Escape Oscillations of a Josephson Junction Switching Out of the Zero-Voltage State

Emmanuel Turlot, Daniel Esteve, Cristian Urbina, John M. Martinis,[a] and Michel H. Devoret
Service de Physique du Solide et de Résonance Magnétique, Centre d'Etudes Nucleaires de Saclay, 91191 Gif-sur-Yvette CEDEX, France

Sebastian Linkwitz and Hermann Grabert
Fachbereich Physik, Universität Essen, D-4300 Essen, Federal Republic of Germany
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The lifetime of the zero-voltage state of a current-biased Josephson junction shunted by a transmission line has been measured in the thermal regime. As the length of the line is increased *in situ*, the lifetime is modulated in an oscillatory pattern. The amplitude, period, phase, and tailing off of the modulation is the first direct evidence for the oscillations of a particle in a well before escape which are implied by Kramers's energy-diffusion model for the escape from a metastable state.

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The thermal escape of a particle from a particle from a potential well is a process of great importance as a model for the decay of a metastable state of a physical system. A simplistic but useful theory pictures the escape process as a free flight over the barrier of a particle whose energy follows a thermal equilibrium Boltzmann distribution. This so-called transition-state theory predicts an escape rate $\Gamma_{TST}=(\omega_p/2\pi)\exp(-\Delta U/k_BT)$, where ΔU is the height of the potential barrier, T is the temperature of the heat bath, and ω_p is the frequency of small oscillations at the bottom of the potential well (see Fig. 1). In 1940, Kramers[1] took explicitly into account the nonequilibrium nature of the escape process and established theoretically that, contrary to the prediction of the transition-state theory, the strength of the coupling to the heat bath does influence the escape rate. For the case of a weak frictional force characterized by a quality factor Q of the small oscillations in the well, the correct value of the escape rate is given in the limit $Qk_BT/\Delta U\gg 1$ by

$$\Gamma=(\alpha\,\Delta U/Qk_BT)\Gamma_{TST}\,, \tag{1}$$

α being a dimensionless factor depending solely on the shape of the potential. According to Kramers's theory the escape process of the underdamped particle is determined by the large-amplitude oscillations it has to undergo before it reaches the top of the barrier. The theory hypothesizes that the energy of these escape oscillations fluctuates diffusively under the influence of the heat bath like the position of a Brownian particle. The escape rate is calculated from the probability that the energy of the escape oscillations diffuses up to the barrier energy ΔU. As the damping decreases, the rate of energy exchange with the heat bath becomes smaller. Consequently, energy diffusion is slowed down and the escape rate is reduced. Although the problem set out by Kramers has been addressed extensively in the past four decades, a clear experimental verification of his predictions is still lacking. Because of the predominance of the exponential Arrhenius factor, it is usually very difficult to extract unequivocally the frictional dependence from the measured escape rate.

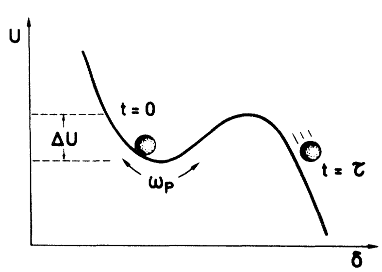

FIG. 1. Potential well in which a Brownian particle is confirmed at $t=0$ and from which it escapes at $t=\tau$.

In this Letter we report escape-rate measurements in which, for the first time, damping alone is modified, and all other parameters are kept constant. The system we have investigated is the current-biased Josephson junction which can be modeled as a particle moving in a one-dimensional tilted washboard potential.[2] When biased with a current below its critical current, the junction is initially in its zero-voltage state with the particle in one potential minimum. Eventually, thermal current fluctuations, induced by the dissipative circuit shunting the junction, cause the particle to escape out of the well. For an underdamped junction the particle then accelerates down the washboard potential, representing the switching of the junction to its voltage state. When shunted by a pure Ohmic resistor the junction provides an ideal realization of Kramers's case of a frictional force proportional to velocity. Various authors[3-5] have reported escape rates compatible with Kramers's predictions or their extension to the moderately underdamped case[6] ($\alpha\,\Delta U/Qk_BT$ of order unity). In particular, Silvestrini *et al.*[5] have interpreted their experiment as a confirmation of the validity of Eq. (1). However, in all of these experiments, the damping, which was not exactly of the pure Ohmic type, was neither measured directly nor changed independently from the rest of the relevant factors.

