Critical Thermal Boundary Resistance of ^{4}He near T_λ

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We present a unified description of the thermal boundary resistance R_K (Kapitza resistance) of ^{4}He above and below T_λ. Using Dirichlet boundary conditions for the order parameter we determine the critical temperature dependence of R_K without an adjustment of parameters. Good agreement with experimental data below T_λ is found. Our local renormalization-group treatment goes beyond the conventional hydrodynamic approach and predicts the nonuniversal pressure dependence of R_K along the λ line.

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Surface critical behavior has been the subject of intense theoretical studies in the past decade.[1] These studies were concerned primarily with static properties, apart from a few cases where the relaxational dynamics was investigated. Very little is known about the critical dynamics at surfaces in the presence of reversible couplings. The latter are of particular interest as they are expected to produce divergent surface transport coefficients which may be accessible to experimental observation. In this Letter we present the first renormalization-group (RG) study of such a divergent surface transport coefficient.

A well suited candidate for this study is the thermal boundary resistance (Kapitza resistance[2]) between superfluid ^{4}He and a solid wall. Favorable experimental conditions such as the exact vanishing of the bulk resistance and the advances in high-resolution thermometry have made possible the detection of a singular contribution R_K to the total boundary resistance $R_K^{\rm tot}$ below T_λ, which was represented as[3]

$$R_K^{\rm tot} = R_K^0 + R_K\,, \tag{1}$$

with a noncritical background term R_K^0. Also above T_λ, deviations from the predicted behavior of the bulk thermal conductivity λ_T have been observed[4,5] which may be attributed to a boundary resistance. Analogous phenomena are expected to exist for other superfluids (^{3}He and superconductors).[3]

The present status of the theory of R_K is rather unsatisfactory. While no theory exists for R_K above T_λ there have been three different[6-9] predictions for the critical exponent of R_K below T_λ, without a quantitative specification of the amplitude of R_K, although a definite identification of this amplitude is indispensible for a conclusive test of the predicted exponents. As a fundamental criticism of the hydrodynamic approach[6-8] we note that it breaks down when applied to phenomena on length scales smaller than the correlation length ξ. This casts serious doubt on the quantitative reliability of the corresponding results for R_K (Refs. 6-8) since the main contribution to R_K arises within a boundary layer of thickness ξ. This point is of general importance to dynamic critical phenomena near surfaces.

In the following we shall present a RG calculation of R_K both below and above T_λ that properly treats spatial variations on length scales smaller than ξ and identifies the amplitude of R_K in terms of known bulk parameters. We shall determine R_K via the temperature variations due to a small heat current Q_0 applied to ^{4}He in half-space geometry. We start from the coupled Langevin equations for the complex order parameter ψ_0 and the entropy variable m_0 of model F (Ref. 10) in the presence of an external heat source W_0:

$$\dot\psi_0 = -2\Gamma_0 \frac{\delta H}{\delta \psi_0^*} + i g_0 \psi_0 \frac{\delta H}{\delta m_0} + \theta_\psi\,, \tag{2}$$

$$\dot m_0 = \lambda_0 \nabla^2 \frac{\delta H}{\delta m_0} + g_0 \nabla j_s^0 + W_0 + \theta_m\,, \tag{3}$$

$$H = \int d^d x \left(\tfrac{1}{2} r_0 |\psi_0|^2 + \tfrac{1}{2} |\nabla \psi_0|^2 + \tilde u_0 |\psi_0|^4 + \tfrac{1}{2} m_0^2 + \gamma_0 m_0 |\psi_0|^2 \right)\,, \tag{4}$$

with $j_s^0 \equiv {\rm Im}(\psi_0^* \nabla \psi_0)$. The ^{4}He liquid is assumed to occupy the half space $z>0$, with the plane $z=0$ representing the solid surface. We assume Dirichlet boundary conditions (Dbc), i.e., vanishing ψ_0 for $z \le 0$, which appears to be fairly realistic.[6,11] Within model F we may interpret $\delta H/\delta m_0 = m_0 + \gamma_0 |\psi_0|^2 \equiv \delta T_0$ as the local temperature deviation from the equilibrium temperature. Hence, we assume δT_0 and m_0 to be continuous at $z=0$. For simplicity we consider Eq. (3) to be valid also for $z<0$.

