Experimental Evidence for the Non-Abelian Nature of QCD from a Study of Multijet Events Produced in e^+e^- Annihilations

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We present evidence for the non-Abelian nature of QCD from a study of multijet events produced in e^+e^- annihilations from $\sqrt{s} = 50$ to 57 GeV in the AMY detector at the KEK storage ring TRISTAN. A comparison of the three-jet event fraction at TRISTAN to the fraction at energies of the DESY storage ring PETRA shows that the QCD coupling strength α_s decreases with increasing Q^2 . In addition, measurements of the angular distributions of four-jet events show evidence for the triple-gluon vertex.

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Quark interactions are described in the standard model by quantum chromodynamics (QCD),¹ a local non-Abelian gauge field theory based on the group symmetry SU(3). In this theory, quark interactions are mediated through the exchange of vector gluons that couple to the color charge of the quarks. Because of the non-Abelian nature of QCD, the gluons themselves carry color charge and, thus, couple to themselves. As a result, the QCD coupling strength a_s decreases^{2,3} with increasing Q^2 and the triple-gluon vertex contributes to the production of four-jet events in e^+e^- annihilations.

Although several of the basic features of QCD, including the existence of spin- $\frac{1}{2}$ colored quarks and spin-1 gluons have been observed, compelling evidence of the non-Abelian nature of QCD is only now becoming available.^{4,5} In this paper, we report evidence for the color charge of gluons from measurements of multijet events in the AMY detector at the KEK e^+e^- storage ring TRISTAN. A comparison of the rate of three-jet events with those from lower-energy experiments^{4,5} at the DESY storage ring PETRA provides evidence for the Q^2 variation of α_s , while measurements of the angular distributions of four-jet events provide evidence for the triple-gluon vertex.

The central feature of the AMY detector is a 3-T solenoidal magnet that allows the detector to be compact while maintaining good momentum resolution. Trajectories of charged particles with $p_t \ge 0.2$ GeV/c are reconstructed with good efficiency over the angular range $|\cos\theta| \le 0.85$ and their transverse momenta are determined with a precision of $\sigma_{p_t}/p_t \simeq 0.7\% p_t$, with p_t in GeV/c. γ rays with energies above 0.15 GeV are detected over the angular range $|\cos\theta| \le +0.73$ with an energy resolution of $\sigma_E/E \simeq 23\%/\sqrt{E} + 6\%$, with E in GeV.

The present analysis is based on 19.3 pb^{-1} of integrated luminosity accumulated at center-of-mass energies from 50 to 57 GeV. The selection criteria and background estimates for the multihadronic event sample are described elsewhere.⁶ In order to select a clean sample of multijet events, we have applied the following additional criteria. To remove low-energy particles which are difficult to associate with the correct jet, we impose a lower-momentum cutoff of 0.25 GeV/c on charged tracks and a lower-energy cutoff of 0.25 GeV on γ rays. To ensure that events are well contained in the solid angle over which the detector is sensitive, we require that the total missing momentum be less than $0.3E_{c.m.}/c$ and that $|\cos\theta_{\text{thrust}}| \leq 0.7$ where θ_{thrust} is the angle between the thrust axis and the beam direction. Finally, to compensate for the fact that the charged-particle coverage extends to $|\cos\theta| \le 0.85$ while the γ -ray coverage only extends to $|\cos\theta| \le 0.73$, we multiply the energy of charged particles in the region $0.73 \le |\cos\theta| \le 0.85$ by a factor of 1.7, since the data show that, in the region where the charged-track γ -ray coverages overlap, 60% of the detected energy is associated with the charged particles. The resulting sample contains a total of 1784 events.

We identify the jet multiplicity of these events by means of the jet-cluster algorithm introduced by the JADE group.⁷ In this algorithm, the scaled invariant mass squared, $y_{ij} = m_{ij}^2/E_{vis}^2$ with $m_{ij}^2 = 2E_iE_j(1 - \cos\theta_{ij})$, is calculated for each pair of particles. If the pair with the smallest value of y_{ij} is smaller than a parameter y_{cut} they are combined into a cluster by summing their fourmomenta. This process is repeated using all combinations of clusters and remaining particles, until the minimum value of y_{ij} exceeds y_{cut} . The number of clusters remaining at this stage is called the jet multiplicity.

