

Spectrum of $J^P = 2^+$ Mesons

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(Received 29 August 1988)

The possibility that the $\theta(1720)$ and the recently reported $\zeta(1480)$ are suitable members of a 36-dimensional multiplet of $(\bar{Q}^2 Q^2)$ mesons is investigated. Decay modes to two pseudoscalars and to four pseudoscalars (via virtual vector-meson pairs) are analyzed. The proposed interpretation is found to have a fair chance of being correct. Further consequences of the model concerning a narrow isoscalar at 1930 MeV and the three doubly charged exotics are presented.

PACS numbers: 14.40.Cs, 12.40.Aa

In addition to the filled nonet, there exist at present two isoscalar, nonstrange mesons with $J^P = 2^+$; namely, the θ at 1720 MeV with width 130 MeV observed in the J/ψ decays^{1,2} and the ζ at 1480 MeV with width 116 MeV reported in $\bar{p}n$ annihilation.^{3,4} The dominant decay mode of $\zeta(1480)$ is to four pions, via a virtual ρ pair. It also has a comparatively small two-pion mode. There is a strong likelihood that the coupling of $\theta(1720)$ to a pseudoscalar pair is also comparatively weak. Indeed, the total absence of the θ in hadronic production experiments [in $\pi^- p \rightarrow K_S^0 K_S^0 n$ (Ref. 5), in $\pi^- p \rightarrow \eta\eta n$ (Ref. 6), and in $K^- p \rightarrow K\bar{K}\Lambda$ (Ref. 7)] would strongly imply this possibility [within the one-pion-exchange (OPE) mechanism]. Thus, θ and ζ appear to possess common characteristics. Could the two states be radial excitations of the established 2^+ nonet? The answer is no; experimental masses are inconsistent with the expected⁸ pattern of masses. Could θ be a glue ball? This possibility is ruled out by the strong suppression of its $\pi\pi$ mode, compared to the $K\bar{K}$, as reported in Ref. 5. The preference for two-vector modes shown by ζ suggests a $(\bar{Q}^2 Q^2)$ interpretation. However, the states concerned do not quite fit into the well-known model of Jaffe⁹: The masses are significantly lower (ζ below $\rho\rho$ threshold) and the widths are appreciably narrower than the predictions of the model. Moreover, the small but significant two-pseudoscalar modes cannot be treated naturally in the model. What one thus needs is a scheme that retains only the general features of the Jaffe model but not its

detailed predictions. Such a scheme has been available for a long time and is, in fact, given by the present author's model^{10,11} of meson multiplets based on the representation $(\bar{6}, 6)$ of the nonchiral flavor group $SU_{\bar{q}}(3) \otimes SU_q(3)$. This then will be the theoretical framework in which we shall operate.

In the notation of Ref. 11, the $(\bar{6}, 6)$ multiplet is described by a tensor $G_{\alpha\beta}^{ab}$ where the upper (antiquark) and the lower (quark) indices take values 1, 2, and 3 corresponding to the quark designations u , d , and s . We identify the ζ with the component $G_{11}^{11} + G_{22}^{22} + 2G_{12}^{12}$ and the θ with $G_{31}^{31} + G_{32}^{32}$. The mass formula^{10,11} gives five equally spaced levels; thus the two input masses determine the spectrum. Degenerate with ζ are isomultiplets $I=2, Y=0$, and $I=1, Y=0$. Degenerate with θ are states $I=1, Y=0$ and $I=1, Y=\pm 2$. States $I=\frac{3}{2}, Y=\pm 1$ and $I=\frac{1}{2}, Y=\pm 1$ occur at 1600 MeV. The level at 1830 MeV has $I=\frac{1}{2}, Y=\pm 1$ and the topmost state G_{33}^{33} is located at 1930 MeV. These estimates are with a quadratic mass formula; the use of a linear formula would alter the position of the two top states to 1840 and 1960 MeV.

