Electroweak Test from the Harder Lepton's Energy Spectrum in $Z^0 \to \tau^+\tau^- \to \mu^+ e^- X$

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The harder lepton's energy spectrum for the decay sequence $Z^0 \to \tau^+\tau^- \to \mu^+ e^- X$ is sensitive to the τ-coupling parameter $\xi\alpha_H$, where ξ is the Michel polarization parameter for $\tau^- \to l^- \nu\bar{\nu}$, and $\alpha_H \simeq -2a_\tau v_\tau/(a_\tau^2+v_\tau^2)$ with a_τ and v_τ describing $Z^0 \to \tau^+\tau^-$ at tree level. Because of a factorization property, radiative corrections to such energy-energy correlations are as tractable as for A_{LR}; however, this test does not require instrumentation of longitudinal beam polarization.

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The τ lepton first seen[1] at the SLAC e^+e^- storage ring SPEAR in 1975 has been extensively studied.[2] While it may be a sequential lepton in the standard model of electroweak interaction, at present, there are serious difficulties[3] in understanding the measurements of its decays into one charged particle, and very little is known[2,4] about the chirality structure of its charged and neutral electroweak couplings.

It might seem that simple precision tests of the chirality of the τ's electroweak couplings must await a programmatic decision at the SLAC Linear Collider (SLC), or the CERN collider LEP, to use polarized beams[5] in e^+e^- collisions at the Z^0. However, we find that for the decay sequence

$$Z^0 \to \tau^+\tau^- \to (\mu^+\nu\bar{\nu})\,(e^-\nu\bar{\nu}) \qquad (1)$$

the muon-energy–electron-energy correlation function will provide a very simple definitive test without beam polarization and with only indirect and calculable effects of initial-state QED radiation. This follows because (i) such functions are independent[6] of the polarization state of the decaying particle, here the Z^0, and because (ii) some corrections due to finite e, μ, and τ masses are negligible in the precision range of current interest at Z^0 energies.

Using (i) and (ii), we have analytically obtained at tree level the full sequential-decay correlation function

$$I(E_{\bar{\mu}},E_e,\cos\psi_{\bar{\mu}e}) = T(E_{\bar{\mu}},E_e,\cos\psi_{\bar{\mu}e})[1+A(E_{\bar{\mu}},E_e,\cos\psi_{\bar{\mu}e})] \qquad (2)$$

in the Z^0 rest frame, where $E_{\bar{\mu},e}$ are the observed charged-lepton energies and $\psi_{\bar{\mu}e}$ is the angle between their momenta. The "analyzing power"

$$A(E_{\bar{\mu}},E_e,\cos\psi_{\bar{\mu}e}) \equiv \xi\alpha_H \frac{U(E_{\bar{\mu}},E_e,\cos\psi_{\bar{\mu}e})}{T(E_{\bar{\mu}},E_e,\cos\psi_{\bar{\mu}e})} \qquad (3)$$

is proportional to the Michel ξ parameter for $\tau^- \to l^-\nu\bar{\nu}$ decay ($\xi=\pm 1$, respectively, for a $V\mp A$ charged-current coupling) and also to

$$\alpha_H \equiv \frac{|t_{-+}|^2-|t_{+-}|^2}{|t_{-+}|^2+|t_{+-}|^2}, \qquad (4)$$

where $t_{\lambda_1\lambda_2,\lambda_Z}$ are the usual helicity amplitudes describing the coupling of the Z^0 to τ^+ and τ^-. The functions T and U are defined below in Eq. (7). Unlike the forward-backward asymmetry for unpolarized beams, here

$$\alpha_H \simeq \frac{-2r[1-(2m_\tau/M)^2]^{1/2}}{1+r^2[1-(2m_\tau/M)^2]} \simeq \frac{-2a_\tau v_\tau}{a_\tau^2+v_\tau^2}, \qquad (5)$$

where $r=a_\tau/v_\tau=(1-4\sin^2\theta_W)^{-1}$ assuming lepton universality. Consequently, α_H is much more sensitive to $\sin^2\theta_W$ than $A_{FB}^{\mu^+\mu^-}$ [$\partial\alpha_H/\partial x_W \simeq 7.8$ for $\sin^2\theta_W=0.23$ and $\partial A_{FB}/\partial x_W = 1.5\alpha_H(\partial\alpha_H/\partial x_W) \simeq 0.238(\partial\alpha_H/\partial x_W)$].

