Determination of $\sin^2\theta_W$ from Measurements of Differential Cross Sections for Muon-Neutrino and -Antineutrino Scattering by Electrons

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Measurements of the combined differential cross sections $d\sigma/dy$ for ν_μ and $\bar{\nu}_\mu$ scattering by electrons yield the value $\sin^2\theta_W=0.195\pm0.018\pm0.013$. This value, at $Q^2\simeq1\times10^{-3}$ (GeV/c)^2, is more precise than and in good agreement with our determination from the ratio of the total cross sections for the same reaction. It is consistent with, but lower than, the value determined from deep-inelastic neutrino scattering.

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The two reactions

$$\nu_\mu+e^-\rightarrow\nu_\mu+e^- \tag{1}$$

and

$$\bar{\nu}_\mu+e^-\rightarrow\bar{\nu}_\mu+e^- \tag{2}$$

provide valuable tests of the electroweak unification model[1] [SU(2)×U(1)]. They involve only leptons, are essentially without the complication of QCD and thresholds, and the radiative corrections to the measured cross sections are small.[2] The kinematics of the reactions and beam energy ensure that they involve low-momentum transfer [$\sim1\times10^{-3}$ (GeV/c)^2 in this experiment], thus providing a check of the theory over a range of 7 orders of magnitude when compared with high-momentum-transfer experiments close to the Z^0 mass.

The differential cross sections for reactions (1) and (2) can be written in a model-independent form as

$$\frac{d\sigma}{dy}=\rho^2\frac{G_F^2m_eE_\nu}{2\pi}\left[(g_V\pm g_A)^2+(g_V\mp g_A)^2(1-y)^2-(g_V^2-g_A^2)\frac{m_ey}{E_\nu}\right], \tag{3}$$

where the upper (lower) sign refers to ν_μ ($\bar{\nu}_\mu$), ρ quantifies the possibility of the neutral current having a different coupling strength than the charged current, G_F is the Fermi constant, g_V and g_A are, respectively, the vector and axial-vector couplings of the electron to the neutral intermediate vector boson, and $y\equiv T_e/E_\nu$, with E_ν and T_e being the energy and kinetic energy, respectively, of the incident neutrino and recoiling electron.

In the standard model[1] of the electroweak unification $g_V\equiv-\frac{1}{2}+2\sin^2\theta_W$, $g_A\equiv-\frac{1}{2}$, and ρ is expected to be unity. With appropriate radiative corrections, $\sin^2\theta_W$ should be a universal constant independent of momentum transfer. As can be seen from Eq. (3), the correlated y dependence between the ν_μ and $\bar{\nu}_\mu$ differential cross sections can provide a sensitive measure of g_V,g_A or of $\sin^2\theta_W$ for these low-Q^2 reactions.

Prior to the experiment reported here these quantities had been extracted either from reactions (1) and (2) by measurement of total elastic cross sections[3] or their ratio.[4] Differential-cross-section measurements present somewhat different systematic errors and only a subset of the data for reaction (1) has been published[5] previously.

The kinematics and small cross sections ($\sim10^{-42}$ cm^2) severely restrict measurement of the differential cross sections because for T_e and E_ν large with respect to m_e the electron scattering angle is constrained as $\theta_e^2<2m_e/T_e$. At the energies of this experiment θ_e is a few milliradians and the last term in Eq. (3) can be neglected. Further, under the same approximations, $\theta_e^2\cong(2m_e/T_e)(1-y)$; thus $d\sigma/dy$ uniquely determines $d\sigma/d\theta_e^2$. This is a valuable feature in a wide-band neutrino beam where the precision required for shape sensitivity is more easily obtained in θ_e than in y. This experiment is unique in that it has sufficient angular resolution and statistics for the reactions (1) and (2).

