Scale Invariance of g_A/g_V in Quark-Confining Potentials

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It is demonstrated that the spin distribution in the nucleon surface determines the value of g_A/g_V. This ratio is shown to be scale invariant; it is sensitive to the surface thickness and to the type of quark confinement (Dirac-scalar or -vector) and vanishes for pure vector confinement. A phase transition is found from confinement to deconfinement when the ratio of vector to scalar confinement falls below a critical value (-1.0). Experiments to probe a Dirac-vector component in the (free or bound) nucleon are proposed in the light of the CPT theorem.

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Within the $V-A$ theory for the charge weak current, the β decay of the free neutron, $n \to p e^- \bar{\nu}_e$, is determined by momentum-independent vector and axial-vector weak-coupling constants g_V and g_A. The neutron-spin–electron-momentum angular correlation is sensitive to g_A/g_V; a recent experiment at the reactor of Institut Laue-Langevin in Grenoble obtains $g_A/g_V = 1.262(5)$.[1] For $q^2=0$ the weak current couples only to those nucleon constituents which build up the spin of the nucleon[2]; the strength of this coupling is g_A/g_V. In confining potentials which follow a simple power law (including the MIT bag model for an infinite power), g_A/g_V is independent of the scale parameter set by the (phenomenological) potential. In such models g_A/g_V depends only on the surface thickness and decreases monotonically with decreasing surface thickness.[3] The nucleon axial-vector–to–vector charge ratio g_A/g_V is therefore of fundamental importance to both weak- and strong-interaction properties; moreover, the Goldberger-Treiman relation links g_A/g_V to the pion-nucleon interaction[4] and the Bjorken sum rule[5] links g_A/g_V to deep-inelastic scattering of polarized electrons or muons off polarized protons.[6,7] It is therefore essential to understand the experimental value for g_A/g_V in quark models. Although the nonrelativistic quark model [within the SU(6) scheme of hadrons] predicts the ratio of proton to neutron magnetic moments to be $\mu_p/\mu_n = -3/2$, very close to the experimental value -1.46, the same model has problems with g_A/g_V, which it predicts to be 5/3 instead of the observed $\sim 5/4$.[1] The latter fact then creates well-known problems in pion-nucleon physics and in deep-elastic scattering.

To find the origin of this discrepancy, without leaving the SU(6) scheme of hadrons, the following relation proves useful[8] (note that this holds for any Dirac-scalar confining potential, including MIT-bag-model wave functions):

$$\frac{g_A}{g_V} = \frac{5}{3}\left(\frac{2E_0}{M}\mu_p^{\text{quark}} - 1\right), \tag{1}$$

with E_0 the $1s_{1/2}$ quark eigenenergy and M the nucleon mass. The nonrelativistic limit is $\mu_p^{\text{quark}} = 3\mu_N$ (μ_N the nuclear magneton) and $g_A/g_V = 5/3$ because $E_0 =$ (constituent quark mass) $= M/3$. In relativistic models which confine quarks inside the nucleon, the pion field is essential to continue the axial-vector current outside the hadron surface. This pion field, however, contributes to μ_p and g_A/g_V in a fundamentally different way: In models where the pion (or any bosonic field) does not contribute to the spin of the nucleon there is no contribution of the pion field to g_A/g_V[9]; in all models, however, the meson fields contribute to the magnetic moment μ_p. From Eq. (1) it is clear that the discrepancy observed in static SU(6) vanishes once the pion field couples to the electromagnetic current: For $g_A/g_V = 5/4$ Eq. (1) yields $\mu_p^{\text{quark}} = \frac{7}{4}(M/2E_0)u_N \approx \frac{7}{4}\mu_N$ [if one assumes $\sim 30\%$ center-of-mass corrections for the three-quark system, and thus $M \approx 2E_0$ rather than $3E_0$ (Refs. 8 and 9)]. In order to reproduce the experimental $\mu_p = 2.793\mu_N$ one needs pionic contributions of the order $\mu_p^{\text{pion}} \approx 1\mu_N$. This is consistent with independent findings in chiral bag models[10] as well as in quark potential models.[8,9]

Now we turn to the calculation of g_A/g_V in models where only the quarks and not the mesons contribute to the spin of the nucleon. The derivation of g_A/g_V is standard,[8,9]

$$\frac{g_A}{g_V} = \frac{5}{3}\left(1 - \frac{4}{3}\frac{\int_0^\infty dr\, f^2(r)}{\int_0^\infty dr[g^2(r)+f^2(r)]}\right), \tag{2}$$

where f (g) is the lower (upper) component of the quark spinor ($1s_{1/2}$ orbits)

$$\psi(\mathbf{r},t) = e^{-iE_0 t}\frac{1}{\sqrt{4\pi}}\begin{pmatrix} ig(r)/r \\ \sigma\cdot\hat{\mathbf{r}} f(r)/r \end{pmatrix}\chi_{1/2}.$$