We proceed to analyze the two-pseudoscalar decays of ζ and θ states. We adopt the scheme of Ref. 11, which has an identical flavor structure as that for the Okubo-Zweig-Iizuka (OZI) superallowed, "fall-apart" modes of Ref. 9, to couple the 36-plet to two nonets. Pseudoscalar mesons are not ideally mixed; we express¹² the physical states in terms of the ideal nonet states via the mixing angle θ_P . We thus obtain the decay amplitudes

$$\begin{aligned}
 A(\theta \rightarrow \pi\pi) &= 0 = A(\zeta \rightarrow K\bar{K}), & A(\theta \rightarrow K^+ K^-) &= \left(\frac{3}{2}\right)^{1/2} A(\zeta \rightarrow \pi^+ \pi^-), \\
 A(\theta \rightarrow \eta\eta) &= (6)^{-1/2} (\sin 2\theta_P - 2\sqrt{2} \cos 2\theta_P) A(\zeta \rightarrow \pi^+ \pi^-), \\
 A(\zeta \rightarrow \eta\eta) &= (2)^{-1/2} (1 + \sin^2 \theta_P - \sqrt{2} \sin 2\theta_P) A(\zeta \rightarrow \pi^+ \pi^-), \\
 A(\theta \rightarrow \eta\eta') &= (3)^{-1/2} (\cos 2\theta_P + 2\sqrt{2} \sin 2\theta_P) A(\zeta \rightarrow \pi^+ \pi^-).
 \end{aligned} \tag{1}$$

The above, with $\theta_P = -10^\circ$ and the experimental masses,² gives the desired partial widths. Thus the ratio $K\bar{K}:\eta\eta$ for the θ is 2.7, which is consistent with the branching ratios quoted by Longacre *et al.*⁵: $B(\theta \rightarrow K\bar{K}) = 0.19^{+0.27}_{-0.01}$ and $B(\theta \rightarrow \eta\eta) = 0.05^{+0.16}_{-0.01}$ (fit I). Equation (1) leads to $\Gamma(\theta \rightarrow \eta\eta) \cong 0.46\Gamma(\zeta \rightarrow \pi\pi)$. This crucial prediction is tested as follows. Following Dover,¹³ the f_2' of Gray *et al.*¹⁴ is identified with the $\pi\pi$ mode of ζ . Use of ex-

perimental¹⁵ branching ratios leads to $\Gamma(\zeta \rightarrow 2\pi) \cong 0.04\Gamma(\zeta \rightarrow 2\rho \rightarrow 4\pi)$. Setting further¹⁶ $\Gamma(\zeta \rightarrow 2\rho \rightarrow 4\pi) \approx \Gamma(\zeta \rightarrow \text{all}) = 116$ MeV, we obtain the rough estimate $\Gamma(\zeta \rightarrow \pi\pi) \cong 4.6$ MeV. Thus finally $\Gamma(\theta \rightarrow \eta\eta) \approx 2.1$ MeV, a result which is a factor of 2 smaller than the lower limit implied by the Longacre result,⁵ which is $4.6 \leq \Gamma(\theta \rightarrow \eta\eta) \leq 40$ MeV. In view of the many uncertainties that are involved, e.g., regarding the $\zeta \rightarrow \pi\pi$ mode,^{14,15} as well as the possibility that $\Gamma(\theta \rightarrow \eta\eta)$ has been overestimated due to contamination from $G(1590) \rightarrow \eta\eta$ events,^{12,17} it would perhaps be fair to conclude that our proposal is not inconsistent with existing data. The absence of $\theta \rightarrow \pi\pi$ and $\zeta \rightarrow K\bar{K}$ is in good accord with the conclusion of Refs. 5 and 14. We consider questions concerning ζ in the J/ψ radiative decays. The estimate $\Gamma(\zeta \rightarrow \eta\eta) \cong 0.2\Gamma(\theta \rightarrow \eta\eta)$ follows from Eq. (1). Assuming comparable production we thus expect a moderate ζ signal in the $\eta\eta$ channel but the effect is going to be difficult to disentangle from the $f'(1525)$. The most logical place to look for ζ would be in the $\rho\rho$ channel; unfortunately here it is masked by the much stronger presence^{18,19} of the iota (1460). The remaining $\pi\pi$ channel is particularly convenient since neither the iota ($J^P = 0^-$) nor the $f'(1525)$ [$f'(1525) \rightarrow \pi\pi$ is strongly suppressed²] can be present here. Interestingly enough, a structure in the $\pi\pi$ spectrum up-