A stationary heat current is produced by a heat source $W_0(z) = Q_0 \delta(z)$ in the plane $z=0$ and by a sink at $z=\infty$. For $Q_0>0$ there exists a temperature profile $\langle \delta T_0 \rangle (z)$ which serves to define

$$R_K = \lim_{z\to\infty} \left\{ \frac{\partial}{\partial Q_0} [\langle \delta T_0\rangle(0) - \langle \delta T_0 \rangle (z)]_{Q_0=0} - R^b(z) \right\}. \tag{5}$$

Here $R^b(z)$ denotes the bulk thermal resistance of a system of length z (with periodic bc). Below T_λ we have $R^b \equiv 0$ while above T_λ, $R^b(z) \sim z/\lambda_T$, where λ_T is the bulk thermal conductivity. From the stationary solution of the averaged Eq. (3) we obtain the representation

$$R_K=\frac{g_0}{\lambda_0}\int_0^\infty dz\frac{\partial}{\partial Q_0}[\langle j_s^0\rangle(z)-\langle j_s^0\rangle(\infty)]_{Q_0=0}, \qquad (6)$$

which is valid above and below T_λ and reflects the physical origin of R_K: It results from the suppression of the superfluid current $\langle j_s^0\rangle(z)$ within a boundary layer which is expected to be of the order of the correlation length.

We shall calculate R_K by means of the field-theoretic RG approach[12] within the minimal subtraction scheme. The renormalized quantities of interest are introduced as[13] $m=Z_m^{-1/2}m_0$ and $\lambda_T^r=Z_m^{-1}\lambda_T$. On the basis of a dissipation-fluctuation theorem and an explicit one-loop calculation above T_λ we have found that the Kapitza resistance is renormalized as

$$R_K^r=Z_mR_K. \qquad (7)$$

This result agrees with the expectation that Dbc do not introduce new Z factors.[1] Dimensional analysis leads to the representation $R_K^r=(\mu\lambda_T^r)^{-1}\tilde{A}$, where $\mu=\xi_0^{-1}$ is the reference wave number of the renormalized theory. The dimensionless amplitude $\tilde{A}$ depends on the various dimensionless parameters w, f, γ, u, and r/μ^2, which are the renormalized counterparts of $w_0=\Gamma_0/\lambda_0$, $f_0=g_0^2/\Gamma_0\lambda_0$, γ_0, $u_0=\tilde{u}_0-\frac{1}{2}\gamma_0^2$, and r_0. Together with Eq. (7) this implies, after integration of the RG equation for R_K^r, that R_K can be represented as

$$R_K=\frac{\xi(t_\pm)}{\lambda_T(t_\pm)}A^\pm \qquad (8)$$

both above and below T_λ. Here $\xi(t)\sim\xi_0t^{-\nu}$ denotes the correlation length above T_λ with $t=(T-T_\lambda)/T_\lambda$ and $t_+=t$, $t_-=-2t$ for $T>0$ and $t<0$, respectively. This settles the question of the type of divergence of R_K, which turns out to be rather weak. All the more important is a calculation of the amplitudes A^+ and A^- for a conclusive comparison with experimental data in a limited temperature range. In lowest order we obtain $A^-=A^-(\theta(l))$ where $\theta(l)$ depends on the effective model-F parameters $u(l)$, $f(l)$, and $w(l)=w'(l)+iw''(l)$ according to

$$\theta(l)^2=\frac{f(l)}{8u(l)}\left[1+\frac{w''(l)^2}{w'(l)^2}\right]^{-1}. \qquad (9)$$