For confining potentials $M(|\mathbf{r}|) = \frac{1}{2}(1+a\gamma_0)c_n r^n$ (i.e., we assume that the current quark mass of 5–10 MeV[11] can be neglected compared to the quark eigenenergy of ~ 500 MeV[9]), it is obvious from the structure of Eq. (2) that $(g_A/g_V)(n;a)$ is independent of the scale c_n.[3] For the special case $a=1$ there is an analytic expression for

g_A/g_V,[3]

$$(g_A/g_V)(n;a=1)=\tfrac{5}{9}(n+3)/(n+1)\,, \qquad (3)$$

which is the same for quarks in $1s_{1/2}$, $2s_{1/2}$, etc. (i.e., independent of the principal quantum number of quarks). Figure 1 shows g_A/g_V as a function of n for three values $a=-1,0,+1$. For $a\to\infty$ (i.e., a pure vector confining potential) g_A/g_V vanishes. Indeed for arbitrary vector potentials $g_A/g_V=0$ (Ref. 3) as a consequence of the Klein paradox.[12]

For $a=0$ and any other value (except for $a=\pm 1,\infty$) there are no known analytic solutions for g_A/g_V. The scale invariance of g_A/g_V, however, can be used to check the reliability of the numerical methods involved.

The scalar-vector harmonic confinement potential (i.e., $a=+1$, $n=2$) yields a complete orthonormal set of Dirac eigenfunctions. In this case the Dirac equation reduces to a simple second-order equation for the upper component. The analytic solutions are given in Ref. 8; they are characterized by a state-dependent scale parameter, unlike the situation known from the nonrelativistic harmonic oscillator. Equation (3) suggests that there might also be analytic solutions for $n\neq 2$.

The n dependence of $(g_A/g_V)(n;a)$ as shown in Fig. 1 reveals that the MIT bag is just the limit (as $n\to\infty$) of a scalar confining potential $M(r)=c_n r^n$ with respect to g_A/g_V. This comes as no surprise since it is well known that $(g_A/g_V)_{\rm MIT}$ is independent of the bag radius (which sets the relevant scale). Figure 1 shows that all classes of confining models (except the vector potentials) give the static SU(6) value $\frac{5}{3}$ for $n\to 0$, as expected [see Eqs. (2) and (3)]. For large n the four classes ($a=-1,0,+1,\infty$) converge to different asymptotic values, $\frac{5}{3}$, 1.0883, $\frac{5}{9}$, and 0.0, respectively. For a given surface thickness (measured by n^{-1}) the admixture of a like- (unlike-) sign vector confining potential decreases (increases) g_A/g_V. Only the quadratic potential reproduces the experimental g_A/g_V without any vector confinement.

Before we discuss the a dependence of $(g_A/g_V)(n;a)$ it seems appropriate to comment on the effect that a nonvanishing current quark mass would have on these findings. A nonzero mass term m_q in the Dirac equation introduces another scale (besides c_n), so that g_A/g_V depends on c_n for $m_q\neq 0$. In fact, g_A/g_V is increased by $\sim M_q/M$ ($\sim$ a few percent for $m_q=5$–10 MeV[11]) which trend is obvious from Eq. (2): Any deviation from the static value $\frac{5}{3}$ is of relativistic origin; a nonzero mass m_q therefore decreases the lower component f compared to the upper component g. I mention that for quark-core radii between 0.5 and 1 fm the dependence of g_A/g_V on c_n due to $m_q=10$ MeV is very weak.

The a dependence of $(g_A/g_V)(n;a)$ for $n=0,1,2,3,4,10,\infty$ is shown in Fig. 2. We find an a-dependent relation generalizing Eq. (1),

$$\frac{g_A}{g_V}=\frac{5}{3}\,\frac{1}{1+2a}\left[\frac{2E_0}{M}\mu_p^{\rm quark}-1\right], \qquad (1')$$

so that $\mu_p^{\rm quark}=(M/2E_0)\mu_N$ for $a=-\frac{1}{2}$. For $a\to -1$ the quark eigenenergy E_0 approaches zero and the quark-core radius becomes infinite; this holds for any $n\geq 0$. Beyond this point ($a\lesssim -1$) the quarks are deconfined. Therefore, we identify two regions in a space: confinement for $a>-1.0$ and deconfinement for $a<-1.0$. From Fig. 2 we read off that a linear confining potential can reproduce the experimental g_A/g_V if $a=+0.42$. Similarly $n=3$, $a=-0.11$ is an acceptable solution, etc.; the MIT bag model would need $a\approx -0.30$ to reproduce the data (see Fig. 2).