The data for this study (Experiment No. E-734) were taken in the wide-band, horn-focused neutrino beam of the Brookhaven National Laboratory Alternating Gradient Synchrotron (AGS). The mean ν_μ ($\bar{\nu}_\mu$) energy was 1.27 (1.23) GeV. The neutrino detector was a 170-ton, fully active (liquid scintillator), highly segmented, electronic device.[6] The data, taken in three running periods, were combined and subjected to a common analysis.[7] Results from the earlier data have been published using the total cross sections and ratio of the cross sections to extract $\sin^2\theta_W$ as mentioned above.[4(a)] Final samples of $160 \pm 17 \pm 4$ [$97 \pm 13 \pm 5$] events for reactions (1) [and (2)] were obtained from total exposures of 3.42×10^{19} [4.74×10^{19}] protons on the production target.

The recoiling-electron candidates were defined to be single showers in the region $210 \le T_e \le 2100$ MeV and in an angular interval $0.0 \le \theta_e \le 0.20$ rad. They were selected by efficient algorithms based upon multiple dE/dx measurements, distinctive shower-energy deposition patterns, and angular and energy resolution as determined from test beam measurements. The angular resolution was found to be well expressed as $\Delta\theta_{x,y} = (13 \pm 1 \text{ mrad})/[E\text{ (GeV)}]^{1/2}$ and the energy resolution to be $\Delta E/E = 0.13/[E(\text{GeV})]^{1/2}$. These selection methods and the reduction of the dominant backgrounds of π^0-decay-photon conversion and ν_e-induced events have been extensively discussed elsewhere.[6,7]

The experimental θ_e^2 distributions resulting from analysis of the ν_μ- and $\bar{\nu}_\mu$-induced exposures are shown in Fig. 1. The data were binned into sixty bins (each of 5×10^{-4} rad^2) so that the width of each bin corresponded to 1σ of angular resolution. Less than 1% of the expected signal extended beyond the region $0.0 \le \theta_e^2 \le 0.03$ rad^2 was used in the fits. Extensive studies of energy and angular distributions in nonsignal regions established θ_e^2 shapes for the backgrounds.

The strongly forward-peaked signal is clearly evident in Figs. 1(a) and 1(b). The cross section in Eq. (3) consists of two components: the first being independent of y and the second varying as $(1-y)^2$. Given the functional dependence between y and θ_e^2, it is clear that the first and second terms in Eq. (3) correspond to two different θ_e^2 distributions. The second term is less sharply peaked in the forward direction since the cross section is suppressed as y approaches 1. The values of g_V and g_A determine the relative contributions of the two terms.

A likelihood fit was made to the data of Figs. 1(a) and 1(b) with the following functions in the parent distributions:

$$n_i = \frac{G_F^2 m_e}{2\pi} F_{\text{norm}}[(g_V \pm g_A)^2 a_1 f_i + (g_V \mp g_A)^2 a_2 s_i] + N_b b_i\,, \qquad (4)$$

where $F_{\text{norm}} = N_{\text{QE}}\langle E_\nu\rangle / \langle\sigma(\text{QE})\rangle$ and the index i runs over the θ_e^2 bins, N_{QE} is the number of normalization

