Slow Bragg Solitons in Nonlinear Periodic Structures

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We show that a new class of optical solitons is possible in nonlinear periodic structures. These waves can propagate undistorted even though their power spectra lie well within the frequency band gap of the periodic structure and even though their carrier frequency is very close to the Bragg resonance. Analysis shows that these Bragg solitons can exhibit velocities which are orders of magnitude than the speed of light in the unperturbed medium.

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Wave propagation in periodic structures is associated with many interesting and potentially useful phenomena. A direct outcome of the Floquet-Bloch theory, which plays a fundamental role in the theory of wave propagation in periodic media, is the existence of forbidden frequency bands or band gaps which are located around the Bragg frequencies.[1] Any wave whose frequency lies within such a forbidden band will undergo Bragg reflection when falling on the periodic structure. In the optical case, if the periodic variation of the refractive index is relatively weak and if the frequency of the light wave is close to the Bragg frequency, then the formalism of coupled-mode theory can be successfully employed.[2] In 1978, Hill *et al.* found that Ge-doped fibers could exhibit a certain photosensitivity process (in the blue-green spectral region) which could be used to write a permanent index grating along the axis of an optical fiber.[3] In principle, these fiber periodic structures can be very long (hundreds of meters) provided that the coherence length of the exposing laser is of the same order.[4] The continuous wave (cw) response of a nonlinear periodic structure was first investigated by Winful, Marburger, and Garmire and was found to involve bistability.[5] Subsequently, Winful proposed the use of nonlinear fiber gratings for optical pulse compression and for soliton propagation.[6] According to his analysis, these are possible as long as the carrier frequency of the optical pulse is within a spectral region where the fiber filter exhibits relatively high transmission and negative dispersion (outside the frequency band gap). Recently, Chen and Mills[7] and Mills and Trullinger[8] have shown that standing-wave (immobile) optical solitons can exist in nonlinear periodic media even though their light frequency lies within the forbidden frequency band. Finally, Sipe and Winful have treated the problem of wave propagation in nonlinear periodic structures in terms of a nonlinear Schrödinger equation.[9] However, their approach implicitly assumes that the structure is weakly dispersive and it does not account for the existence of reflected waves. Thus this latter formalism is perhaps more suitable for waves whose carrier frequency lies far away from a Bragg resonance.

In this Letter, we show that a new class of (mobile) optical solitons is possible in nonlinear periodic structures. These waves can propagate undistorted even though their carrier frequencies are very close to the Bragg resonance and even though their power spectra fall well within the frequency band gap. Our analysis shows that these solitons can move at speeds which are much lower than the velocity of light in the unperturbed medium. Thus, here we will call them slow Bragg solitons.

Let us consider a lossless, single-mode fiber periodic structure. Furthermore, let us assume that this fiber is polarization preserving and that its refractive index increases (instantaneously) with the optical intensity. Thus the refractive index of this fiber periodic structure is given by

$$n = n_0 + n_1\cos(2\pi z/\Lambda) + n_2|E|^2, \tag{1}$$

where n_0 is the effective index of the unperturbed waveguide, n_1 is the depth of the periodic index modulation of the structure and is taken to be rather weak ($n_1 \ll n_0$), Λ is the spatial period of the index grating, n_2 is the nonlinear Kerr coefficient of the material, and E is the electric field of the optical wave. In Eq. (1) we have assumed for simplicity a sinusoidal index grating.

Let us decompose the electric field into a forward and backward wave, i.e.,

$$E = E_f(z,t)\exp[i(\beta_0 z - \omega_0 t)] + E_b(z,t)\exp[-i(\beta_0 z + \omega_0 t)], \tag{2}$$

where E_f and E_b are, respectively, the envelopes associated with forward and backward components of the optical field,

$\beta_0 = n_0\omega_0/c$ is the unperturbed propagation constant, $\omega_0 = 2\pi c/\lambda_0$ is the carrier angular frequency of the wave, and λ_0 its free-space wavelength.