Is there any evidence for a Dirac-vector part in the confining potential? Nonrelativistic models cannot distinguish between scalar and vector confinement. Therefore, evidence must be sought under conditions where relativistic effects show up. If the origin of the Dirac-

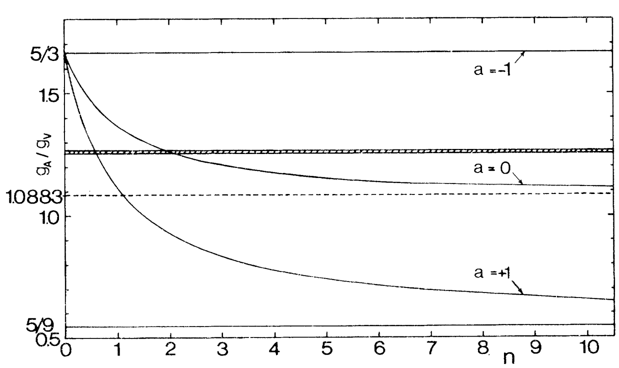

FIG. 1. g_A/g_V as a function of $n=r(d/dr)\ln M(r)$ for potentials $\frac{1}{2}(1+a\gamma_0)M(r)$, $M(r)=c_n r^n$, in the Dirac equation. The shaded area is the experimental value (Ref. 1). A pure vector confinement gives $g_A/g_V\equiv 0$ (see text). The dashed line corresponds to the MIT-bag-model value for $m_q=0$.

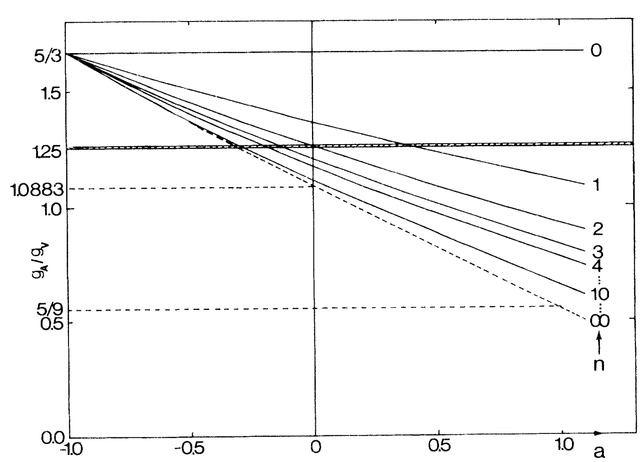

FIG. 2. g_A/g_V as a function of a; otherwise as in Fig. 1.

vector confinement is even under the C transformation (such as the vacuum state) then the vector-type potential will appear to the antiquarks with the opposite sign as compared to the quarks, thereby giving a different radius to the antinucleon. The CPT theorem[13] requires that particles and antiparticles have the same mass and opposite charges. The CPT theorem, however, does allow for a different size of nucleon and antinucleon [if the $3q$ ($3\bar{q}$) system cannot be described by a local field theory]. The presence of an additional Dirac-vector confining potential inside the nucleon would also imply different weak decay properties (i.e., g_A/g_V) for the antineutron.

In order to find experimental evidence for a vector part in the quark confinement of nucleons one could scatter "ultracold" antiprotons [currently being developed at the low-energy antiproton ring (LEAR) facility at CERN[14]] off atomic electrons; very much as thermal-neutron-on-atomic-electron scattering has been used to accurately measure the charge form factor of the neutron close to $-q^2=0$.[15] Even a very small momentum transfer would suffice to determine the derivative of the antiproton charge form factor at $q^2=0$ which is proportional to $\langle r_{\bar{p}}^2 \rangle$.

With present day technologies (Penning-trap experiments at LEAR[14]) it should soon be possible to measure the weak decay of the antineutron. If the nucleon is bound in a nucleus, the many-body forces could induce a Dirac-vector component into the individual nucleon's internal quark-confinement potential. Polarization measurements on nuclei are sensitive to relativistic effects and could signal possible Dirac-vector components.

Summarizing, I have shown that the weak axial-vector–to–vector coupling g_A/g_V is suitable to point out certain general features of bag-type models; these features should be kept in mind when trying to improve on hadron phenomenology. The ratio g_A/g_V is scale invariant and measures the internal spin structure of the nucleon; therefore, g_A/g_V is sensitive to the surface thickness and decreases with decreasing thickness. Moreover, g_A/g_V is a decreasing function of the Dirac-vector component in the quark confinement. This has implications for quark models to be used in nuclear physics.

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