One manifestation of the gluon's color charge is the decrease of the coupling strength with increasing Q^2 . In second-order perturbation theory, this dependence is given by⁸

$$\alpha_s(Q^2) = \frac{12\pi}{(33 - 2N_f)\ln(Q^2/\Lambda_{\overline{\text{MS}}}^2)} \left(1 - 6\frac{153 - 19N_f}{(33 - 2N_f)^2} \frac{\ln[\ln(Q^2/\Lambda_{\overline{\text{MS}}}^2)]}{\ln(Q^2/\Lambda_{\overline{\text{MS}}}^2)} \right)$$

where N_f is the number of quark flavors and $\Lambda_{\overline{MS}}$ parametrizes the fundamental scale of the theory (\overline{MS} denotes the modified minimal-subtraction scheme). Since in electron-positron collisions the Q^2 scale is proportional to the square of the center-of-mass energy $E_{c.m.}^2$, the measured value of α_s should decrease with increasing $E_{c.m.}^2$. While attempts to measure this dependence by comparing values of α_s at different values of $E_{c.m.}$ have been ambiguous,⁹ Bethke *et al.*⁴ have developed a method, described below, that is experimentally feasible and not dependent on model calculations.

In QCD, the differential cross section for three-jet events is given in first order by 10

$$\frac{d^2\sigma}{\sigma_0 dx_1 dx_2} = \frac{\alpha_s}{2\pi} C_F \left(\frac{y_{23}}{y_{13}} + \frac{2y_{12}}{y_{13}y_{23}} + \frac{y_{13}}{y_{23}} \right),$$

where C_F is the color factor and y_{ij} is the scaled invariant mass of the *ij*th parton pair. Integrating over the y_{ij} 's between the limits $y_{ij} = y_{\min}$ to $y_{ij} = 1$ yields the 1714

fraction of three jets, $R_3 = C_1(y_{\min})\alpha_s$ where C_1 is a function of y_{\min} . Since the only Q^2 dependence is contained in α_s , with y_{\min} fixed, the Q^2 dependence of R_3 depends only upon the Q^2 dependence of α_s . This result is also true in second order¹¹ where $R_3 = C_1(y_{\min})\alpha_s$ $+ C_2(y_{\min})\alpha_s^2$.

The parameter y_{cut} in the jet-cluster algorithm described above is similar to the parameter y_{min} that appears in the theoretical formula. The primary difference between the two is that y_{cut} is applied to particles that are observed in the final state while y_{min} is applied to partons. To verify the correspondence between these two parameters, we have examined the ratio of R_3 determined by applying the jet-cluster algorithm at the particle level with $y_{cut} = 0.08$, to R_3 determined by applying the algorithm at the parton level with $y_{min} = 0.08$. For the purposes of this comparison, we have assumed an ideal detector with 4π geometry and have generated events without initial-state radiation. We have determined by applying the algorithm to the parton level with $y_{min} = 0.08$.



FIG. 1. Measured values of the three-jet fraction R_3 with $y_{\text{cut}} = 0.08$ vs $E_{\text{cm.}}^2$. The solid line is based on a second-order calculation by Kramer and Lampe (Ref. 11).

mined this ratio using three different Monte Carlo event generators: LUND 6.3 with a leading-logarithm parton shower, ¹² LUND 6.2 with the Guthrod-Kramer-Schierholz (GKS) second-order matrix elements, ¹³ and LUND 6.2 with the Gottschalk-Shatz second-order matrix elements. ¹⁴ For all three cases, the ratio is constant to within $\pm 1\%$ as a function of $E_{c.m.} \ge 28$ GeV.

The measured value of R_3 will be affected by initialstate radiation, event-selection cuts, and detector acceptance. We find that these corrections approximately cancel and that the net correction is small; the eventselection criteria that we use tend to reject events with hard initial-state radiation and the use of E_{vis} rather than $E_{c.m.}$ in the determination of y_{ii} compensates for the missed particles and the finite resolution of the detector. Using the LUND 6.3 Monte Carlo parton shower generator, we have compared R_3 determined with the AMY detector resolution, geometry, and event-selection cuts and including initial-state radiation, with R_3 determined by a ideal detector without initial-state radiation. The difference in these values, $R_3^{\text{ideal}} - R_3^{\text{AMY}}$ is less than $\pm 1\%$ for any $y_{cut} > 0.04$. For $y_{cut} = 0.08$, the difference is $0.1\% \pm 0.3\%$, which is negligible compared to the statistical error of the R_3 measurement.