Furthermore, let us assume that this fiber filter is very long (say hundreds of meters). In that case, by employing the formalism of coupled-mode theory and by assuming slowly varying envelopes, E_f and E_b obey the following pair of coupled evolution equations[6]:

$$i\left(\frac{\partial E_f}{\partial z}+\frac{1}{v}\frac{\partial E_f}{\partial t}\right)+\kappa E_b\exp(-2i\delta z)+\gamma(|E_f|^2+2|E_b|^2)E_f=0\,,$$
$$-i\left(\frac{\partial E_b}{\partial z}-\frac{1}{v}\frac{\partial E_b}{\partial t}\right)+\kappa E_f\exp(2i\delta z)+\gamma(|E_b|^2+2|E_f|^2)E_b=0\,, \tag{3}$$

where $v = c/n_0$ is the speed of light in the unperturbed waveguide, $\kappa = \pi n_1/\lambda_0$ is the coupling coefficient of the structure,[2,6] and $\delta = (n_0/c)(\omega_0 - \omega_B)$ is a parameter which measures how detuned the system is from the Bragg angular frequency ω_B. The Bragg frequency ω_B is given by $\omega_B = \pi c/n_0\Lambda = 2\pi c/\lambda_B$, where λ_B is the Bragg free-space wavelength. γ in Eq. (3) is the nonlinear coefficient, $\gamma = \pi n_2/\lambda_0$. The nonlinear terms in Eq. (3) contain self- and cross-phase modulation terms and they implicitly account for nonreciprocity effects.[4] In Eq. (3) we have neglected any material and/or waveguide dispersive effects. It is well known that the dispersion arising from the periodic structure itself dominates near a Bragg resonance.[10]

The width of the frequency band gap can be readily obtained by assuming that $\omega_0 = \omega_B$ or $\delta = 0$ (Bragg resonance) and by linearizing Eq. (3). In that case, the forward envelope E_f obeys a relativistic Klein-Gordon equation, that is,

$$\frac{\partial^2 E_f}{\partial z^2}-\frac{1}{v^2}\frac{\partial^2 E_f}{\partial t^2}=\kappa^2 E_f\,, \tag{4}$$

with a similar equation for E_b. If we write E_f as a time harmonic field, i.e., $E_f = \exp[i(Kz - \Omega t)]$, we can then obtain the dispersion equation for the structure $K = [(\Omega/v)^2 - \kappa^2]^{1/2}$ which shows that K becomes imaginary for all frequecies $\omega = \omega_B + \Omega$ within the range $\omega_B - \Delta\omega/2 < \omega < \omega_B + \Delta\omega/2$, where $\Delta\omega$ is the width of the forbidden frequency band and is given by $\Delta\omega = 2\kappa c/n_0$. Thus for frequencies within the band gap, the forward wave will decay and its energy will be transferred to the backward wave through the process of Bragg reflection.

By adopting the following dimensionless variables, $x = (\kappa/2)(z + vt)$, $y = (\kappa/2)(z - vt)$, $\sigma = \delta/\kappa$, $U = (\gamma/\kappa)^{1/2}E_f$, and $V = (\gamma/\kappa)^{1/2}E_b$, Eq. (3) takes the normalized form

$$i\frac{\partial U}{\partial x}+V\exp[-2i\sigma(x+y)]+(|U|^2+2|V|^2)U=0\,,$$
$$-i\frac{\partial V}{\partial y}+U\exp[2i\sigma(x+y)]+(|V|^2+2|U|^2)V=0\,. \tag{5}$$

If one neglects the self-phase modulation terms ($|U|^2U$ and $|V|^2V$) in Eq. (5) and assumes Bragg resonance, e.g., $\omega_0 = \omega_B$ or $\sigma = 0$, then Eq. (5) reduces to the fully integrable massive Thirring model which exhibits soliton solutions.[11] The massive Thirring field is known to be to some extent relevant[11] to the sine-Gordon equation which is typically employed in nonlinear optics as a simple model equation for the phenomenon of self-induced transparency[12,13] (SIT). It would thus seem that there is some kind of correspondence between the phenomenon of SIT and what we are investigating here. As it will be shown, both processes exhibit slow soliton waves and some sort of resonance. In our opinion, the role of the two-level absorption resonance involved in SIT is played here by the Bragg reflection process. A similar point was made by Chen and Mills.[7] Even though the soliton solutions of Eq. (5) can be obtained in general for any arbitrary detuning parameter σ, these solutions are rather involved when $\sigma \neq 0$ and we will report on them elsewhere. Here we treat only the simple case of Bragg resonance, i.e., $\omega_0 = \omega_B$ or $\sigma = 0$.