ward of $f(1270)$ is apparently being seen¹⁸ in the four experiments Mark II, Crystal Ball, Mark III, and DM2. *It is tempting to identify this structure with the 2π mode of $\zeta(1480)$.* To be more specific, consider a model for J/ψ radiative decays in which the photon is radiated by the initial state thereby projecting out the flavor singlet component²⁰ of the tensor mesons. The ideal mixing parameters^{9,11} now give for the ζ to θ production ratio the value 1.5 (uncorrected by phase space). This, together with the estimate $\Gamma(\zeta \rightarrow \pi\pi) \cong 0.8\Gamma(\theta \rightarrow K\bar{K})$ that also follows from Eq. (1), leads to

$$\frac{B(J/\psi \rightarrow \gamma\zeta)B(\zeta \rightarrow \pi\pi)}{B(J/\psi \rightarrow \gamma\theta)B(\theta \rightarrow K\bar{K})} = 1.2 \frac{\Gamma(\theta \rightarrow \text{all})}{\Gamma(\zeta \rightarrow \text{all})} \cong 1.3. \quad (2)$$

Experimental information regarding the above prediction is extracted from the data of Ref. 5. From the fitted curve in Fig. 3(c) and the data points in Fig. 3(b), we estimate this ratio to be 1.3 ± 0.3 , which is interesting.

The decay of a tensor meson into four pseudoscalars via a virtual vector-meson pair $2^+ \rightarrow 1^- + 1^- \rightarrow 4\ 0^-$ will now be calculated. Write the first vertex as $G\epsilon_\mu\epsilon^\mu\epsilon^\nu$ in terms of the relevant polarization tensors. The $1^- \rightarrow 0^- + 0^-$ vertex is written $g\epsilon_\mu(P_1 - P_2)^\mu$; here the P 's are pseudoscalar four-momenta. It is then not difficult to derive the following expression²¹ for the partial width:

$$\Gamma(2^+ \rightarrow 1^- + 1^- \rightarrow 4\ 0^-) = \frac{C}{M} G^2 g_A^2 g_B^2 \int_{(m_1+m_2)^2}^{(M-m_1-m_2)^2} ds_{12} \int_{(m_3+m_4)^2}^{[M-(s_{12})^{1/2}]^2} ds_{34} \frac{P}{M} F \frac{P^* 3}{s_{12}^{3/2}} \frac{1}{(s_{12}-m_A^2)^2 + m_A^2 \Gamma_A^2} \times \frac{Q^* 3}{s_{34}^{3/2}} \frac{1}{(s_{34}-m_B^2)^2 + m_B^2 \Gamma_B^2}, \quad (3)$$

where

$$P = \frac{\lambda^{1/2}(M^2, s_{12}, s_{34})}{2M}, \quad P^* = \frac{\lambda^{1/2}(s_{12}, m_1^2, m_2^2)}{2(s_{12})^{1/2}}, \quad Q^* = \frac{\lambda^{1/2}(s_{34}, m_3^2, m_4^2)}{2(s_{34})^{1/2}}, \quad (4)$$

$$F = (2P^2 + 5s_{12})(2P^2 + 5s_{34}) + 5s_{12}s_{34}, \quad \lambda(x, y, z) = x^2 - 2x(y+z) + (y-z)^2. \quad (5)$$

