Excitation of an Electron Bernstein Wave in a Magnetized Plasma by the Optical Mixing of Two Microwave Beams

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The first experimental evidence of the excitation of an electron Bernstein wave by the nonlinear coupling of two microwave beams in a magnetized plasma is reported. The dependence of the excited wave on the electron density and magnetic field is found to be in excellent agreement with the dispersion relation of the Bernstein wave. The level measured with an electric probe is typically 3×10^{-16} W for a mean incident power of 10 W. This value is about 10 dB below the theoretical value, but still within the uncertainty limit. The resonance width is consistent with the theory.

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Optical mixing in plasmas has been of interest for the past ten years, particularly with a view to achieving local nonperturbative measurements in plasmas. Many theoretical works have been devoted to the coupling of two electromagnetic waves, both in unmagnetized and magnetized plasmas.[1,2] A few recent experiments carried out in unmagnetized plasmas describe the optical mixing of either two laser beams[3] or two microwave beams.[4,5] The electron density can be obtained by this method with a fair accuracy. This Letter reports the first experimental observation of an electron Bernstein wave excited by the nonlinear mixing of two microwave beams in a magnetized plasma. In magnetized plasmas, the excitation of a wave very sensitive to the magnetic field direction, such as the Bernstein wave, may give an interesting method of measuring the total magnetic field in tokamaks.

Both incident waves are launched perpendicularly to the external magnetic field $\mathbf{B}_0$, with their electric field parallel to $\mathbf{B}_0$ (ordinary polarization). Their frequencies ω_1 and ω_2 are much above ω_p and ω_{ce}, the plasma and cyclotron frequencies, and their wave vectors make the angle $(\mathbf{k}_1,\mathbf{k}_2)=\phi$. For suitable values of the plasma parameters, the frequency $\omega=\omega_1-\omega_2$ and the wave vector $\mathbf{k}=\mathbf{k}_1-\mathbf{k}_2$ of the beat wave are those of an eigenmode, for instance the Bernstein wave.[6] A theoretical estimation of the coupling efficiency can be obtained from a previous work.[2] For simplicity, we only outline the case of a homogeneous plasma, although the calculation was done for an inhomogeneous plasma as well.

Since the beat wave is nonlinearly excited by two intense pump waves, there is no power threshold, and the excited power increases proportionally to the product of the incident powers. If the beat wave is weakly damped, the sensitivity of the experiment is then determined by the noise level of the overall experimental setup.

The second-order nonlinear current driven by the high-frequency waves is

$$\mathbf{J}_{\rm nl}=-e\{n_0\boldsymbol{\mu}\cdot[\mathbf{v}\times\mathbf{B}+(m/e)\mathbf{v}\cdot\nabla\mathbf{v}]+n\mathbf{v}\}\,,$$

where $\boldsymbol{\mu}$ is the mobility tensor, n_0 is the unperturbed density, and n, $\mathbf{v}$, and $\mathbf{B}$ are the first-order density, velocity, and magnetic field, respectively.

Let $\mathbf{B}_0$ be along the z axis and $\mathbf{k}$ along the y axis. We introduce the angle $\theta=(\mathbf{k}_1,\mathbf{k})$; hence $\mathbf{k}_1=(k_1\sin\theta, k_1\cos\theta,0)$ and $\mathbf{k}_2=(k_2\sin\theta,-k_2\cos\theta,0)$. The previous expression becomes

$$\mathbf{J}_{\rm nl}=[\epsilon_0\omega_p e A_1 A_2/2m\omega_1\omega_2(\omega_{ce}^2-\omega^2)]\begin{bmatrix}\omega(k_1-k_2)\sin\theta-i\omega_{ce}(k_1+k_2)\cos\theta\\ i\omega_{ce}(k_1-k_2)\sin\theta+\omega(k_1+k_2)\cos\theta\\ 0\end{bmatrix},$$

where A_1 and A_2 are the electric field amplitudes of each pump wave. This current is excited in a finite-interaction region of length R and induces an electric field,

$$\mathbf{E}=\mu_0\omega R\mathbf{D}^\dagger\cdot\mathbf{J}_{nl}/(\partial\,\det\mathbf{D}/\partial k)\,,$$

where $\mathbf{D}^\dagger$ is the adjoint of the dispersion tensor $\mathbf{D}$ of the magnetoplasma, and $\det\mathbf{D}$ is the determinant of $\mathbf{D}$. Let $\boldsymbol{\epsilon}^h$ be the Hermitian part of the dielectric tensor $\boldsymbol{\epsilon}$; the power flux T of the excited wave can then be written[7]

$$T=\tfrac{1}{4}\epsilon_0\mathbf{E}^*\cdot[\partial(\omega\boldsymbol{\epsilon}^h)/\partial\mathbf{k}]\cdot\mathbf{E}\,.$$

