Matter Parity, Intermediate Scale Breaking, and $\sin^2\theta_W$ in Calabi-Yau Superstring Models

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(Received 30 November 1988)

Consequences of matter-parity invariance on intermediate scale breaking in three-generation Calabi-Yau superstring models are discussed. It is shown that values of $\sin^2\theta_W$ in conformity with the current experimental limits can arise in the Calabi-Yau models when the extra families and mirror families gain superheavy masses through nonrenormalizable interactions. A small number of exotic leptons with electroweak-scale masses remain and may be accessible at accelerator energies.

PACS numbers: 12.10.Dm

I. Introduction.—The Calabi-Yau compactification[1] of the $E_8\times E_8'$ heterotic string[2] possesses many appealing features as a viable candidate for a unified theory of particle interactions in four dimensions. The compactification leaves intact one $N=1$ supersymmetry at the compactification scale M_c. Further, the Calabi-Yau manifold $CP^3\times CP^3/Z_3$ contains three quark-lepton generations which belong to the nonet representations of $SU(3)_C\otimes SU(3)_L\otimes SU(3)_R$. This $[SU(3)]^3$ symmetry is expected to break further to the standard-model gauge-group symmetry $SU(3)_C\otimes SU(2)_L\otimes U(1)_Y$ at an intermediate scale[3,4] M_I due to the vacuum-expectation-value (VEV) growth of the SO(10)-singlet field N and the SU(5)-singlet field ν^c in the nonet representation $(1,3,\bar{3})$.

The analysis of this Letter is within the framework of matter-parity invariance[4,5] which is useful in eliminating some of the most dangerous proton-decay interactions.[6] (See Refs. 7 and 8 for analyses without this invariance.) The three-generation Calabi-Yau manifold depends on nine complex parameters and the Yukawa couplings here are not known. For the zero-parameter (symmetric) Calabi-Yau manifold, Yukawa couplings are known[9] but only modulo the normalizations of the fields. (However, see Ref. 10 for recent progress on the full determination of the coupling structure.) In this analysis we use the symmetries and the spectrum of the $CP^3\times CP^3/Z_3$ manifold but determine the constraints on the couplings in order that they be in conformity with experiment.

II. Matter-parity invariance and N and ν^c VEV growth.—The massless spectrum of the theory just below the compactification scale consists of the following nonets of leptons, quarks, conjugate quarks, and mirrors: $9L+6\bar{L}$, $7Q+4\bar{Q}$, and $7Q^c+4\bar{Q}^c$, where $L=(1,3,\bar{3})$, $Q=(3,\bar{3},1)$, $Q^c=(\bar{3},1,3)$, and $\bar{L}=(1,\bar{3},3)$, etc. in $[SU(3)]^3$ representation. The $[SU(3)]^3$-invariant superpotential of the theory has the general form (suppressing the generation indices)

$$W=(27)^3+(\overline{27})^3+\lambda M_c^{-2n+3}(27\times\overline{27})^n+\cdots, \quad (2.1a)$$

$$(27)^3=\lambda^1\det Q+\lambda^2\det L+\lambda^3\det Q^c-\lambda^4\det(QLQ^c), \quad (2.1b)$$

where $M_c\sim M_{\rm Pl}$ and $M_{\rm Pl}=2.4\times10^{18}$ GeV and a formula similar to Eq. (2.1b) holds for $(\overline{27})^3$. As discussed above, matter-parity[4,5] invariance is an important superstring model building constraint. Matter parity M_2 is defined so that $M_2=CU_z$ where C is the transformation,

$$C=(1,1,\sigma)\otimes(1,1,\sigma),\quad \sigma=\begin{pmatrix}0&1\\1&0\end{pmatrix},$$

which acts on the Calabi-Yau coordinates $(x^0,x^1,x^2,x^3)\otimes(y^0,y^1,y^2,y^3)$ and U_z is given by

$$\mathrm{diag}(U_z)=(1,1,1)\otimes(-1,-1,1)\otimes(-1,-1,1)$$

is an element of $[SU(3)]^3$. The components of the 27-plet[11] have the following transformations under U_z: $q^\alpha,l^\alpha,u^c,d^c,e^c,\nu^c$ (U_z odd) and $H^\alpha,H_\alpha',D,D^c,N$ (U_z even). It is useful to form linear combinations which are eigenstates of C as is shown in Table I.[4]