For $\sigma = 0$, it can be shown that Eq. (5) exhibits the following soliton solution (strictly speaking it is a solitary wave):

$$U = \hat{A}\,\mathrm{sech}^{1/2}(\tau/\tau_0)\exp(i\Theta)\,, \tag{6}$$

$$V = -\mu^2\hat{A}\,\mathrm{sech}^{1/2}(\tau/\tau_0)\exp(i\Phi)\,, \tag{7}$$

where

$$\Theta = \Psi_0 + \frac{1}{2}\,\frac{\mu^8 - 4\mu^4 - 3}{\mu^8 + 4\mu^4 + 1}\tan^{-1}\left[\sinh\left(\frac{\tau}{\tau_0}\right)\right], \tag{8}$$

$$\Phi = \Psi_0 + \frac{1}{2}\,\frac{3\mu^8 + 4\mu^4 - 1}{\mu^8 + 4\mu^4 + 1}\tan^{-1}\left[\sinh\left(\frac{\tau}{\tau_0}\right)\right]. \tag{9}$$

$$\hat{A}^2 = \frac{4\mu^2}{\mu^8 + 4\mu^4 + 1}\,, \tag{10a}$$

$$\tau = t - \frac{z}{v_e}\,, \tag{10b}$$

$$\mu^4 = \frac{v - v_e}{v + v_e}\,, \tag{11a}$$

$$v_e = \frac{\pm v}{(1 + 4\kappa^2 v^2 \tau_0^2)^{1/2}}\,, \tag{11b}$$

$$\kappa^2 v^2 \tau_0^2 = \frac{\mu^4}{(1 - \mu^4)^2}\,. \tag{11c}$$

τ in the above equations is a time coordinate frame moving at the effective group speed v_e of the wave. Equation (11b) shows that the speed v_e of the wave is always lower than v ($|v| > |v_e|$) and that it can be positive (forward solitons) or negative (backward solitons). τ_0 is the temporal pulse width of the wave which determines through Eqs. (11) the speed of the wave and the auxiliary parameter μ. The parameter μ is real and the quantity μ^4 in Eq. (11a) is always positive since $|v| > |v_e|$. $\hat{A}$ is associated with the amplitude of this wave and Ψ_0 is an arbitrary phase constant. The total field of this wave can be obtained by using Eq. (2), and is

$$E=(\kappa/\gamma)^{1/2}\hat{A}\operatorname{sech}^{1/2}(\tau/\tau_0)\{\exp[i(\Theta+\beta_0 z)]-\mu^2\exp[i(\Phi-\beta_0 z)]\}\exp(-i\omega_0 t)\,, \tag{12}$$

and hence its intensity is given by

$$|E|^2=(\kappa/\gamma)\hat{A}^2\operatorname{sech}(\tau/\tau_0)[1+\mu^4-2\mu^2\operatorname{sech}(\tau/\tau_0)\cos(2\beta_0 z)-2\mu^2\tanh(\tau/\tau_0)\sin(2\beta_0 z)]\,. \tag{13}$$

In deriving Eq. (13) we have used the relations $\cos(\Phi-\Theta)=\operatorname{sech}(\tau/\tau_0)$ and $\sin(\Phi-\Theta)=\tanh(\tau/\tau_0)$, which can be readily obtained from Eqs. (8) and (9).

