Peeling U(1)-Gauge Cosmic Strings

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We numerically investigate collisions of cosmic strings carrying different winding numbers. We find that for strings with winding numbers n_1 and n_2, intercommutation occurs by peeling a string of winding number $|n_1-n_2|$ from the string with the larger winding number. The resulting string connects the original colliding strings to form a state of three joined strings, but because of the peeling the eventual result is a reduction in the winding numbers of the network. Stable astrophysical strings with large winding number are thus unlikely to persist. All simulations have gauge/scalar-field mass ratio $=2$.

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Since the introduction of the cosmic-string theory,[1] it has been clear that the process of chopping off loops from infinite strings via intercommutations needs to be investigated. Loops breaking away from the main network are required to form seeds around which matter begins to accrete.[2] A numerical treatment seems unavoidable because the nonlinearity of the theory prevents making definite conclusions from analytical approaches.

Numerical studies[3-5] of intercommutation have shown that local and global strings exchange partners under most circumstances. Recently,[6,7] we have extended the numerical analysis to superconducting-bosonic-string interactions and found the same result. In all these numerical simulations each of the colliding strings had winding number equal to unity. It has been suggested[8] that by allowing the strings to possess unequal winding numbers, the intercommutation could become more complicated because the wrapping of the scalar-field phase prevents a string with no branches from changing its winding number between different points along the string.

We have performed numerical simulations to investigate how the outcome of intercommutation is changed when the colliding strings have unequal winding numbers. The interacting string consists of a complex scalar field $\Phi\equiv Re^{i\psi}$ minimally coupled to U(1) gauge vector field A_a, with the scalar field self-interacting through the standard "Mexican-sombrero" potential. The Lagrangian for the system reads

$$\mathcal{L}=\tfrac{1}{2}\nabla_a R\nabla^a R+\tfrac{1}{2}R^2(\nabla_a\psi+eA_a)\nabla^a\psi+eA^a) + (\lambda/8)(R^2-\eta^2)^2+\tfrac{1}{4}F_{ab}F^{ab}, \qquad (1)$$

where $F_{ab}\equiv\nabla_a A_b-\nabla_b A_a$ and e, λ, and η are positive constants. In cylindrical coordinates (ρ,ϕ,z), consistent static coupled-fields solutions are found to have the form $R=R(\rho)$, $\psi=n\phi$, and $A_a=(n/e)[P(\rho)-1]\nabla_a\phi$. Here n represents the winding number. Since the vacuum manifold arising from the Mexican-sombrero potential is not simply connected, n counts the twists in the phase of the scalar field.

The interstring potential is determined by $\alpha\equiv e/\sqrt{\lambda}$, the relative strength of the gauge and scalar fields. As for flux lines in the Ginsburg-Landau theory of superconductors, U(1)-gauge cosmic strings can be classified by the value of α. If $\alpha>1$, the equations describing static strings are identical to those of type-I superconductivity. Parallel vortices of this type attract each other because the magnetic penetration depth of the U(1)-gauge (repulsive) field is smaller than the coherence length of the scalar (attractive) field. The situation reverses for $\alpha<1$. Here parallel strings are like vortices in type-II superconductors and have repulsive interactions. Thus one intuitively concludes that only type-I vortices with winding number $n>1$ are stable. Indeed, studies of classical solutions[9] verify that $n>1$ type-I (type-II) strings are stable (unstable). Therefore, analysis of $n>1$ string collisions must be performed on type-I vortices.

We have carried out numerical string-collision simulations for 45°, 90°, and 135° incident angles and $v/c\gtrsim 0.75$ initial velocities since strings typically reach relativistic speeds when friction becomes negligible.[10] Each colliding string, labeled R (L) right (left) with winding number n_R (n_L) is a type-I vortex. We have chosen $\alpha=2$ because in general one expects the parameters e and $\sqrt{\lambda}$ to be of the same order. Results are similar for all cases considered.

The simulations used Matzner's code[4] for string interactions adapted to allow collisions of strings with unequal winding numbers. Initial data are constructed by superposed boosted static strings.[4] The Lorentz gauge is used to simplify the wave equations for both the gauge and dynamic variables. One may verify that the boosted static solutions for A_a satisfy this gauge. The evolution uses an explicit staggered leapfrog difference scheme, second order in both space and time.[11] Boundary conditions are obtained from the initial data, and they behave as if the strings extended infinitely out of the 64^3 computational grid. We find very small $(\lesssim 0.01)$ relative differences between the interior evolved region near the boundary and the analytic boundary forms until interior effects reach the boundaries. Energy conservation was tested by computing the sum of the time-time component of the stress tensor throughout the cube; it is maintained at ≤ 0.05 relative until interactions reach the boundary. The Lorentz-gauge condition is preserved during the evolution of ≤ 0.04 relative change. A simpler test of energy conservation is to compute the kinetic energy of a single moving string segment. We find ≤ 0.01 relative change in the kinetic energy upon evolving the string completely across the computational cube. The code has also been tested by comparing 64^3 and 128^3 computations, for which identical results (relative changes ≤ 0.003) are obtained on sampling back the computation to 64^3.