In our experiment the influence of damping on the escape out of the zero-voltage state was measured in a novel setup. The circuit shunting the junction has been designed to be both adjustable and amenable to characterization. The junction is coupled to a delay line which is the microwave analog of a Fabry-Perot cavity and whose length l can be varied *in situ* without affecting the barrier height or the temperature. This circuit provides damping which is a function of both l and the oscillation frequency ω. Kramers's theory has been extended to the case of frequency-dependent damping[7] and the result can be cast in the form

$$\Gamma=(\Delta U/k_BT)\left[\int_0^{\omega_p} g(\omega)Q(\omega)d\omega\right]^{-1}\Gamma_{\mathrm{TST}}, \qquad (2)$$

where $g(\omega)$ is the frequency distribution of the escape oscillations which depends solely on the junction and where $Q^{-1}(\omega)$ is the dimensionless damping function which depends on the admittance $Y(\omega)$ shunting the junction. When $Y(\omega)$ does not vary too rapidly compared with $g(\omega)$, $Q(\omega)$ varies like $1/\mathrm{Re}[Y(\omega)]$. The function $g(\omega)$ can be well approximated by a Gaussian whose peak frequency is $\omega_e=\omega_p/[1+\ln(\Delta U/k_BT)/2\pi]$ and whose standard deviation is $\omega_e^2/2\omega_p$. The full numerical theory of Ref. 7 computes the precise shape of $g(\omega)$, includes the contribution of higher harmonics of the escape oscillations, and incorporates a correction extending the validity of the theory into the moderately underdamped regime. For the case of the delay line, $Q(\omega)$ is a sinusoidal function of $l\omega$. Thus, by varying the length l, we can change the damping of the escape oscillations and determine its influence on the escape rate.

The coordinate of the particle modeling a Josephson tunnel junction biased at a constant current I is the phase difference δ between the two superconductors across the barrier, and the potential in which δ evolves is given by $U(\delta)=U_0(-s\delta-\cos\delta)$. Here $U_0=I_0\Phi_0/2\pi$ and $s=I/I_0$, where I_0 is the junction critical current and $\Phi_0=h/2e$ is the flux quantum. For $s\lesssim 1$, the particle escapes from a cubic well (see Fig. 1) characterized by the barrier height $\Delta U=U_0'(1-s)^{3/2}$, where $U_0'=4\sqrt{2}/3U_0$, and the frequency of the small oscillations at the bottom of the well (plasma frequency) $\omega_p=(2\pi I_0/\Phi_0 C)^{1/2}(1-s^2)^{1/4}$, where C is the junction capacitance.[8]

The practical realization of the junction and its coupling to a delay line is shown in Fig. 2(a). The 18×22-μm^2 Nb-NbO_x-PbIn tunnel junction is patterned in a cross strip on a silicon chip (inset). The junction is connected to a horn-shaped $Z_c=50$-Ω coplanar transmission line which is impedance matched to the delay line built from a printed circuit board. A second line, opposite to the delay line, has been designed on the chip in order to excite the junction with a microwave current at or near the plasma frequency. This irradiation line, which is weakly coupled to the junction, is used in additional experiments needed to measure the junction parameters and has no influence on the escape in the absence of microwaves. A microwave-absorbing block, referred to in the following as "the load," is pressed on the delay line at a distance L from the edge of the chip. The delay line is dc connected through a series of filters[4] to a current source in parallel with a fast voltmeter. The microwave connection between the irradiation line and the microwave generator has been designed to eliminate any spurious frequency-dependent attenuation.

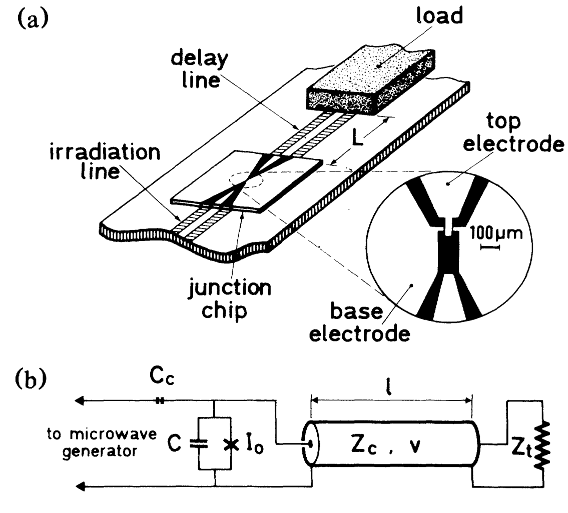

FIG. 2. (a) Josephson junction (see inset) connected to a delay line and capacitively coupled to an irradiation line. A movable microwave-absorbing block (load) terminates the delay line. (b) Equivalent circuit at microwave frequencies. The capacitor C_c represent the high-impedance capacitive coupling between the junction and irradiation line.