For μ^+ and e^- back to back, Figs. 1 and 2 show the tree-level contour plots of T and A. As the opening angle of $\psi_{\bar{\mu}e}$ decreases we find as expected that both the available phase space and I of Eq. (2) decrease rapidly. Here, however, as $\psi_{\bar{\mu}e}$ decreases, in contrast to generalizations[7] of Yang's parity test, there is only a slight dependence on $\cos\psi_{\bar{\mu}e}$ due to approximate helicity conservation in the $Z^0 \to \tau^+\tau^-$ amplitudes, so A remains positive in the region approximately bounded by $E_{\bar{\mu}}=E_e \simeq M/4$. Therefore, $\cos\psi_{\bar{\mu}e}$ can be integrated out, and after folding events about the diagonal $E_{\bar{\mu}}=E_e$, the energy of the lighter lepton can also be integrated out. The resulting harder lepton's energy spectrum

$$I(x_H)=T(x_H)[1+A(x_H)], \quad x_H=E_H/E_{\max}, \qquad (6)$$

is shown in Figs. 3 and 4.

The function $I(x_H)$ at tree level for $M \gg m_\tau$ can be easily checked by many readers. Conceptually it is simple, which should help in accessing the limitations from anticipated systematic errors for detectors at SLC or LEP. For 10^7 Z^0 events, using all $\tau^\pm \to \mu^\pm\nu\bar{\nu}$ and $e^\pm\nu\bar{\nu}$ channels, the statistical errors are $\delta\alpha_H=0.016$ and $\delta\xi=0.063$. Since the τ decays into μ and e have the least background, they are the cleanest decay channels for α_H determination. In principle, the $\tau \to \pi\nu$ and $\tau \to \rho\nu$ channels among others are also good decay channels to study. Since they are two-body modes the τ rest frames for two-body pairs are accessible[7] and, depending on many factors including possibly new physics, it might be useful to include some of them in a test (the statistical errors reduce to $\delta\alpha_H=0.0053$ and $\delta\xi=0.026$). Mea-

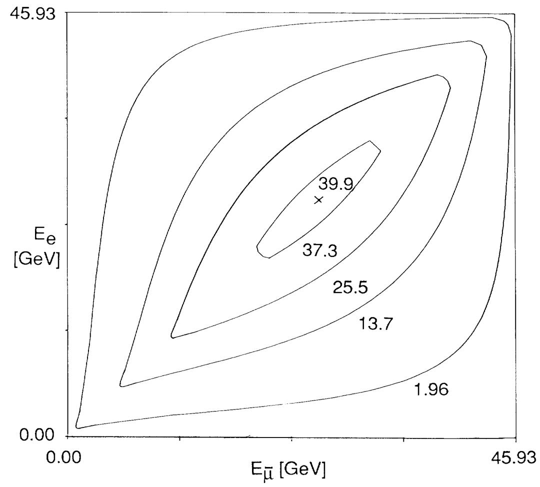

FIG. 1. For $\bar{\mu}$ and e back to back, the contour plot of $T(E_{\bar{\mu}},E_e)$ for a Z^0 mass $M=91.9$ GeV.