In the above, m_i ($i=1-4$) are the 0^- masses, m_A and m_B are the 1^- (labeled A and B) masses and Γ_A and Γ_B are the corresponding widths, M is the decaying 2^+ mass, and C is an overall constant. The constants g_A and g_B are determined from relevant $1^- \rightarrow 0^- + 0^-$ partial widths; G is given up to an overall constant by the coupling scheme.¹¹ Computing the integral numerically we can thus calculate the desired partial widths to within an overall constant. We thus compute

$$\Gamma(\theta \rightarrow K^* K^* \rightarrow K^+ K^- 2\pi^0) \cong 0.1\Gamma(\zeta \rightarrow \rho^0 \rho^0 \rightarrow 2\pi^+ 2\pi^-)$$

and hence

$$\Gamma(\theta \rightarrow K^* \bar{K}^* \rightarrow K\bar{K} 2\pi) \cong 0.6\Gamma(\zeta \rightarrow \rho\rho \rightarrow 4\pi) \approx 70 \text{ MeV},$$

a result of the right order. If $K^* \bar{K}^*$ is the prominent mode for θ , as this result would indicate, then the most promising setup for hadronic production of θ would be in the $K\bar{K} 2\pi + \Lambda$ channel in the $K^- p$ collisions.

We proceed to discuss further consequences of our proposal. The topmost state G_{33}^{33} is at 1930 MeV (at 1960 MeV if linear mass formula is used). Its dominant decay mode would have been to $\phi\phi \rightarrow 2K 2\bar{K}$, which is kinematically forbidden. Thus it must be very narrow. As regards its decay into a 0^- pair, the modes $\pi\pi$ and $K\bar{K}$ are forbidden.¹¹ For the allowed modes we obtain [as in Eq. (1)]

$$A(G_{33}^{33} \rightarrow \eta\eta) = 2 \frac{1 + \cos^2\theta_P + \sqrt{2}\sin 2\theta_P}{\sin 2\theta_P - 2\sqrt{2}\cos 2\theta_P} A(\theta \rightarrow \eta\eta), \quad A(G_{33}^{33} \rightarrow \eta'\eta') = -\sqrt{2}A(\theta \rightarrow \eta\eta),$$

$$A(G_{33}^{33} \rightarrow \eta'\eta) = 2 \frac{1 + \sin^2\theta_P - \sqrt{2}\sin 2\theta_P}{\sin 2\theta_P - 2\sqrt{2}\cos 2\theta_P} A(\theta \rightarrow \eta\eta). \quad (6)$$

The above three modes add up to a contribution of $2.84\Gamma(\theta \rightarrow \eta\eta)$ to the G_{33}^{33} width. Taking $\Gamma(\theta \rightarrow \eta\eta) \approx 5$ MeV (say), we estimate this contribution to be around 14 MeV. Because this contribution is so small, it is necessary to estimate also the small contributions to the width arising out of decays to channels other than those considered, notably $\phi\eta$ and $K^*(890)K$ channels; but we do not attempt it here. This state is expected in the J/ψ radiative decays; the production ratio for this particle to the θ is 0.5 in the model²⁰ mentioned before [remarks preceding Eq. (2)]. The experimental situation in the 2-GeV region is shrouded in mystery; recall the conflicting claims^{22,23} made by Mark III and DM2 regarding the $\xi(2230)$.

The isospinor state G_{3i}^{33} ($i=1,2$) at 1830 MeV also deserves mention. Its dominant mode is to $\pi K K \bar{K}$ via the virtual $K^*\phi$ pair. The partial width for this mode, calculated from Eq. (3), is 0.2 times the partial width $\Gamma(\zeta \rightarrow 4\pi)$, i.e., about 25 MeV. Thus this is also very narrow.