Figure 1 shows the evolution for $(n_L, n_R) = (2,1)$, 90°, and $v/c = 0.75$. The string with larger n has larger core radius. Stability of the $n_L = 2$ string is verified since we compute the linear energy density for $n_L = 2$ and find it 7.9% smaller than twice the linear energy density for $n_L = 1$. When the strings approach each other, the $n_L = 2$ string starts splitting into two $n = 1$ string branches in the region close to the interaction. After the collision, the $n_R = 1$ string reconnects with one of the branches of $n_L = 2$; the direction of the U(1)-gauge magnetic flux determines the direction of reconnection and which of the string branches intercommutes with the $n_R = 1$ string. The other $n = 1$ branch continues joining the two ends of the $n_L = 2$ string. As the evolution proceeds, the joining point of the two $n_R = 1$ strings peels outward, apparently at the speed of light. The final configuration consists of a network of two reconnected strings joined by an $n = 1$ string-bridge, but with the joints peeling outward into the $n_L = 2$ strings.

Figure 2 shows the case $(n_L, n_R) = (3,1)$, 90°, and $v/c = 0.75$. Here again $n_L = 3$ stability is verified since the linear energy density for $n_L = 3$ is 11.3% smaller than 3 times the linear energy density for $n_L = 1$. The system evolves similarly to that of the previous case, peeling off the $n_L = 3$ string by the $n_R = 1$ string. However, the string-bridge now has winding number $n = 2$. Figure 2(c) shows when the string-bridge branches temporarily into two $n = 1$ strings; they subsequently merge back to a single $n = 2$ string-bridge.

As a consistency check, we obtained sections of the signs of the imaginary part of the scalar field. [In cylindrical coordinates $\Phi_{\text{imag}} = R(\rho)\sin(n\phi)$.] Figure 3 shows the signs of Φ_{imag} after intercommutation for Figs. 1(d) and 2(d); in each case two parallel slices were taken across the L string, one near the boundary and the other across the string-bridge. The centers of the corresponding strings are indicated with zeros.

As we move around the zero in Fig. 3(a) that denotes the position of the L string for the case $(n_L, n_R) = (2,1)$, there are four regions where Φ_{imag} changes sign, meaning that indeed the L string has winding number $n_L = 2$.