The M_2 properties of all the particles and mirror par-

TABLE I. List of C-even and C-odd families in the notation of Ref. 4 [$L_{1\pm}=(L_1\pm L_2)/\sqrt{2}$, etc.].

C even	C odd
$L_{1+},L_{3+},L_5,L_7,L_{8+}$	L_{1-},L_{3-},L_6,L_{8-}
$Q_1,Q_2,Q_3,Q_{4+},Q_{6+}$	Q_{4-},Q_{6-}
$Q_1^c,Q_2^c,Q_3^c,Q_{4+}^c,Q_{6+}^c$	Q_{4-}^c,Q_{6-}^c
$\bar{L}_1,\bar{L}_2$	$\bar{L}_3,\bar{L}_4,\bar{L}_5,\bar{L}_6$
$\bar{Q}_{1+},\bar{Q}_{3+}$	$\bar{Q}_{1-},\bar{Q}_{3-}$
$\bar{Q}_{1+}^c,\bar{Q}_{3+}^c$	$\bar{Q}_{1-}^c,\bar{Q}_{3-}^c$

ticles can be obtained from U_z transformations and Table I. Since N is U_z even while ν^c is U_z odd, matter-parity invariance below the intermediate scale requires that $\langle \nu^c_{C\,\mathrm{even}}\rangle = 0 = \langle N_{C\,\mathrm{odd}}\rangle$. This implies that matter-parity invariance in the intermediate-mass-scale breaking at M_I requires at least two lepton multiplets, one C even (L_1) and the other C odd (L_2). It is N_1 contained in L_1 and ν_2^c contained in L_2 that generate the desired breaking of $[\mathrm{SU}(3)]^3$ to the standard model. In the present analysis we shall consider the minimal model with just the lepton multiplets L_1, L_2, and a nonrenormalizable interaction of the form

$$W_{\rm nr} = \sum_{\lambda=1,2} \lambda_i^2 (M_c)^{-2n_i+3} (L_{ir}^l \bar{L}_{ir}^l) n_i \,. \tag{2.2}$$

As shown in Ref. 12, matter-parity invariance is then automatically preserved by the intermediate-mass-scale breaking, for the $\mathrm{SU}(2)\times\mathrm{U}(1)$-invariant solution.

The effective potential which governs the symmetry breaking at the intermediate mass scale consists of $V = V_m + V_F + V_D$, where V_F is obtained from Eq. (2.2). V_m is a mass term and V_D is the D term generated from the $\mathrm{SU}(3)_L \otimes \mathrm{SU}(3)_R$ gauge transformations. The soft (mass)^2 terms of V_m arising from supersymmetry (SUSY) breaking (for example, from gaugino masses) can turn negative through renormalization-group effects. This can happen sufficiently close to the compactification scale to trigger intermediate scale breaking at a rather high scale.[13] We take V_m of the form

$$V_m = -\sum_i m_i^2 L_{ir}^{l\dagger} L_{ir}^l - \sum_i \bar{m}_i^2 \bar{L}_{ir}^{l\dagger} \bar{L}_{ir}^l \,, \tag{2.3}$$

which is the most general structure consistent with $[\mathrm{SU}(3)]^3$ invariance. V_D is given by

$$V_D = \tfrac{1}{8} \sum_{\alpha=1}^{8} (D_L^\alpha D_L^{\alpha\dagger} + D_R^\alpha D_R^{\alpha\dagger}) \,, \tag{2.4a}$$