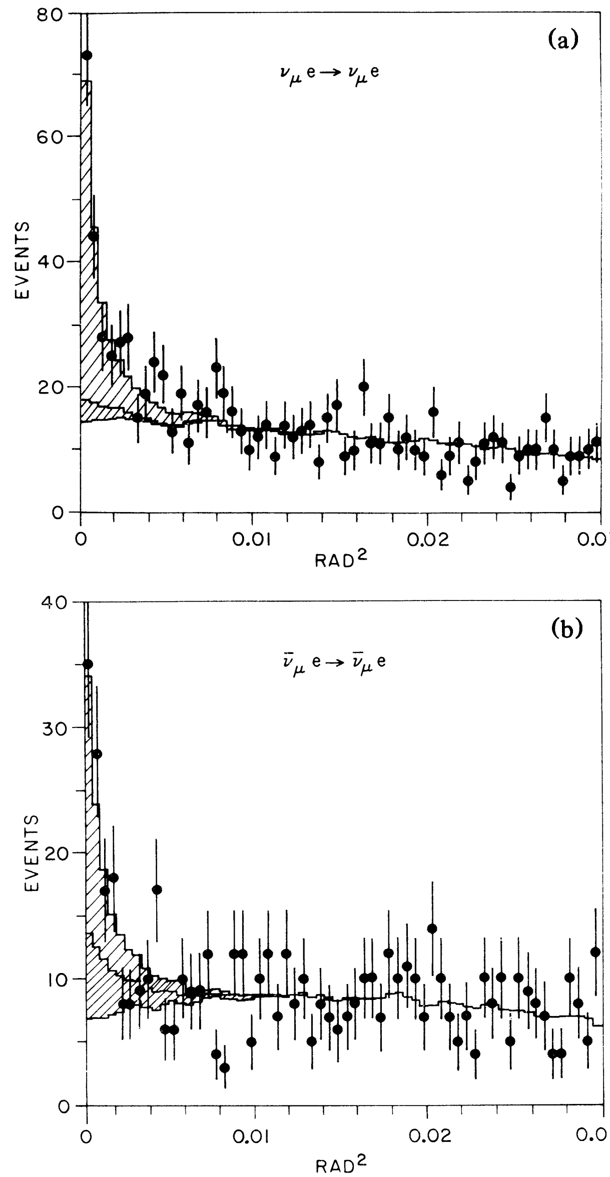

FIG. 1. (a) Differential distributions in θ_e^2 for the neutrino data. (b) Differential distributions for the antineutrino data. Data are points with error bar; the y-independent term is light shaded, and the $(1-y)^2$ term is dark shaded while background is unshaded.

events from the simple quasielastic charged-current reactions $\nu_\mu + n \to \mu^- + p$ and $\bar{\nu}_\mu + p \to \mu^- + n$. $\langle E_\nu\rangle$ and $\langle\sigma(\text{QE})\rangle$, respectively, are the incident neutrino energy and cross section averaged over the appropriate beam and acceptance distributions. The functions f_i, s_i, and b_i are the normalized θ_e^2 distributions corresponding to the first, second, and background terms, respectively. The quantities a_1 and a_2 are acceptances; N_b is the total number of background events, and the upper [lower] sign is for reaction (1) [(2)]. The functions f_i, s_i, and b_i, which included all relevant detector characteristics and resolutions, were Monte Carlo generated.

A negative log-likelihood function was calculated assuming Poisson statistics in each bin. The program MINUIT[8] was used to minimize the function by varying g_V, g_A, and N_b. The likelihood function was computed for the neutrino and antineutrino data separately and an additional simultaneous fit of both data sets was performed by adding the two data sets and minimizing the result. The results of the three fits are internally consistent, and the number of events extracted as signal and background agree with those determined from the total cross sections.[7] Results from the simultaneous fits are presented here in Figs. 1 and 2.

In Fig. 1 the light-shaded, dark-shaded, and unshaded histograms are the y-independent, y-dependent terms, and the background contributions, respectively, from the best fit. The solid histograms are the complete fits for all three terms.

The 68%-likelihood contours for the separate fits to the neutrino and antineutrino data and the combined fits are shown in Fig. 2. In this analysis we have set $\rho = 1$ in Eq. (3). Only one of the four solutions remains when results of other experiments are taken into account. All but solution D are eliminated by the $\bar{\nu}_e$-e reactor[9] and e^+-e^- collider[10] results. The choice of solution D yields

$$g_V = -0.107^{+0.035}_{-0.036}(\text{stat.})^{+0.029}_{-0.028}(\text{syst.}), \quad g_A = -0.514^{+0.023}_{-0.023}(\text{stat.})^{+0.029}_{-0.027}(\text{syst.}), \tag{5}$$