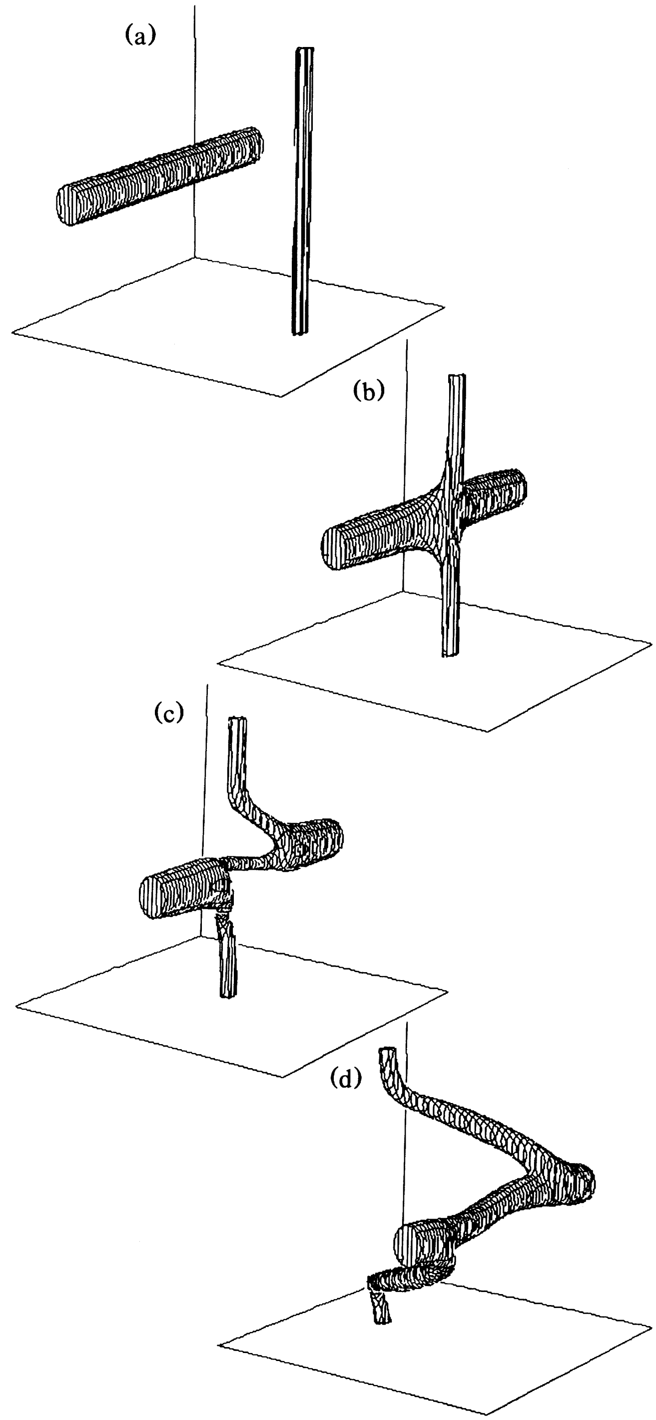

FIG. 1. Evolution of $||\Phi| - \eta|/\eta$ for $(n_L, n_R) = (2,1)$. The fatter (left) string has $n = 2$. The contour label is 60% of the maximum value, which occurs at the string core.

From Fig. 3(b) the string-bridge clearly has $n = 1$ since, when circling the bridge, there are two regions with different signs. Similarly, for $(n_L, n_R) = (3,1)$, Fig. 3(c) shows the L string having $n_L = 3$, and Fig. 3(d) the

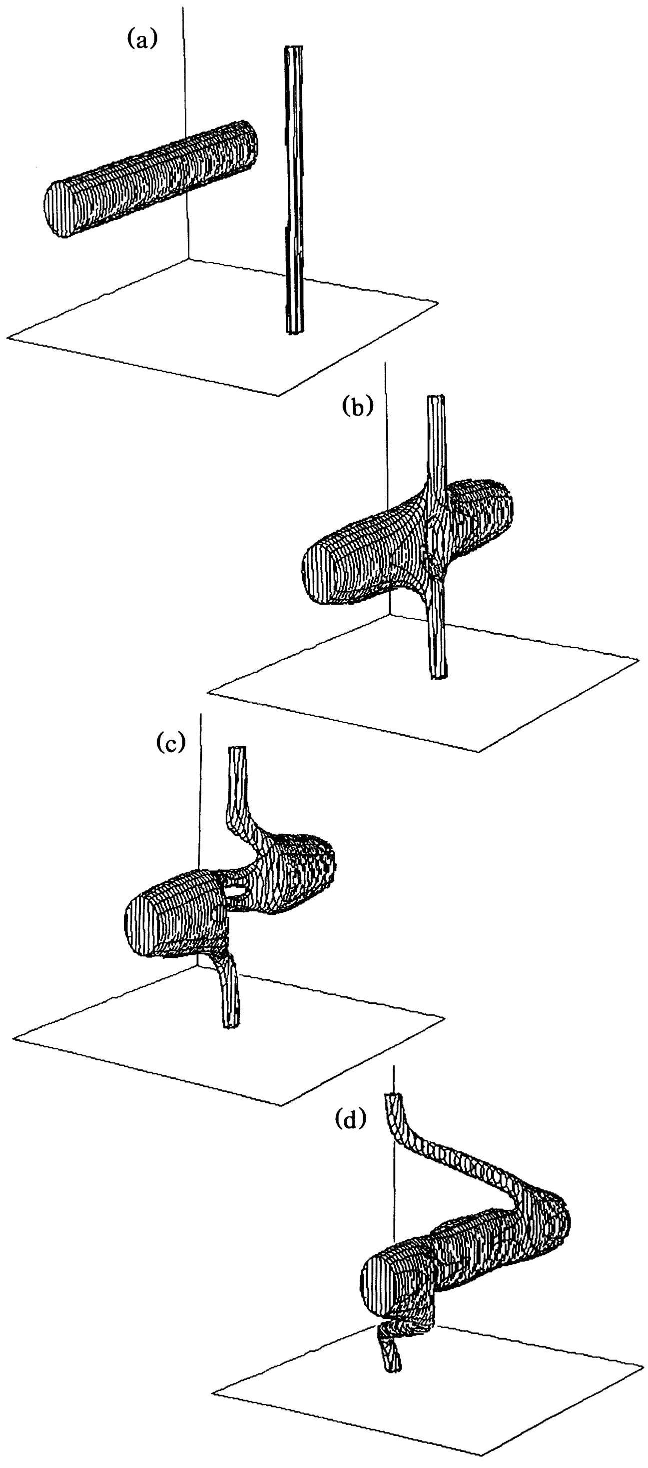

FIG. 2. Evolution of $||\Phi|-\eta|/\eta$ for $(n_L,n_R)=(3,1)$. The fatter (left) string has $n=3$. The contour label is 60% of the maximum value, which occurs at the string core.

string-bridge with $n=2$. Such behavior is also observed for the other cases not shown, corroborating the general result that the string-bridge has a winding number $|n_L-n_R|$.