In the low-frequency range where the current source and voltmeter operate, the load and filters play no role in the biasing circuit and the junction experiences only a very high impedance. However, in the microwave range, the portion of delay line covered by the load behaves as lossy transmission line. In the frequency range $\omega_p/4<\omega<2\omega_p$, the circuit shunting the junction can be modeled as an ideal delay line of characteristic impedance Z_c terminated by a resistor Z_t [see Fig. 2(b)]. The contribution of the filters to the impedance seen by the junction is made negligible by the attenuation of the lossy transmission line. The ideal delay line is also characterized by its propagation velocity v and its length $l=L+D$, where the dead length D accounts for the length of the horn and for impedance discontinuities at the chip and load boundaries. Microwave reflection and attenuation measurements of the delay line with the load but without the junction chip yielded $Z_c = 50 \pm 2\ \Omega$, $Z_t = 30 \pm 3\ \Omega$, $v = (1.1 \pm 0.1) \times 10^8$ m/s. We calculated a value 5 fF $< C_c <$ 20 fF for the coupling capacitance between the irradiation line and the junction. The dead length D, which determines the precision of l, was estimated to be 6 ± 1 mm. The microwave circuit and the filters connecting it to room-temperature electronics were placed in a pumped helium bath whose temperature was regulated to better than 2 mK.

The lifetime of the junction in its zero-voltage state was determined by square-wave modulating the bias current and recording the times τ that elapsed between the leading edge of each pulse and the consecutive voltage rise signaling the switching of the junction to the voltage state. We recorded typically 10^4 switching events in 10 s. The variations of the average lifetime $\Gamma^{-1} = \langle \tau \rangle$ are plotted in Fig. 3(a) as a function of the length l for $T = 1.37 \pm 0.01$ K and $I = 21.67 \pm 0.01\ \mu$A. The reproducibility of Γ^{-1} throughout the duration of the experiment was as expected for a Poisson process and the standard deviation is indicated in Fig. 3(a) by the diameter of the dots. As l increases, the lifetime is modulated in a damped sinusoidlike pattern. We repeated the experiment at 4 K to check that the line did not influence the escape rate through factors other than the damping of the junction. At this temperature the damping due to quasiparticles overrides the damping due to the line. The modulation of the lifetime with the length of the delay line was indeed suppressed.

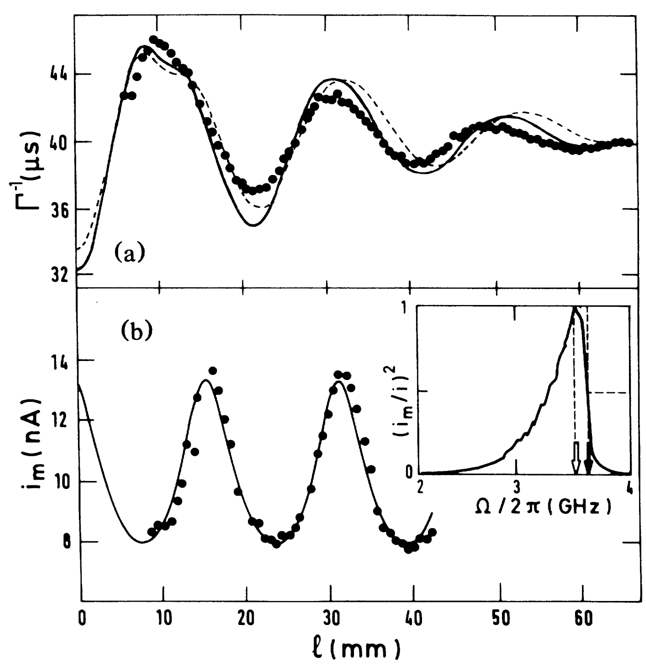

FIG. 3. (a) Lifetime Γ^{-1} of the zero-voltage state in the absence of microwaves as a function of the total length l of the delay line. Solid line and dashed line are from the theory of Ref. 7. (b) Microwave current needed to decrease Γ^{-1} by a factor of 2 at the frequency corresponding to the maximum of the resonant activation curve, plotted as a function of l. Inset: Curve taken at $L = 0$; open arrow shows frequency of maximum, solid arrow shows position of plasma frequency. Solid line is theory based on Refs. 9 and 11.

