance fluctuations $S_R(f)$ was measured in the spectral range 0.05–10 Hz with the five-terminal ac bridge technique.[9]

Figure 1 shows the normalized noise power $fS_R(f)/R^2$ evaluated at 1 Hz for two 90-nm films and one 11-nm film, as a function of temperature. The two 90-nm films were prepared in the same evaporation, and their noise powers scale inversely with their volume. The peak in the noise just below room temperature was previously noted.[10] The novel observation in the present work is that the noise does not decrease monotonically as the

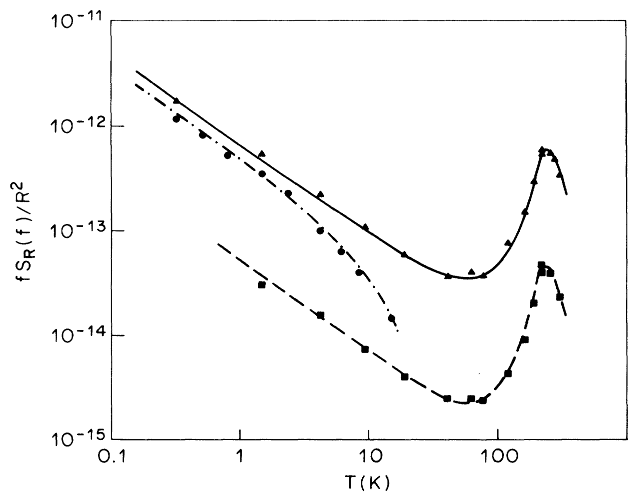

FIG. 1. Log-log plot of normalized noise power, $fS_R(f)/R^2$, evaluated at $f=1$ Hz, vs temperature, for three Bi samples. Sample dimensions are the following: 10 μm $\times$ 100 μm $\times$ 90 nm (■); 2 μm $\times$ 20 μm $\times$ 90 nm (▲); 10 μm $\times$ 100 μm $\times$ 11 nm (●).

temperature is lowered, but instead it reaches a minimum at $T \approx 70$ K, and then rises dramatically as $T \rightarrow 0$. The 11-nm Bi film, measured only at low temperature, shows the same asymptotic temperature dependence. The spectral shape of the noise is very close to $1/f$ over the entire low-temperature range. The lowest temperatures shown represent the limit at which electron heating could be neglected. Since the sample resistances are essentially constant below 70 K, the normalization has no effect on the temperature dependence of the noise power.

Figure 2 shows the conductance noise $S_G(f) \equiv S_R(f)/R^4$ as a function of magnetic field for an 11-nm Bi sample at two different temperatures, 0.5 and 1.5 K. The noise decreases in the presence of the magnetic field, reaching an asymptotic value of about $\frac{1}{2}$ its zero-field value. The characteristic field for the reduction of the noise depends strongly on temperature. The sample has a positive magnetoresistance of a few percent, due to weak localization in the presence of strong spin-orbit scattering.[11] This small magnetoresistance is not the cause of the conductance noise decrease in Fig. 2.

The striking low-temperature results of Figs. 1 and 2 do not appear consistent with theories of $1/f$ noise based on local electron scattering from defects.[12] We believe that they can be explained by electron quantum interference as described by Feng, Lee, and Stone.[7] There are four aspects to the noise which we discuss: (1) the anomalous temperature dependence, $T^{-0.9 \pm 0.1}$ below a few K, (2) the sensitivity to a magnetic field, (3) the noise magnitude, and (4) the spectral characteristics. The derivation of the Feng, Lee, and Stone theory proceeds in two steps. First one calculates δG_1, the magnitude of the conductance fluctuation in a "coherence volume" (i.e., a cube of dimensions $L_{\rm IN}$ in the 3D case or a box of sides $L_{\rm IN}$ and height equal to the sample thickness in the 2D case),

$$\delta G_1^2 = C \left(\frac{e^2}{h} \right)^2 \frac{L_{\min}^2 l_e}{(k_f l_e)^2} n_S(T) \alpha(k_f \delta r) . \quad (1)$$

$L_{\min}$ is the smaller of $L_{\rm IN}$ or L_T, the thermal length $(\hbar D/k_B T)^{1/2}$ where D is the conduction electron diffusivity. L_T enters only if $k_B T > \hbar D/L_{\rm IN}^2$, due to energy averaging. The function $\alpha(x) = 1 - (\sin\frac{1}{2}x/\frac{1}{2}x)^2$ expresses the phase shift of an electronic wave function due to the displacement δr of a defect. The constant C contains a factor of $\frac{1}{4}$ due to the strong spin-orbit scattering in Bi, and also a numerical factor of order unity given by Lee, Stone, and Fukuyama.[3] Equation (1) is valid for $d=2$ and 3, and assumes independent contributions from scatterers in the coherence volume, where $n_S(T)$ is the density of fluctuating scattering centers.[7] The conductance fluctuation in the entire sample is obtained by the addition of the contributions from coherence volumes classically,

$$(\delta G_{\rm total}/G_{\rm total})^2 = (1/N)(\delta G_1/G_1)^2 , \quad (2)$$

where N is the number of coherence volumes in the sample, and depends on temperature through $L_{\rm IN}$. The temperature-dependent terms for the total conductance variance are given by

$$\delta G_{\rm total}^2 \propto n_S(T) L_{\min}^2 L_{\rm IN}^{4-d} . \quad (3)$$