We now interpret these results. When $2\kappa v\tau_0 \gg 1$, then $|v_e/v| \ll 1$ (slow solitons) and $\mu\approx 1$. If, on the other hand, $2\kappa v\tau_0 \ll 1$, then $|v_e| \lesssim |v|$ and thus $\mu \to 0$ or ∞. The range $0<\mu<1$ corresponds to forward Bragg solitons, whereas for $1<\mu<\infty$ the solitons move backwards. If $\mu=1$, then in principle (for lossless media) an immobile Bragg soliton is possible which is related to that found by Chen and Mills[7] and Mills and Trullinger[8]. From the above analysis, the two components (E_f and E_b) of this soliton wave move together and in doing so they produce interference effects in the wave's intensity profile. For a given nonlinear periodic structure, as τ_0 increases not only is it the case that $|v_e/v| \ll 1$ (or $\mu\approx 1$) but also the power spectrum of the soliton wave is well confined within the frequency band gap and furthermore its central frequency is close to ω_B. More specifically, for $\mu \approx 1$ ($2\kappa v\tau_0 \gg 1$)

$$E\approx(4\kappa/3\gamma)^{1/2}\operatorname{sech}^{1/2}(\tau/\tau_0)\exp[i(\tfrac{1}{2}\pi+\Psi_0-\omega_0 t)]$$
$$\times\{\sin(\beta_0 z)[1+\operatorname{sech}(\tau/\tau_0)]^{1/2}-\operatorname{sgn}(\tau)\cos(\beta_0 z)[1-\operatorname{sech}(\tau/\tau_0)]^{1/2}\}\,. \tag{14}$$

The maximum intensity involved in this slow wave occurs at $\tau\approx 0$ and it is approximately equal to $|E|^2_{\max}\approx 8\kappa/3\gamma$ or $n_2|E|^2_{\max}\approx \frac{8}{3}n_1$. Thus, the Bragg filter will become "transparent" if the maximum nonlinear index change is comparable to n_1. If, on the other hand, τ_0 decreases, then v_e starts to approach v from below and thus the Bragg soliton becomes considerably more mobile. This regime corresponds to $\mu\to 0$ for forward solitons and to $\mu\to\infty$ for backward waves. From Eqs. (12) and (13), as $\mu\to 0$ the forward wave dominates and the interference effects are suppressed. For $\mu\to\infty$ the backward wave dominates. However, as τ_0 decreases, the spectral width of the wave is no longer contained within the frequency gap and furthermore the central frequency of this soliton can be far away from ω_0 or ω_B. This is due to the strong chirp involved in the phases of U and V and in effect these waves will behave as if they had been excited at a frequency ω_0' which is far away from ω_B. This explains the high mobility of these latter solitons and the small amplitude of the back-reflected signal. What is interesting, however, is that their field profiles vary as $\operatorname{sech}^{1/2}(\tau/\tau_0)$ and not as hyperbolic secants that a nonlinear Schrödinger formalism would have predicted. Strictly speaking slow Bragg solitons can occur only when $2\kappa v\tau_0 \gg 1$.

As an example, consider a single-mode fiber periodic structure which is to be operated at $\lambda_0=\lambda_B=0.52$ μm. This fiber has an effective core area of $S\approx 10$ μm^2 and its refractive index is $n_0=1.46$. The Kerr nonlinearity of glass is $n_2=1.2\times10^{-22}$ $(\text{m/V})^2$ and the index modulation n_1 is assumed to be $n_1=10^{-6}$. Therefore the coupling coefficient of the structure is $\kappa\simeq 6$ m^{-1} and the frequency width of the gap is $\Delta\omega/2\pi\approx 400$ MHz. The velocity of light in the unperturbed medium (c/n_0) is $v=2.055\times10^8$ m/s. If the temporal width of the Bragg soliton is taken to be $\tau_0=10$ ns, then $v/v_e\approx 25$. From Poynting's theorem, the peak power of this wave can be evaluated and it is approximately 423 W. Furthermore, the spatial extent of the Bragg soliton, $z_0=v_e\tau_0$, is ~ 8.3 cm.

In conclusion, we point out that there are still many interesting issues regarding these Bragg solitons that require further investigation. These include their stability, their dynamics, and of course the initial conditions necessary to excite these slow "bullets" of light.

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