$$D_L^\alpha = \tfrac{1}{2} g_L \sum_i (L_{ir}^{l\dagger} L_{ir}^{l'} - \bar{L}_{ir}^l \bar{L}_{ir}^{l'\dagger})(\lambda^\alpha)_{l'}^l \,, \tag{2.4b}$$

$$D_R^\alpha = -\tfrac{1}{2} g_R \sum_i (L_{ir}^{l\dagger} L_{ir'}^l - \bar{L}_{ir}^l \bar{L}_{ir'}^{l\dagger})(\lambda^\alpha)_r^{r'} \,, \tag{2.4c}$$

where λ^α are the Gell-Mann matrices. Using the above effective potential one finds that the solutions to the extrema equations show that the $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$-preserving solutions with lowest energy automatically preserve matter parity. The nonvanishing VEV's are, to leading order,

$$\langle N_1 \rangle \simeq \left[\frac{\Sigma_1^2 M_c^{4n-6}}{2n_1^2 (2n_1 - 1)\lambda_1^4} \right]^{1/(4n_1-4)} ,$$
$$\langle \nu_2^c \rangle = \left[\frac{\Sigma_2^2 M_c^{4n-6}}{2n_2^2 (2n_2 - 1)\lambda_2^4} \right]^{1/(4n_2-4)} , \tag{2.5}$$

and $\langle \bar{N}_1 \rangle \simeq \langle N_1 \rangle$, $\langle \bar{\nu}_2^c \rangle \simeq \langle \nu_2^c \rangle$ where $\Sigma_i^2 = m_i^2 + \bar{m}_i^2$. In Eq. (2.5) we have not displayed the $O(m_i, \bar{m}_i)$ contributions which mark the deviation from D flatness and play an important role in determining the lowest-energy vacuum solutions. The full results are given in Ref. 12. From Eq. (2.5) it is easily seen that with an appropriate set of parameters, i.e., $\Sigma_i \sim 10^3$ GeV, $\lambda_i \sim 10^{-2}$, and $n_i > 2$, values of N_1 and ν_2^c VEV's of order 10^{15} GeV can be achieved.

III. Low-energy spectrum and $\sin^2\theta_W$.—Spontaneous symmetry breaking at the intermediate mass scale reduces the $\mathrm{SU}(3)_C \otimes \mathrm{SU}(3)_L \otimes \mathrm{SU}(3)_R$ gauge symmetry to the standard-model gauge symmetry $\mathrm{SU}(3)_C \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$. Consequently twelve vector bosons become superheavy by absorbing twelve linear combinations of the L_i and $\bar{L}_i$ multiplets. Further, an analysis of the scalar boson mass growth from the D term of Eq. (2.4) shows that twelve linear combinations of L_i and $\bar{L}_i$ become superheavy with mass of $O(M_I)$. This removes $12+12$ components out of a total of 72 real components of $L_i, \bar{L}_i$. Of the remaining 48 components, 32 components belong to the Higgs doublet fields H_i^α, $H'_{\alpha i}$, $\bar{H}_i^\alpha$, and $\bar{H}'_{\alpha i}$. An analysis of the cubic interactions shows that one has an interaction of the type $\lambda_{AB1}^3 H_A^\alpha H'_{\alpha B} \langle N_1 \rangle$ which implies that all Higgs doublets for which λ_{AB1}^3 are nonzero would become superheavy. The fact that we need to break $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ at the electroweak scale requires that at least one pair of Higgs doublets remain massless below the intermediate scale. An analysis of the cubic couplings at least on the symmetric $CP^3 \times CP^3/Z_3$ Calabi-Yau manifold[9] shows that this is indeed the case. We shall assume that the cubic couplings leave only one pair of Higgs doublets massless which is just the standard set. Thus the leptons from the 72 components of L_i, $\bar{L}_i$ (on eliminating the 32 components of H_i^α, $H'_{\alpha i}$, $\bar{H}_i^\alpha$, and $\bar{H}'_{\alpha i}$) which remain light are 16 in number: $72-(12+12+32)$. The light modes are[14] the singlet fields $\mathrm{Im}(\nu_2^c + \bar{\nu}_2^c)$, $\mathrm{Im}(N_1 + \bar{N}_1)$, N_2, $\bar{N}_2$, $\mathrm{Re}(N_1 + \bar{N}_1)$, $\mathrm{Re}(\nu_2^c + \bar{\nu}_2^c)$, and the $\mathrm{SU}(2)_L$ lepton doublets l_2^α and $\bar{l}_2^{\alpha\dagger}$. Actually the lightness of the two doublets depends on the λ^3 coupling structure since the ν_2^c VEV growth generates a mass term of the form $\lambda_{A2B}^3 H_A^\alpha l_{\alpha B} \langle \nu_2^c \rangle$. Thus, for example, the lepton doublet l_2^α would remain light if λ_{221}^3 vanishes but would become superheavy if λ_{221}^3 is nonzero.