The potential of the excited wave is deduced from the electric field. Figure 1 displays its theoretical variation as a function of the interaction angle ϕ between the incident beams. The calculation was performed with incident frequencies around 70 GHz, and $f_p=0.94$ GHz, $f_{ce}=0.6$ GHz, $R=3$ cm, and the incident flux normalized to 1 W/cm^2 for each beam. The potential and the coupling efficiency diverge as the frequency approaches the upper hybrid frequency; of course, the homogeneous approximation is no longer valid as inhomogeneity effects limit the field amplitude near the resonance. In any case

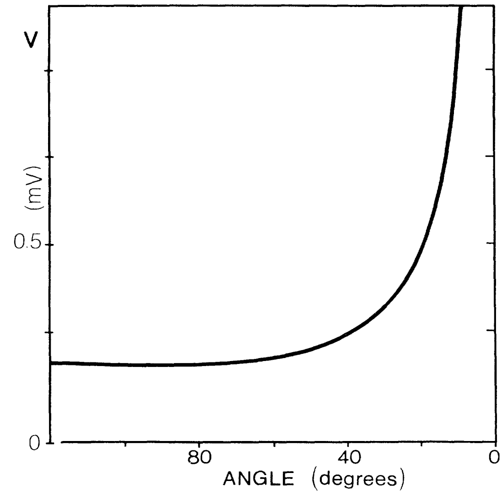

FIG. 1. Theoretical evolution of the potential V in the excited wave as a function of the angle between the two microwave beams: The increase at low angles is partly related to the divergence of the electric field when approaching the upper hybrid frequency.

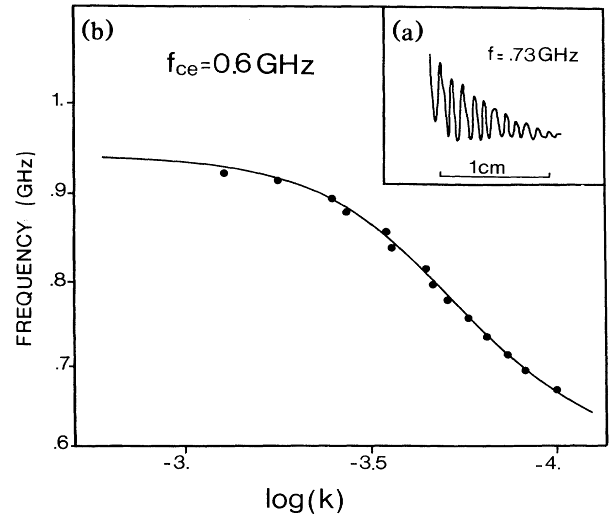

FIG. 2. (a) A typical radial interferogram obtained with the electrostatic wave excited by the antenna, and (b) the dispersion curve for the Bernstein mode, with a cyclotron frequency $f_{ce}=0.6$ GHz. The upper hybrid frequency is close to 950 MHz.

the efficiency, defined as the ratio of the excited power flux to the mean incident flux, increases near the upper hybrid frequency, up to values of the order of 10^{-11}.

The finite size of the interaction region gives rise to a broadening of the frequency spectrum of the excited wave. This convective resonance width is $\Delta\omega = v_g/R$ where v_g is the group velocity. Similarly, in an inhomogeneous plasma with a density scale length L, the resonance width can be estimated as

$$\Delta\omega = \tfrac{1}{2}(R/L)(\omega_p^2/\omega),$$

where ω_p is the plasma frequency at the center. These expressions will be compared hereafter with the measured width.