The flow parameter $l\sim(-2t)^\nu$ introduces a weak temperature dependence of $A^-(\theta(l))$. The calculation of the θ dependence of $A^-(\theta)$ is parallel to that of Ref. 7, which uses the representation (5) and leads to

$$A^-(\theta)=\frac{\Gamma(\alpha_+)\Gamma(\alpha_-)}{\Gamma(\alpha_++\frac{1}{2})\Gamma(\alpha_-+\frac{1}{2})}, \qquad (10)$$

with $\alpha_\pm=\theta+\frac{1}{4}\pm(\theta^2+\frac{1}{16})^{1/2}$. Equations (8)–(10) have the same structure as the result obtained by the hydrodynamic approach where effective (critical) bulk parameters have been substituted in a phenomenological way.[6-9] Unlike the hydrodynamic result, however, which contains the thermal conductivity $\varkappa$ below T_λ as an *unknown quantity*,[8] our result, Eqs (8)–(10), is expressed entirely in terms of parameters that are well known from bulk theory[13] and experiment[14] *above* T_λ. This permits testing of the "hydrodynamic" result, Eqs. (8)–(10), by means of a comparison with the measured total resistance,[3] Eq. (1) after an adjustment of $R_K^0(=0.434$ cm^{2}K/W). The discrepancy between the dashed line and the data in Fig. 1 clearly reveals the inadequacy of both the hydrodynamic approach and the RG approximation that has led to Eqs. (9) and (10).

In the following we propose a basic improvement of the theory by focusing the RG treatment on the spatially varying response function

$$S^0(z,r_0)\equiv\frac{g_0}{\lambda_0}\frac{\partial}{\partial Q_0}[\langle j_s^0\rangle(z)]_{Q_0=0}, \qquad (11)$$

which determines R_K via the integral representation (6). It is natural to expect that such a *local* treatment takes into account more efficiently the short-distance properties within finite order of renormalized perturbation theory. From the bulk relation $\lambda_T^{-1}=\lambda_0^{-1}+S^0(\infty,r_0)$ it follows that S^0 needs multiplicative and additive renormalizations. Accordingly, we introduce the renormalized response function

$$S(z,r)=Z_m[S^0(z,r_0)-S^0(z,r_{0b})], \qquad (12)$$

where $r=Z_r^{-1}r_0$ and $r_{0b}=Z_rr_b$, with $r_b=\mu^2$ for $T>T_\lambda$

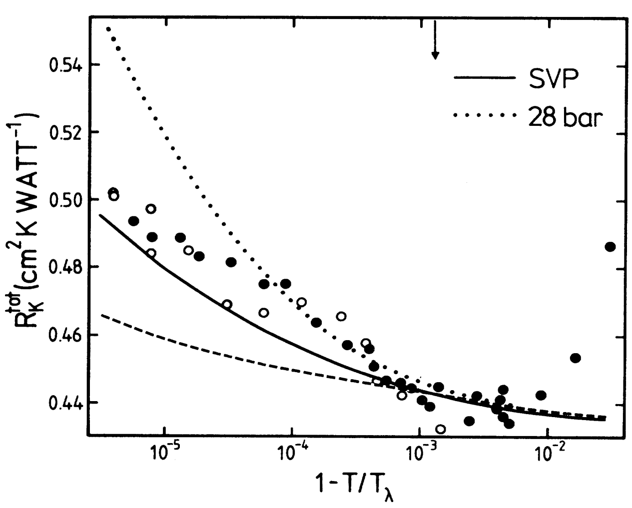

FIG. 1. Kapitza resistance R_K^{tot}, Eq. (1), vs $(T_\lambda-T)/T_\lambda$. The solid line is our theoretical result [Eqs. (13)–(16)]; the dashed line represents the "hydrodynamic" result [Eqs. (8)–(10)]; both lines and the data (Ref. 3) are for saturated vapor pressure. The dotted line is our prediction for 28 bars according to Eqs. (13)–(16). The arrow indicates the temperature at which R_K^0 has been adjusted.