The analysis of the mass matrix from the remaining generations and mirror generations shows the presence of three massless generations as expected. However, a straightforward extension of Eq. (2.2) where the sum runs over all families of quarks and leptons and mirror families yields too many light particles, with masses of order of the electroweak-mass scale. Such a situation produces a value of $\sin^2\theta_W$ which is too large. It was pointed out in Ref. 14 that superheavy mass growth for the unwanted extra families and mirror families can occur from the nonrenormalizable sector of the theory. This would occur naturally if the degree of the nonrenormalizable interactions responsible for mass growth for the unwanted sector is lower than n_i of Eq. (2.2). Thus,

TABLE II. Values of $\sin^2\theta_W(M_W)$, $\alpha_3(M_c)$, and $\alpha_{3L}(M_c)$ in one-loop approximation using $\alpha_{em}(M_W)=\frac{1}{128}$ and $\alpha_3(M_W)=0.11$.

M_I (GeV)	M_I/M_c	$\sin^2\theta_W(M_W)$	$\alpha_3(M_c)$	$\alpha_{3L}(M_c)$
10^6	1	0.244	0.0407	0.0478
	0.5		0.0456	0.0568
	0.1		0.0634	0.1008
5×10^{16}	1	0.237	0.0395	0.0511
	0.5		0.0441	0.0615
	0.1		0.0605	0.1165
10^{17}	1	0.235	0.0300	0.0526
	0.5		0.0434	0.0638
	0.1		0.0592	0.1250

for example, the interaction

$$\Delta W_{\rm nr}=\sum_{A\neq i}\lambda_A M_c^{-2n_A+3}(L_i\bar{L}_i)^{n_A-1}(L_A\bar{L}_A) \qquad (3.1)$$

can generate masses $\sim10^{14}$-10^{15} GeV, when $n_i\gtrsim3$, $n_A=2$, $\lambda_A=1$, $\Sigma_i=10^3$ GeV, and $\lambda_i\simeq10^{-2}$. Thus the superpotential of Eqs. (2.2) and (3.1) shifts the masses of the unwanted generations close to the intermediate mass scale and eliminates them from the low-energy domain. Further, the cubic interactions of Eq. (2.1b) generate superheavy masses for all the Higgs triplet fields D^a, D_a^c (including the ones from the three massless generations) when N_1 develops a VEV from the $\lambda^4 D^a D_a^c N_1$ interaction. Thus when Eq. (3.1) holds, the low-energy theory will consist of the spectrum of the minimal $N=1$ SUSY theory with only the additional exotic leptons stated above.