The scale of $L_{\rm IN}$ for Bi films is known from extensive analysis of magnetoresistance data.[13] Below 2 K, $L_{\rm IN}$ is greater than the sample thickness; thus the $d=2$ case applies. In this region $L_{\rm IN}$ is controlled by electron-electron scattering[11] and it as well as L_T (which have comparable magnitudes, 100–200 nm at 1 K) vary as $T^{-1/2}$. If we make the assumption that the scattering centers are two-level tunneling systems with nearly constant density of states, $n_S(T) \propto T$. Thus below 2 K the $1/f$ noise increases as T^{-1} because the temperature-dependent length factors overwhelm the freezeout of active scattering centers.

Above 2 K the situation is complicated by crossover from 2D to 3D near 2 K and 20 K for the 90- and 11-nm films, respectively. Contributions from the electron-phonon interaction cause $L_{\rm IN}$ to vary as $T^{-1.25}$ above 5 K.[11] The stronger T dependence above 5 K of the thin sample reflects this change. The thick samples, well into the 3D limit, show the expected weaker T dependence. Additional uncertainty exists at higher temperatures because a crossover of defect dynamics to activated behavior proably occurs above 10 K.

A consequence of the interpretation in terms of electron quantum interference[6,7] is that the noise should be reduced by a factor of 2 in a magnetic field.[14] This results from the breaking of time-reversal symmetry on the

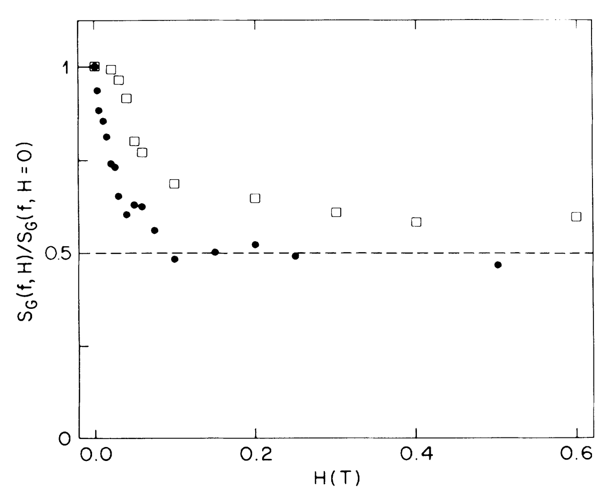

FIG. 2. Conductance noise power, $S_G(f) \equiv S_R(f)/R^4$, as a function of magnetic field, normalized by its zero-field value, of a Bi sample of dimensions 10 μm$\times$1.3 μm$\times$11 nm and resistance 4.8 kΩ. The data are for $T=0.5$ K (●) and $T=1.5$ K (□). The magnetic field is applied perpendicular to the plane of the film.

scale of L_{IN}; hence the magnetic field scale is comparable to the scale of the weak-localization contribution to the magnetoresistance. If one defines H^* as that field where the noise is reduced to $\frac{3}{4}$ of its zero-field value, then $H^* = c^*(h/e)L_{IN}^2$, where $c^* = 0.1$–0.2.[15] We thus obtain $L_{IN} \approx 0.20$ μm at 0.5 K, and 0.12 μm at 1.5 K. We have observed the magnetic field noise reduction for T as high as 20 K, and the data are consistent with $L_{IN} \propto T^{-0.8}$ between 1.5 and 20 K.

The theoretical result for the total fluctuation due to the noise, summarized in Eqs. (1) and (2), is only valid if δG_1 does not exceed the universal value e^2/h. We can obtain the magnitude of the universal conductance fluctuations, δG^{UCF}, by measuring the static conductance fluctuations as a function of magnetic field. Figure 3 shows the magnetofingerprint at 0.3 K for the same sample shown in Fig. 2. The top trace was taken while sweeping the field up at a rate of 3 T/h, and the bottom trace was taken immediately after while sweeping the field down at the same rate. The fingerprint is highly reproducible on a time scale of a few hours, but changes appear over long times, as shown in the shaded part of the figure.