The experiments have been carried out in a magnetized multipolar plasma device described elsewhere.[8] Two large multipolar chambers (1 m in diameter) are joined together by a column (1.3 m in length, 30 cm in diameter) surrounded by a solenoid. The electron density goes up to 2×10^{11} cm^{-3}, and the electron temperature is $T_e = 3$ eV at a pressure of 4×10^{-4} Torr. The magnetic field can reach 1200 G; most experiments have been done at 214 G, which corresponds to $f_{ce}=600$ MHz. The central section of the column is made of a polytetrafluorethylene (PTFE) cylinder (15 cm long and 1.5 cm thick) transparent to the microwaves, so that the interaction can be made at any coupling angle. The pump sources are an extended-interaction oscillator (EIO) (30 W, 70 GHz) and a backward-wave oscillator (BWO) (10 W, 69.05 GHz). The power emitted by the latter is chopped by a ferrite modulator in order to perform a synchronous detection of the excited wave. Each incident wave is focused at the center of the column by a horn-lens system perpendicular to the magnetic field. The angle between the microwave beams is 18°, which corresponds to an excited wave vector of 455 m^{-1}. The beams are incident normally on the PTFE cylinder and cross it with very little perturbation. Only a small part of the incident power (3% to 4%) is reflected on the PTFE cylinder and absorption is negligible. The incident power flux in the vacuum vessel was determined by accurate calorimetric measurements; typical values are 4 W (BWO beam) and 25 W (EIO beam). The interaction region is roughly an ellipsoid whose major axis is perpendicular to the magnetic field. The interaction length is about 3 cm, i.e., about two excited wavelengths. The induced fluctuations are measured by an electric probe moving along and across the magnetic field. After filtering and amplification, a heterodyne detection is performed. A further enhancement (30 dB) of the signal-to-noise ratio is supplied by a lock-in amplifier whose reference signal drives the ferrite modulator. The overall sensitivity of the system is then -135 dBm.

The optimal plasma conditions for resonant coupling are deduced from the dispersion relation of test waves launched into the plasma by a point antenna located at the center of the interaction region: The density must be adjusted so that the excited wavelength (related to the angle between the beams) matches the Bernstein wavelength. Figure 2(a) shows a typical record and Fig. 2(b) displays the experimental dispersion relation, in very good agreement with the theoretical curve for the Bernstein mode for $f_p=0.74$ GHz, $f_{ce}=0.6$ GHz, and $T_e=3$ eV. This value of T_e is also in agreement with experi-

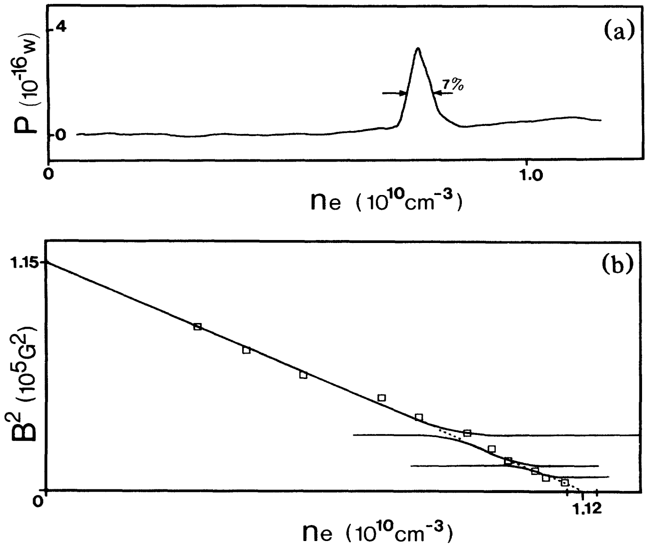

FIG. 3. (a) Probe signal received in the center of the interaction region as a function of the plasma density, with a magnetic field strength $B=190$ G. (b) The squared magnetic field vs the resonance density, exhibiting a good fit with the data. The critical density at 950 MHz ($n_c=1.12\times10^{10}$ cm^{-3}) is obtained with accuracy of 4%.