and $r_b = -\mu^2/2$ for $T < T_\lambda$. The subtraction at the noncritical temperature r_{0b} is taken to be z dependent in order to preserve the Dbc for S, i.e., $S(0,r)=0$. Integration of the (inhomogeneous) RG equation for $S(z,r)$ and substitution into Eq. (6) leads to $R_K \equiv R_K(r_0)$ in the form

$$R_K(r_0) = R_K(r_{0b}) + R_K^s , \tag{13}$$

where $R_K^s \equiv R_K^s[r,\mu,v]$ is represented as

$$R_K^s = \int_{l_\pm}^1 dl' \frac{G(l')}{l'\nu(l')} \exp\left[-\int_{l'}^1 \zeta_m(l'')\frac{dl''}{l''} \right], \tag{14}$$

$$G(l') = -\left\{ r_b \frac{\partial}{\partial r_b} R_K^s[r_b,\mu,v(l')] \right\}_{\mu=\mu l'} . \tag{15}$$

Here we have employed the usual RG functions[13] $\zeta_j = (\mu\partial_\mu \ln Z_j^{-1})_0$, $j=m,r$, with $\nu(l)^{-1} = 2-\zeta_r(u(l))$ and have abbreviated the model-F parameters (u,γ,w,f) by v. The flow parameter $l_\pm$ is identified as $r(l_+) = \mu^2 l_+^2$ and $-2r(l_-) = \mu^2 l_-^2$ above and below T_λ, respectively, which implies $l_\pm \sim t_\pm^\nu$. To leading order we find below T_λ

$$G(l') = \frac{A^-(\theta(l'))}{2\lambda_T[l']\mu l'} , \tag{16}$$

where $\lambda_T[l'(t'_-)] \equiv \lambda_T(t'_-)$ is taken at the appropriate temperature *above* T_λ. Equations (14)-(16) fully include the nonuniversal critical effects which are relevant in dynamics and also in statics at higher pressure.[15] The critical exponent of Eq. (14)-(16) agrees with that of Eq. (8), as expected. The effective amplitude, however, differs significantly and leads to strikingly improved agreement with the data[3] [solid line in Fig. 1; an adjustment of the background term $R_K(r_{0b}) + R_K^0 = 0.439$ $\mathrm{cm^2\,K/W}$ is understood].

The reason for this improvement can be elucidated by rederiving Eqs. (8)-(10) via a different renormalization of S^0: instead of Eq. (12) one may perform a z-independent (bulk) subtraction which suffices to cancel just the pole terms (in $\epsilon = 4-d$) for $z>0$. This replaces Eq. (12) by

$$\bar{S}(z,r) = Z_m[S^0(z,r_0) - \lambda^{-1}(Z_m^{-1} - Z_\lambda)] , \tag{17}$$

where $\lambda = Z_\lambda \lambda_0$. Substitution of Eq. (17) into Eq. (6) leads to Eq. (7) and, in lowest order of perturbation theory, indeed reproduces Eqs. (8)-(10). From $S^0(0,r_0)=0$ it follows that $\lim_{z\to 0}\bar{S}(z,r) \neq 0$; hence $\bar{S}(z,r)$ does not satisfy the Dbc, in contrast to $S(z,r)$ defined by Eq. (12). In fact, $\bar{S}(z,r)$ is not well behaved in the limit $z \to 0$ and $\epsilon \to 0$, contrary to $S(z,r)$. This clearly indicates that the z-dependent subtraction (12) is better adapted to Dbc.