We next discuss the evaluation of $\sin^2\theta_W$. Below the intermediate mass scale, in the domain $m_W\leq\mu\leq M_I$, the gauge coupling constants $g_i=g_3,g_2,g_Y$ corresponding to the gauge group $SU(3)_C\otimes SU(2)_L\otimes U(1)_Y$ obey the following one-loop renormalization-group equations: $\mu(dg_i/d\mu)=b_i g_i^3/16\pi^2$, where $b_3=-9+(n_Q+\bar{n}_Q)$, $b_2=-6+\frac{1}{2}(n_A+\bar{n}_A)$, and $b_Y=\mathrm{Tr}(Y_L^2+Y_R^2)$. Here n_Q ($\bar{n}_Q$) is the number of color triplets from the families (mirror families), n_A ($\bar{n}_A$) is the number of quark, lepton, and Higgs $SU(2)_L$ doublets from the families (mirror families), and the trace in b_Y runs over all the particles and mirror particles which are light. Since in the domain $\mu<M_I$, the spectrum of light states is just the spectrum of $N=1$ minimal SUSY theory along with the additional exotic leptons, one has $n_Q=6$, $\bar{n}_Q=0$, $n_A=15$, $\bar{N}_A=1$, and $b_Y=12$. Above the intermediate mass scale in the domain $M_I\leq\mu\leq M_c$, the gauge group is $SU(3)_C\otimes SU(3)_L\otimes SU(3)_R$ and the massless modes consist of the full spectrum of families and mirror families on $CP^3\times CP^3/Z_3$, i.e., $9L+6\bar{L}$, $7Q+4\bar{Q}$, and $7Q^c+4\bar{Q}^c$. The b_3 for the color coupling constant in this domain is given by $n_Q=21$, $\bar{n}_Q=12$ while $b_{3L}=-9+n_L+\bar{n}_L$, where $n_L=24$ and $\bar{n}_L=15$ and an identical relation holds for b_{3R}. Using the boundary conditions $\alpha_3(M_W)=0.11$, $\alpha_{em}(M_W)=\frac{1}{128}$, and the value of $\frac{2}{8}$ for $\sin^2\theta_W$ at $\mu=M_I$, we may compute the value of $\sin^2\theta_W$ at $\mu=M_W$ and the values of α_3 and α_{3L} (α_{3R}) at the compactification scale M_c. The results are exhibited in Table II.

We note that at the one-loop approximation the value of $\sin^2\theta_W(M_W)$ is decoupled from the physics above M_I. {This is due to the fact that we have assumed $\alpha_{3L}=\alpha_{3R}$ at $\mu\rightarrow M_c$. The $[SU(3)]^3$ invariance then maintains this equality down to $\mu=M_I$ so that $\sin^2\theta_W$, which depends on the ratio of these coupling constants, does not evolve above $\mu=M_I$.} From Table II one finds values of $\sin^2\theta_W$ consistent with the current experimental bound[15] of $\sin^2\theta_W=0.230\pm0.005$, for $M_I\gtrsim10^{16}$ GeV. This is due to the fact that M_I plays the role of the grand-unification (GU) scale, and for the standard SUSY model, one needs $M_{\rm GU}\simeq10^{16}$ GeV to obtain agreement with experiment. For the symmetric Calabi-Yau manifolds, M_I is generally close to M_c, i.e., for a wide range of parameters[13] $0.1\lesssim M_I/M_c\lesssim1$ with M_I often very close to M_c. That is, the intermediate scale breaking occurs quite rapidly due to the large number of fields entering the renormalization-group equations in the region $M_I\leq\mu\leq M_c$. Table II favors $M_I/M_c\gtrsim0.5$, where the remaining disparity between $\alpha_3(M_c)$ and $\alpha_{3L}(M_c)$ could be accounted for by higher loop corrections (or the existence of a soliton sector which can destroy equality of coupling constants at the compactification scale[16]).