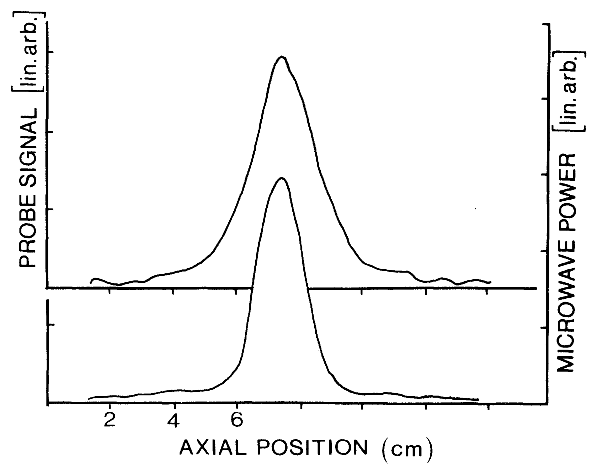

FIG. 4. Axial profile of the excited power superimposed on the axial profile of the pump power, whose width at half maximum is slightly lower.

ments involving ion-acoustic waves.

In the coupling experiment itself, the moving probe is put into the interaction region. The frequency difference is fixed to 950 MHz. The probe signal around the difference frequency is recorded as a function of the electron density. A typical record is displayed in Fig. 3(a). The excited power detected by the probe is -110 dBm. The theory then predicts an electric field of 0.25 V/m. This value yields the related density fluctuation and the electrostatic power density. The high-frequency current collected by the probe can then be deduced only if the probe efficiency is known. For an ideal probe (efficiency of 1), the theory leads to an excited power level of -100 dBm, i.e., 10 dB above the measured value. It can be shown that the inhomogeneity effects mentioned above reduce the level by less than 3 dB. Thus, this discrepancy is mostly due to the assumption of an ideal probe efficiency at such a high frequency. The actual value is rather difficult to estimate because anisotropic sheath effects and radiation patterns are involved. Besides, it is quite impossible to accurately determine by an independent method the field amplitude of an electron plasma wave. From our test-wave experiments, it results that the probe efficiency varies from 10^{-1} to 10^{-2} depending on the frequency and direction with respect to the magnetic field. Furthermore, the presence of the probe inside the interaction region slightly disturbs the incident electric fields, generating weak standing-wave patterns that can alter the signal. To sum up, the 10-dB discrepancy turns out to be lower than the overall uncertainty limit of the experimental arrangement. In fact, only a Thomson scattering experiment could possibly lead to a more accurate estimate of the optical mixing efficiency.

In Fig. 3(a), the experimental resonance width $\Delta n/n$ can be estimated to 7%. The theoretical convective width is 4% to 6%. For a gradient length $L=80$ cm, the inhomogeneity broadening is $\Delta n/n \simeq 5\%$. Both evaluations are compatible with the measured value. Figure 4 shows the excited power profile along the axis superimposed on the incident power profile. The former is slightly broader, due to a weak axial component of the group velocity (the group velocity of the Bernstein wave is mostly pointing towards the perpendicular direction).

The wave number and frequency tuning depends on both density and magnetic field. When the latter is changed, the electron density must be adjusted in order to keep the upper hybrid frequency slightly above the (fixed) working frequency. It results that the relationship between n_e and ω_{ce}^2 is almost linear, except for a small region around each harmonic crossing, where thermal effects are important. This density dependence is shown in Fig. 3(b). The agreement between the experimental points and the theoretical curve is quite good, especially for the fundamental ($\omega_{ce} < \omega < 2\omega_{ce}$). Points on the higher-harmonic branches are less accurate, since for decreasing values of the magnetic field, the cyclotron harmonics are crossed more and more quickly. The cutoff density ($\mathbf{B}_0=0$) can still be estimated with a good precision ($\Delta n/n=4\%$); its value is useful to calibrate the Langmuir probe which measures the electron density. The results displayed on Fig. 3(b) clearly demonstrate that the mode excited by the nonlinear interaction is the electron Bernstein mode.

Summarizing, we have reported the first experimental observation of an electron Bernstein wave excited by the nonlinear mixing of two electromagnetic waves. Provided that the excited wave is measured by a remote operation (e.g., Thomson scattering), such a method could allow local measurements in hot plasmas. For instance, the excitation of the Bernstein wave is expected to depend strongly upon its direction with respect to the magnetic field. Therefore, the poloidal-field component (hence, the current profile) in a tokamak plasma could be determined. Whether this technique can actually be utilized rests on the existence of sources powerful enough to overcome the noise of the plasma.

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