The remainder of this paper will be devoted to the exotic states in the spectrum. These are the $I=2, Y=0$ state at 1480 MeV; $I=\frac{3}{2}, Y=\pm 1$ at 1600 MeV; and $I=1, Y=\pm 2$ at 1720 MeV. Each of these isomultiplets has a doubly charged member, denoted here as $f_2^{++}(1480)$, $f_2^{++}(1600)$, and $f_2^{++}(1720)$. The relation

$$\Gamma(f_2^{++}(1480) \rightarrow \rho^+\rho^+ \rightarrow 2\pi^+2\pi^0) \\ = 4\Gamma(\zeta \rightarrow 2\rho \rightarrow 4\pi) (\cong 446 \text{ MeV})$$

$$\frac{\sigma(\pi^+p \rightarrow f_2^{++}(1480)n)}{\sigma(\pi^-p \rightarrow f(1270)n)} \cong \left(\frac{g_{f_2^{++}\pi^+\pi^+}}{g_{f\pi^+\pi^-}} \right)^2 = 0.6 \frac{\Gamma(f_2^{++}(1480) \rightarrow \pi^+\pi^+)}{\Gamma(f(1270) \rightarrow \pi^+\pi^-)}. \quad (7)$$

Taking $\Gamma(f(1270) \rightarrow \pi^+\pi^-) = 99$ MeV (Ref. 2) and our estimate for $f_2^{++}(1480) \rightarrow \pi^+\pi^+$ we obtain for the above ratio the value 0.11. However, this is a *very* difficult experiment. The related annihilation experiment $\bar{p}n \rightarrow \pi^+f_2^{--}(1480)$ looks somewhat less difficult. Prospects are much brighter for the other two exotics. The π^+p collision may also be used to look for $f_2^{++}(1600)$, in the $K3\pi\Lambda^0$ final states. As before, we obtain

$$\sigma(\pi^+p \rightarrow f_2^{++}(1600)\Lambda^0) \cong 0.3\sigma(\pi^-p \rightarrow K^*(1430)\Lambda^0). \quad (8)$$

The state $f_2^{++}(1720)$ can appear in K^+p collisions leading to $2K2\pi\Lambda^0$ final states. We estimate

$$\sigma(K^+p \rightarrow f_2^{++}(1720)\Lambda^0) \cong 0.17\sigma(K^-p \rightarrow f'(1525)\Lambda^0). \quad (9)$$

In making the above estimates, we used² $\Gamma(K^{*0} \rightarrow K^+\pi^-) = 29.7$ MeV and $\Gamma(f'(1525) \rightarrow K^+K^-) = 35$ MeV.

A possible (\bar{Q}^2Q^2) interpretation for $\zeta(1480)$ was considered earlier by Li and Liu²⁴ in connection with the photon-photon collision data. Here we have attempted this interpretation for $\zeta(1480)$ and $\theta(1720)$ and analyzed available data using what we believe to be a minimal theoretical input. Considering the many uncertainties in the data, our results are not too bad. The narrow isoscalar at around 1930, if present, would require a downward revision of the $\theta \rightarrow \eta\eta$ branching ratio. Probably, only in this way will the conflict with hadroproduction data be resolved. All this, however, is rather in-

direct, as far as the (\bar{Q}^2Q^2) structure is concerned. A direct test is provided by the doubly charged particles whose hadronic production we discussed.

$$\Gamma(f_2^{++}(1600) \rightarrow \rho^+K^{*+} \rightarrow K+3\pi) \cong 336 \text{ MeV}.$$

Its partial width to $K^+\pi^+$ is about 15 MeV. Thus this state is also fairly broad. In the same fashion we obtain

$$\Gamma(f_2^{++}(1720) \rightarrow K^{*+}K^{*+} \rightarrow 2K2\pi) \cong 120 \text{ MeV}.$$

Its partial width for K^+K^+ decay is about 12 MeV. Thus $f_2^{++}(1720)$ should be fairly narrow. The comparative largeness of the 0^- -pair modes of these exotics is due to a large factor (=12 as compared to $\zeta^0 \rightarrow \pi^0\pi^0$) arising out of our coupling scheme.¹¹ This will cause a corresponding enhancement of the hadronic production cross sections, which we now proceed to consider.