In Fig. 1, we plot the value of R_3 with $y_{cut} = 0.08$ from AMY together with previously published values at lower energies from JADE (Ref. 4) and TASSO (Ref. 5) as a function of $E_{c.m.}^2$. The value of the AMY measurement is $R_3 = 19.6\% \pm 0.9\%$ at an average value of $E_{c.m.}^2$ of 3032 GeV². Since Q^2 is proportional to $E_{c.m.}^2$ the AMY measurement approximately doubles the Q^2 range. Over this range, the measured values of R_3 decrease with increasing Q^2 . The χ^2 for a fit to constant α_s is 23.3 for six degrees of freedom. The χ^2 for a fit to a second-order calculation¹¹ with $Q^2 = E_{c.m.}^2$, shown in the figure, is 6.1 for six degrees of freedom.



FIG. 2. Diagrams that yield four-parton states.

A further manifestation of colored gluons is the contribution of the triple-gluon vertex, corresponding to the diagram shown in Fig. 2(a), to the production of four-jet events. To see the effect of this diagram, we compare non-Abelian QCD to an Abelian version in which the color factor of the quarks is retained but the color of the gluons is turned off. In QCD the dominant contribution, 66%, to the four-jet cross section is from the triple-gluon vertex corresponding to Fig. 2(a), with Fig. 2(b) contributing only 4%. In the Abelian model, Fig. 2(a) is forbidden while Fig. 2(b) contributes 51%. The remaining second-order process contributing to four-jet production is a double-gluon bremsstrahlung, shown in Fig. 2(c), and is common to both models.

In order to select four-jet events, we require that each jet satisfy the following criteria: the energy of the jet ≥ 3 GeV, the number of particles in the jet ≥ 2 , and $|\cos\theta_{jet}| \leq 0.85$. In addition, we apply the selection criteria for multijet events described above except that we remove the requirement that $|\cos\theta_{thrust}| \leq 0.7$. We set $y_{cut}=0.022$ for $E_{c.m.} > 52$ GeV and $y_{cut}=0.026$ for $E_{c.m.} \leq 52$ GeV. With these values, the total number of events in the four-jet sample is 139 and the estimated three-parton background in the selected sample is 22%.

Bremsstrahlung gluons are polarized in the $q\bar{q}g$ plane.¹⁵ When these gluons couple to two quarks, as in



FIG. 3. Distribution of θ_{BZ}^{*} in four-jet events. Also shown are the LUND 6.2 predictions for QCD (solid line) and for the Abelian model (dashed line).



FIG. 4. Distribution of $|\cos\theta_{NR}^*|$ in four-jet events. Also shown are the LUND 6.2 predictions for QCD (solid line) and for the Abelian model (dashed line).

Fig. 2(b), the direction of the quarks tends to be perpendicular to the direction of this polarization.¹⁶ On the other hand, when the coupling is to a pair of gluons as in Fig. 2(a), the two gluons have a slight preference for being along the line of the polarization. Bengtsson and Zerwas¹⁷ (BZ) have suggested that the angle between the plane defined by the two highest-energy jets and that defined by the two lowest-energy ones, θ_{BZ} , would be sensitive to this effect. Figure 3 shows the distribution of this angle for the data and for the Monte Carlo prediction for both OCD and the Abelian model. In both cases, the LUND 6.2 Monte Carlo with the second-order calculation of GKS (Ref. 13) was used to generate the events. The calculations were normalized to yield the same number of four-jet events for both models. The three-parton background is also included in these calculations. The fits with one degree of freedom give $\chi^2 = 0.3$ for QCD and $\chi^2 = 6.5$ for the Abelian model.

In determining θ_{BZ} , the angles between the jets that define the planes are required to be less than 160°. This requirement eliminates about 50% of the four-jet events. In order to avoid this loss of events, Bengtsson¹⁸ suggested using the angle between the vector difference of the momentum of the two highest-energy jets and the vector difference of the momentum of the two lowest-energy jets θ_{NR}^* . The measured distribution of $|\cos\theta_{NR}^*|$ is shown in Fig. 4, along with the Monte Carlo expectations for the non-Abelian and Abelian models. Here again, the data show a clear preference for QCD. The fits with three degrees of freedom give $\chi^2 = 0.5$ for QCD and $\chi^2 = 7.3$ for the Abelian model.

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