The charged exotic leptons of Eq. (3.1) would be accessible at accelerator energies if their masses are sufficiently low. Thus if the masses of the charged exotic leptons are ≤100 GeV, they would be produced at the CERN e^+e^- collider LEP II via $e^+e^-\rightarrow\gamma,Z^0\rightarrow l\bar{l}$. Further, since the exotic lepton doublets would couple in a normal way with W bosons, production of a single-charged exotic lepton can occur through the decay of an off-shell W boson at the Fermilab Tevatron or the Superconducting Super Collider; e.g., $p+p\rightarrow W^{\pm}\rightarrow l^{\pm}+X$.

This work was supported in part under National Science Foundation Grants No. PHY-8706873 and No. PHY-8706878.

[1]P. Candelas, G. T. Horowitz, A. Strominger, and E. Witten,

Nucl. Phys. **B256**, 46 (1985).

[2]D. Gross, J. Harvey, E. Martinec, and R. Rohm, Phys. Rev. Lett. **55**, 502 (1985).

[3]M. Dine, V. Kaplunovsky, M. Mangano, C. Nappi, and N. Seiberg, Nucl. Phys. **B259**, 519 (1985).

[4]B. Greene, K. H. Kirklin, P. J. Miron, and G. G. Ross, Phys. Lett. B **180**, 69 (1986).

[5]M. C. Bento, L. Hall, and G. G. Ross, Nucl. Phys. **B292**, 400 (1987).

[6]S. Weinberg, Phys. Rev. D **26**, 287 (1982).

[7]B. A. Campbell, K. A. Olive, and D. Reiss, Nucl. Phys. **B296**, 129 (1986); J. E. Ellis, K. Enqvist, D. V. Nanopoulos, and K. A. Olive, Phys. Lett. B **188**, 415 (1987).

[8]M. Dine and C. Lee, Phys. Lett. B **203**, 371 (1988).

[9]P. Candelas, Nucl. Phys. **B298**, 458 (1988); P. Candelas and S. Kalara, Nucl. Phys. **B298**, 493 (1988); B. R. Greene, K. H. Kirklin, P. J. Miron, and G. G. Ross, Phys. Lett. B **192**, 111 (1987); J. Distler, B. G. Greene, K. H. Kirklin, and P. J. Miron, Phys. Lett. B **195**, 41 (1987).

[10]J. Distler and B. Greene, Cornell University Report No. CLNS 88/834, Harvard University Report No. HUTP-88/A020, 1988 (to be published); D. Gepner, Princeton University Report No. PUPT 1093, 1988 (to be published); C. A. Lutken and G. G. Ross, CERN Report No. CERN-TH. 5/32/88, 1988 (to be published).

[11]The 27 multiplet decomposes so that $27 = L(1,3,3) \otimes Q(3,\bar{3},1) \otimes Q^c(\bar{3},1,3)$, where L contains H^a, H'_a, e^c, ν^c, l^a, and N, Q contains q^{aa}, D^a and Q^c contains u^c_a, d^c_a, and D^c_a. Here l^a, H^a, H'_a, and q^{aa} are the $SU(2)_L$ lepton, Higgs, and quark doublets, while D^a, D^c_a are the Higgs color triplets. N and ν^c are SU(5) singlets.

[12]P. Nath and R. Arnowitt, Phys. Rev. D **39**, 2006 (1989).

[13]F. del Aguila and C. D. Coughlan, CERN Report No. CERN-TH.5143/88, Universidad Autónoma de Barcelona Report No. UAB-FT-195/88 (to be published).

[14]R. Arnowitt and P. Nath, Center for Theoretical Physics, Texas A&M University Report No. CTP-TAMU-74/88, Northeastern University Report No. NUB 2960 (to be published).

[15]U. Amaldi *et al.*, Phys. Rev. D **36**, 1385 (1987); G. Costa *et al.*, Nucl. Phys. **B297**, 244 (1988).

[16]K. Choi and J. E. Kim, Phys. Lett. B **176**, 103 (1986).