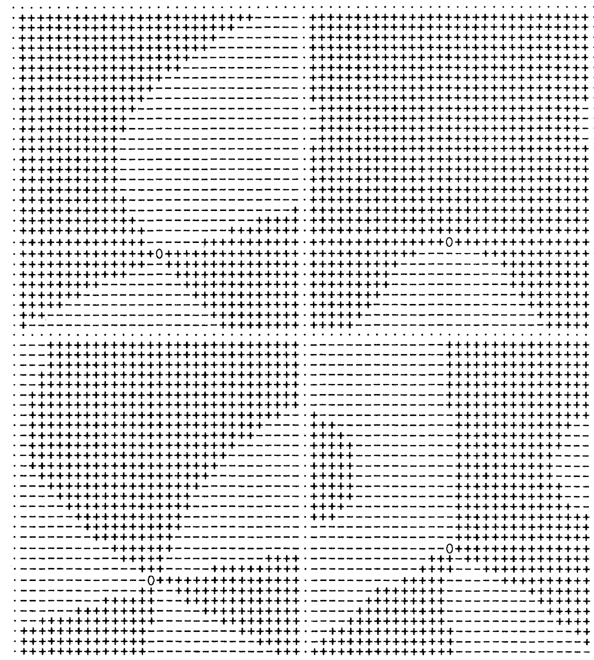

FIG. 3. Sections of the $\Phi_{\rm imag}$ signs. Sections (a) and (b) correspond to $(n_L,n_R)=(2,1)$ shown in Fig. 1(d), and sections (c) and (d) to $(n_L,n_R)=(3,1)$ in Fig. 2(d). Slice (a) was taken near the boundary across the $n_L=2$ string, and section (b) cuts across the string-bridge. Similarly, the slice (c) cuts across the $n_L=3$ string and section (d) across the string-bridge. Zeros denote the string position.

For U(1)-gauge strings arising in field theories with the gauge-vector mass greater than the scalar-field mass, it is possible that $n>1$ stable strings can exist. The usual theory of formation of such objects assumes that a Kibble mechanism,[12] when the phase wraps by multiples of 2π, generates them. But there seems to be no reason why a multiply wrapped string would form rather than a somewhat randomly oriented collection of $n=1$ strings. Further, if the result after the phase transition is of a jumble of $n=1$ strings, there is no obvious way they could merge into a large-winding-number object. Forces between these gauge strings fall off exponentially with distance, so while a lower energy state exists, the path to it is via a very flat part of the interaction potential energy, and many fluctuations may interfere with the formation of multiply wound strings.

The kinetic energy of the interacting strings, which in the early Universe is expected to be considerable, and the disruption of the axial symmetry due to the second string overcome the binding which normally prevents $n>1$ type-I strings from decaying. The result is that, for instance, an $n=1$ string catalyzes as $n=2$ string (putting in energy) to break up by peeling, leaving three $n=1$ strings which can further catalyze the process. If intercommutation of strings with different winding occurs with considerable frequency, the peeling process would also complicate the branching structure of the string network because of the appearance of string-bridges after each collision. Since it is commonly believed[9,13-15] that typically strings have comparable parameters e and $\sqrt{\lambda}$ $(10 > \alpha > 0.1)$ and that the collisions occur with relativistic incident velocities, we can conclude from our simulations that interactions between unequal winding-number strings will reduce the maximum winding number of the surviving strings. Therefore, the large winding-number strings are in general unfavored dynamically, even for choices of parameters that make them energetically stable. These computations have not considered the strongly bound case, that is, the attracting limit $(\alpha \gg 1)$. It is possible that in that case, with very small collision velocities and small crossing angles, "reverse peeling" could occur. We only note that Shellard,[3] in numerical work on the global-string case $(\alpha = \infty)$, finds that for angles not equal to 0 or π approaching strings tend to antialign near segments in a way which leads to annihilation of those segments and reconnection, especially for small angles and slow velocities. These suggest that $(\alpha \gg 1)$ strings may be unstable to non-z-symmetric perturbations. This question needs to be investigated further, but if it is the case, then for the large-α, as well as the $\alpha = 2$ case considered here, only $n = 1$ strings are of serious astrophysical interest.

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