In order to compare quantitatively this modulation of the lifetime with the predicted one,[7] we measured the parameters ω_p and $\Delta U / k_B T$. The plasma frequency ω_p was determined by measuring τ in the presence of a microwave excitation of angular frequency Ω. Because of the high-impedance coupling proveded by the capacitor C_c [see Fig. 2(a)], this excitation is well described by an ac current bias of the junction and its shunt admittance. The load was placed at $L = 0$ mm for which the shunt admittance can be modeled, in the range of variation of Ω, as being frequency independent. We recorded as a function of the frequency Ω the microwave amplitude $i(\Omega)$ for which Γ^{-1} is reduced by 2 from its value in the absence of microwaves. In the inset of Fig. 3(b) we show the resonance curve plotted as $i^{-2}(\Omega)$ and normalized to its maximum. The properties of this resonant activation response are now well established.[9-11] In particular, numerical experiments[9] have shown that the plasma frequency is given by the half-maximum frequency Ω_p on the high-frequency side of the resonance [see solid arrow in inset of Fig. 3(b)]. Thus we found $\omega_p / 2\pi = 3.580 \pm 0.005$ GHz. We then determined I_0 from the variations of Γ^{-1} with I, as described in Ref. 4. We obtained $I_0 = 22.94 \pm 0.30\ \mu$A, the error resulting mainly from the theoretical uncertainty in the prefactor Γ/Γ_{TST}. Together with I, this yielded $\Delta U / k_B T = 10 \pm 3$, a value precise enough to compute the *relative* changes of Γ with the line length.

A further experiment was performed to provide an independent calibration of the delay line at the plasma frequency in the actual conditions of the experiment. We recorded at each load position the resonance curve $i^{-1}(\Omega)$. The frequency Ω_p as well as the microwave amplitude i_m corresponding to the frequency Ω_m of the maximum [see open arrow in inset of Fig. 3(b)], were then determined as a function of l. Both Ω_p and i_m varied periodically with l. In Fig. 3(b) we only show the variations of i_m, the variations of Ω_p consisting of a small shift $\delta\Omega_p = \Omega_p - \omega_p$ of maximum value $\omega_p / 100$. Accoridng to theory,[9,11] $\delta\Omega_p$ and i_m should scale, respectively, as $\mathrm{Im}[Y(\Omega_p)]$ and $\mathrm{Re}[Y(\Omega_m)]$. Theory also predicts $\Omega_m \approx \Omega_p [1 + 1/2Q(\Omega_p)]$, where $Q(\omega) = C\omega_p / \mathrm{Re}[Y(\omega)]$. The theoretical curve, calculated assuming $C_c = 10$ fF, is plotted in Fig. 3(b). Data and theory concord, thus confirming that the model of Fig. 2(b) correctly describes the damping experienced by the junction in the vicinity of ω_p. Neglecting the small difference of less than 2% between Ω_m and ω_p, the data in Fig. 3(b) directly display the wavelength at the plasma frequency, as well as the quality factor $Q(\omega_p)$ of the plasma oscillations which varied between $Q_{min} = 30$ and $Q_{max} = 80$. Note that the conductance $\mathrm{Re}[Y(\omega)]$ and reactance $\mathrm{Im}[Y(\omega)]$ of the delay line, although of the same magnitude, do not affect the junction equally: The resonance frequency Ω_p of the junction plus line system is essentially determined by C, the reactance of the line being only a small perturbation. On the other hand, $Q_{\min}$ and $Q_{\max}$ are determined by the conductance of the line, the junction's intrinsic conductance being negligible at 1.37 K.

The prediction of the modulation of the escape time (full numerical calculation of Ref. 7) resulting from the determination of the parameter ω_p is plotted in Fig. 3(a) (solid line) with $\Delta U/k_B T = 11.0$ ($I_0 = 23.06$ μA) chosen so that the experimental and theoretical values of τ coincide at $l = 65$ mm. This value of $\Delta U/k_B T$ is compatible with our previous determination. The dashed line corresponding to $\Delta U/k_B T = 14$ shows the effect on the prefactor of a 30% change in the value of $\Delta U/k_B T$. The effect of the uncertainty in ω_p is too small to be represented in the figure. The amplitude of the predicted modulation is comparable to the variation of Γ^{-1} calculated using the theory of Ref. 6 with a frequency-independent quality factor varying between $Q_{\min}$ and $Q_{\max}$. Although the amplitude of the observed modulation agrees qualitatively with the predicted one, there is still a quantitative discrepancy, which we attribute to a slight frequency dependence of Z_t for $\omega > 2\omega_p$ which has been neglected in the theory, and to the approximation made in the correction accounting for the junction being only moderately underdamped. There is, however, quantitative agreement between theory and experiment for the period, phase, and tailing off of the modulation. These features reveal the frequency distribution of the escape oscillations. Comparison between Figs. 3(a) and 3(b) shows that the distribution peak frequency is smaller than the plasma frequency, as theory predicts: During the escape oscillations the particle comes very close to the barrier top where its motion is highly nonharmonic.

In conclusion, the results of our experiment directly show that a variation of the damping affects the escape rate and confirms Kramers's hypotheses for the escape in the underdamped regime. Additionally, they provide a detailed measurement of the probability of occurrence of the higher-energy oscillations during an escape event.

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[a]Present address: National Institute for Standards and Technology, 325 Broadway, Boulder, CO 80303.

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