In order to further support our result given in Eqs. (13)-(16) we briefly discuss a similar *static* problem related to the surface energy density

$$E_s = \int_0^\infty dz [\langle \varphi_0^2 \rangle (z) - \langle \varphi_0^2 \rangle (\infty)] \tag{18}$$

for an n-component order parameter φ_0 (with Dbc at $z=0$). If we use a renormalization of $\langle \varphi_0^2 \rangle (z)$ analogous to Eq. (17) we arrive at a renormalized quantity $\bar{E}_s$ which is identical with the result of Goldschmidt and Jasnow (GJ).[16] Their expression is finite in $d=4$ but exhibits a pole $\sim (d-3)^{-1}$ in $d=3$ as noted recently.[17] This is an artifact that has not been discussed in the literature although it raises serious doubt concerning the quantitative applicability of the GJ result to $d=3$. Here we point out that the $d=3$ pole is a spurious result which can be avoided by a local renormalization analogous to Eq. (12), as introduced by Dietrich and Diehl.[18] Using their renormalization of $\langle \varphi_0^2 \rangle (z)$ we obtain the renormalized energy density in two-loop order (for $t>0$, $t\to 0$),

$$E_s^r = \frac{c}{(1-\rho)\rho}[t^{1-\rho} + (1+\rho)r/\mu^2 - \rho][1+2(n+2)u^*] , \tag{19}$$

where c is a geometric factor and $\rho = \alpha + \nu$. In Eq. (19) the pole $(d-3)^{-1}$ of GJ is replaced by $(d-3+\zeta_r^*)^{-1}$ $= \nu/(1-\rho)$ which renders E_s^r finite in $d=3$. Further evidence in favor of the local renormalization scheme can be obtained from a comparison between the corresponding surface heat capacities, $\bar{C}_s$ and C_s^r. We find in two-loop order, apart from geometric prefactors,

$$\bar{C}_s \propto t^{-\rho}[1+8(n+2)u^*] , \tag{20}$$

$$C_s^r \propto t^{-\rho}[1+2(n+2)u^*] . \tag{21}$$

We see that the $O(u^*)$ correction in Eq. (20) is prohibitively large (of the order of 1) in $d=3$, in contrast to Eq. (21). These examples demonstrate rather convincingly that our result, Eqs. (13)-(16), is based on a more reliable approximation than the hydrodynamiclike expression, Eqs. (8)-(10).

As a further test of our theory we propose to measure the pressure dependence of R_K along the λ line. After adjustment of a pressure-dependent background term R_K^0 our result provides quantitative predictions for all pressures. As an example we present our prediction for 28 bars as the dotted line in Fig. 1 (for simplicity we have taken the saturated-vapor-pressure value of R_K^0).

Finally, we turn to the critical Kapitza resistance above T_λ. It results solely from fluctuations that cannot be treated by a hydrodynamic approach at all. In one-loop order we find Eqs. (13)-(15) with

$$G(l') = \frac{A^+(x(l'),y(l'))}{2\lambda_T[l']\mu l'} , \tag{22}$$

$$A^+(x,y) = \tfrac{1}{6}x^{-1} + \tfrac{1}{4}\pi x^2 - \tfrac{1}{2}x + \tfrac{1}{2}xy\ln[(1+y)/x] , \tag{23}$$

where $x = |w'/w|$ and $y = |w''/w|$. It would be interesting to perform a quantitative comparison between this result and experimental data for R_K above T_λ. Unfortunately, a direct measurement of $R_K^{\rm tot}$ above T_λ does not seem possible because of the finite bulk resistance R_b. The presently available data for the thermal conductivity exhibit size-dependent effects whose interpretation is as yet controversial.[4,5] A preliminary analysis indicates that the measured deviations from the expected bulk behavior are much larger than suggested by the surface effect predicted by Eqs. (13)–(15), (22), and (23). More accurate experimental information would be highly desirable.

In conclusion, we have presented a unified description of the thermal boundary resistance above and below the λ transition of ^{4}He and have explained the recently detected singular temperature dependence below T_λ. An extension of the theory to the nonlinear regime at finite heat current[3,19] as well as to the critical Kapitza resistance in other superfluids would be of considerable interest.

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