From the autocorrelation function of the data in Fig. 3, we extract both the size and the characteristic magnetic field scale of the fluctuations.[16] These data yield a fluctuation magnitude $(\delta G^{UCF})^2 \approx 2\times10^{-6}(e^2/h)^2$ and a correlation field $H_c = 0.04$ T. Since $H_c = (h/e)L_{IN}^2$, we obtain $L_{IN} \approx 0.3$ μm at $T = 0.3$ K (and 0.2 μm at 1.5 K) in good agreement with the value obtained from H^*. We can calculate $(\delta G^{UCF})^2$ using Eq. (2) and $(\delta G_1^{UCF})^2 = (0.61\times\frac{1}{2}e^2/h)^2$, where the factor of $\frac{1}{2}$ again arises from spin-orbit scattering.[3] Using the above

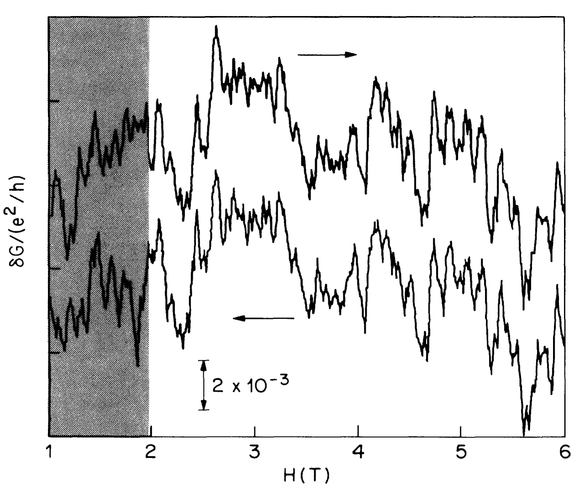

FIG. 3. Conductance fluctuation δG normalized to e^2/h as a function of magnetic field (magnetofingerprint), for the sample shown in Fig. 2. The shaded region indicates where the two traces differ substantially, and represents a time difference of 3–4 h. The traces are offset for clarity.

value of L_{IN} we obtain $(\delta G^{UCF})^2 = 10\times10^{-6}(e^2/h)^2$, in remarkable agreement with experiment. Extraction of L_{IN} is more difficult at higher temperatures because H_c becomes comparable to the available field.

The total fluctuation due to the $1/f$ noise is just the integral of the noise power times frequency: $\delta G^2 = \int_0^\infty S_G(f)\,df$. If the noise is strictly $1/f$ over a wide bandwidth, then the integral contains equal contributions from each decade of frequency. At high fields, the noise power at 0.5 K is $fS_G(f) \approx 3\times10^{-9}(e^2/h)^2$, while the saturated UCF have an amplitude (obtained from the magnetofingerprint at 0.5 K) $(\delta G^{UCF})^2 \approx 1.5\times10^{-6} \times(e^2/h)^2$. It would therefore require about 200 decades of $1/f$ noise to saturate the UCF. This observation is important for two reasons. It validates the assumptions behind Eq. (1), and it implies that the observed $1/f$ spectral shape of the noise is due to the underlying microscopic distribution of rates, and has not been distorted by saturation of the fluctuations at the UCF amplitude.[17]

To understand the changes that occasionally occur in the magnetofingerprint, we have measured the sample resistance as a function of time over several hours at fixed magnetic field. We do observe occasional large, slow, changes which could account for the evolving magnetofingerprint, but whose contribution to the noise power in their frequency bandwidth does not exceed the $1/f$ contribution. No evidence is seen for sudden discrete jumps of the kind recently reported[18] in Bi samples larger than ours.

Recently, interesting $1/f$ noise data have been reported[19] on CCu composites near the metal-insulator transition. In contrast to our results, the conductance noise in CCu is temperature independent below 10 K. Unfortunately, little is known about the magnitude and temperature dependence of the relevant length scales in this material. Presumably L_{IN} is very short, which is why neither the magnetofingerprint nor the full reduction of the noise in a field are observable.

Even though tunneling systems have been invoked as the source of elastic scattering at low temperatures, we consider the connection to $1/f$ noise to be highly qualitative. The most convincing argument is the requirement of a $1/f$ spectrum which is provided by the distribution of rates that follows from the standard tunneling model.[7] The estimate of the noise amplitude is more tenuous. Taking Bi as a case in point, we can estimate from transport $k_f \approx 0.9$ nm^{-1} and $l_e \approx 5$ nm.[8] Taking $\delta r \approx 0.1$ nm as an upper limit, we get $\alpha(k_f\delta r) < 10^{-2}$. From Eqs. (1) and (2), taking the value $n_S(T) = 10^{17}$ cm^{-3} as the density of centers with energies less than 1 K and relaxation rates faster than 1 Hz for *glasses*, and $L_{IN} = 0.1$ μm, we find rough agreement with our experimental magnitudes. However, there is little information on tunneling centers in polycrystalline metals and even less on their relaxation rates near 1 Hz. The most relevant experiments are recent low-frequency acoustic measurements between 0.1 and 10 K on polycrystalline alloys which indicate a tunneling system density of states about $\frac{1}{10}$ that of metallic glasses.[20]

The minimum in the noise spectral density at 70 K separates the quantum interference regime from a "local interference"[12] regime at higher temperatures. At this temperature we estimate that L_{IN} becomes comparable to l_e, so that the fundamental requirement for coherence, $L_{IN} > l_e$, is violated. It is remarkable nonetheless that coherent electronic transport should persist to such high temperatures.

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