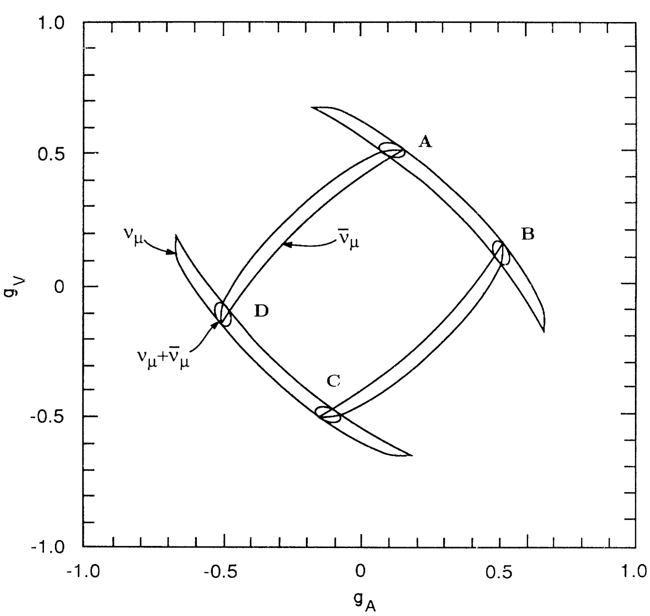

FIG. 2. Likelihood contours for fits to the neutrino and antineutrino data. Regions inside contours are allowed. The small regions labeled A–D are for the simultaneous fit to reactions (1) and (2). The projections to the axes give the 68%-likelihood intervals for g_V and g_A (statistical only).

or, in the context of the standard model,

$$\sin^2\theta_W = 0.195 \pm 0.018(\text{stat.}) \pm 0.013(\text{syst.}). \tag{6}$$

These values are consistent with our previous results[5,11] obtained from a much smaller and partial portion of the total, final sample analyzed here.

It is instructive to combine this result with the CHARM determination,[4(b)] which used the cross-section ratio method, since the combined E-734 and CHARM experiments represent more than 80% of the world's data on reactions (1) and (2), and both have well-controlled systematic errors. Combining the statistical and systematic errors from the two experiments in quadrature, one has

$$(\sin^2\theta_W)_{\text{combined}} = 0.199 \pm 0.019. \tag{7}$$

Because reactions (1) and (2) have negligible radiative corrections[2] ($< 1\%$) to the value of $\sin^2\theta_W$ extracted from them, it is of interest to compare the value of $\sin^2\theta_W$ from Eq. (5) to the value obtained from radiatively corrected[2] measurements[12] of the intermediate-vector-boson masses via $M_W^2 = (37.281)^2/\sin^2\theta_W(1-\Delta r)$ and $\sin^2\theta_W = 1 - M_W^2/M_Z^2$, where Δr is expected[2] to be 0.0713 ± 0.013 including all one-loop diagrams. Amaldi *et al.*[13] in a recent comprehensive analyses of the relevant high-Q^2 experiments have found $\sin^2\theta_W$ values of 0.229 ± 0.008, 0.230 ± 0.011, 0.228 ± 0.007, and 0.233 ± 0.003 from use of M_Z, M_W, M_W/M_Z, and deep-inelastic neutrino scattering, respectively. While all of these determinations of $\sin^2\theta_W$ are consistent, the value from neutrino- (and antineutrino-) electron scattering is lower by 0.03 ± 0.02.

In conclusion, this is the first determination of $\sin^2\theta_W$ from the combined use of the differential-cross-section measurements of reactions (1) and (2). The value obtained is consistent with that obtained in the same experiment from the individual total cross sections and the ratio of the total cross sections. The systematic errors enter differently in each of these determinations. The value of $\sin^2\theta_W$ is lower but within errors consistent with the values obtained from the high-Q^2 measurements of the intermediate-vector-boson masses or of deep-inelastic neutrino scattering.

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