The state $f_2^{++}(1480)$ could appear in π^+p collisions leading to $2\pi^+2\pi^0n$ final states. To estimate the size of the effect we proceed as in Ref. 12. We assume the OPE mechanism and express the production cross section in terms of that of $f(1270)$ in π^-p collision at the same incident energy. Then

In conclusion, we should justify our model. The original motivation for $SU_{\bar{q}}(3) \otimes SU_q(3)$ came from the utility of its $(\bar{\mathbf{3}}, \mathbf{3})$ representation in explaining the $(\bar{Q}Q)$ no-net structure.¹¹ With the advent of QCD as the theory of strong interactions, and the development of various QCD-based models for hadrons, the physical basis of our proposal become clear. In a model for conventional hadrons built upon the one-gluon exchange plus a long-range confining potential (whose form is prescribed by lattice QCD), De Rújula, Georgi, and Glashow²⁵ explicitly

show the emergence of supermultiplets of $SU_{\bar{q}}(6) \otimes SU_q(6) \otimes O(3)$ and thus of $SU_{\bar{q}}(3) \otimes SU_q(3)$ for multiplets of fixed spin. There are many variants of this model, including the ones with $(\bar{Q}^2 Q^2)$ mesons²⁶, all have the flavor group $SU_{\bar{q}}(3) \otimes SU_q(3)$, which is also the classifying group of the Jaffe model⁹ as well as of a model based on one-gluon exchange plus dual unitarization.²⁷ We can understand the emergence of $SU_{\bar{q}}(3) \otimes SU_q(3)$ in QCD as follows. The flavor group $SU_{\bar{q}}(3) \otimes SU_q(3)$, needed for the enumeration of color singlet states of an assembly of quarks and antiquarks, evidently becomes a symmetry if the quark-antiquark forces are disregarded (and quark mass differences neglected). The inclusion of the quark-antiquark force does not change the conclusion; the force depends, in the first approximation, on the color configuration and not at all on flavor. In higher order, an effective flavor-dependent force that breaks the symmetry down to a diagonal $SU(3)$ arises via quark-antiquark virtual annihilation into two or more gluons but this has a significant effect *only* on the pseudoscalar mesons.²⁵ In QCD, the remaining freedom to break the flavor symmetry is through the quark mass differences but this has the exact tensorial behavior $(\mathbf{8}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{8})$ under the product group and leads to our mass formula.^{10,11} The splitting within our $(\bar{\mathbf{6}}, \mathbf{6})$ multiplet is about 120 MeV which is roughly the mass difference between the s and u, d quarks. The overall mass scale is difficult to establish in a model independent way. The bag-model prediction⁹ for $\zeta(1480)$ is 1650 MeV; the discrepancy, although leading to important phenomenological consequences, is nonetheless about 10%. Actually, a mass slightly below the threshold is exactly what is expected if $\zeta(1480)$ were an s -wave $(\rho\rho)$ molecule in the fashion of Ref. 28. All in all, the overall mass scale appears roughly correct.

We thank N. N. Biswas, N. Cason, and V. Barger for conversations. Throughout, we received help from Z. Ajaltouni concerning the experimental data. Numerical evaluation of integrals was done by Sudesh Bose to whom we are much indebted.

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¹⁶The only other two-vector mode for ζ allowed by our coupling is to $\omega\omega$. A rough estimate for this is

$$\Gamma(\zeta \rightarrow \omega\omega \rightarrow 6\pi) / \Gamma(\zeta \rightarrow \rho\rho \rightarrow 4\pi) \approx 9\Gamma^2(\omega \rightarrow 3\pi) / \Gamma^2(\rho \rightarrow 2\pi),$$

the factor coming from the coupling scheme. Thus the 6π mode is very small. A more detailed consideration does not change this conclusion.

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