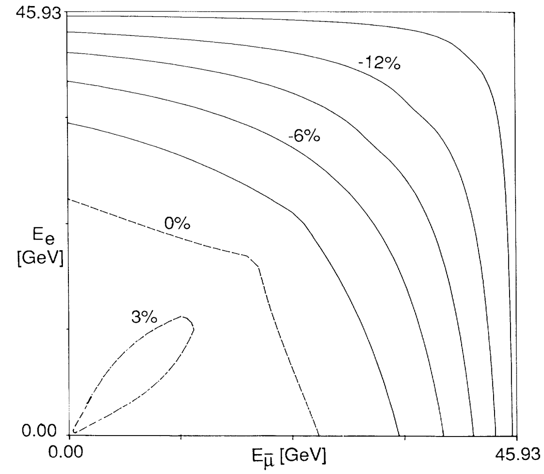

FIG. 2. The analyzing power $A(E_{\bar{\mu}},E_e)$ for $\sin^2\theta_w=0.23$, $\xi=+1$.

surement of charged-particle energy correlation functions for various one-prong τ modes in $Z^0\to\tau^+\tau^-$ might also be a helpful constraint in resolving the τ missing-decay-modes problem.

For a precision test of α_H to be feasible, a prime requirement of course is that all radiative-correction effects and Γ_Z/M effects be correctly included in the helicity formalism,[6,7] so $I(x_H)$ has a precision of $\sim 10^{-3}$ or better.[8,9] By time-reversal invariance, $t_{\lambda_1\lambda_2,\lambda_Z}=\tilde{t}_{\lambda_Z,\lambda_1\lambda_2}$, where the latter is the helicity amplitude for $l^+l^-\to Z^0$. Thus, "oblique corrections" can be included in α_H as in the starred renormalization approach of Kennedy and Lynn,[10] which has been used in a state-of-the-art calculation[11] of $A_{LR}^{\mu^+\mu^-}$ for a polarized e^- beam. So, from $\alpha_H\simeq -A_{LR}$, using for instance Refs. 5, 10, or 11, one can estimate the sensitivity level of $I(x_H)$ to m_t, m_H, and to new physics. Next,[11] the $t_{\lambda_1\lambda_2,\lambda_Z}$ amplitudes are modified to include the known weak vertex corrections and the final QED corrections[10-12] for $Z^0\to\tau^+\tau^-$. Next, in the factors for $\tau^\pm\to l^\pm\nu\bar{\nu}$, the electroweak corrections[13] to $S(x)\simeq(E_l)^2(-1+2x)$, $R(x)\simeq(E_l)^2(3-2x)$ will induce corrections $\sim 10^{-3}$, much as for the electron spectrum in muon decay, since

$$T=R(E_1^\tau)R(E_2^\tau)+\xi^2S(E_1^\tau)S(E_2^\tau)\cos\theta_1^\tau\cos\theta_2^\tau\,,$$
$$U=S(E_1^\tau)R(E_2^\tau)\cos\theta_1^\tau+R(E_1^\tau)S(E_2^\tau)\cos\theta_2^\tau\,, \qquad (7)$$

where $\theta_{1,2}^\tau$ is the polar angle of $l^\pm$ in the $\tau^\pm$ rest frame. Finally, this correlation function $I(E_{\bar{\mu}},E_e,\cos\psi_{\bar{\mu}e})$ must be convoluted with the Z^0 resonance line shape[11,14] to include the Γ_Z/M effect and the initial-state QED corrections, and then the remaining integrations per-

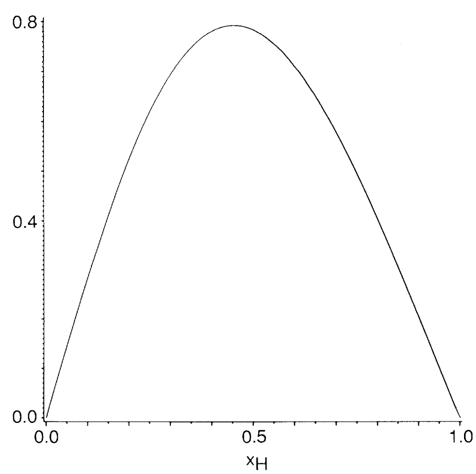

FIG. 3. The $T(x_H)$ factor in the harder lepton's energy spectrum, $I(x_H)=T(x_H)[1+A(x_H)]$.

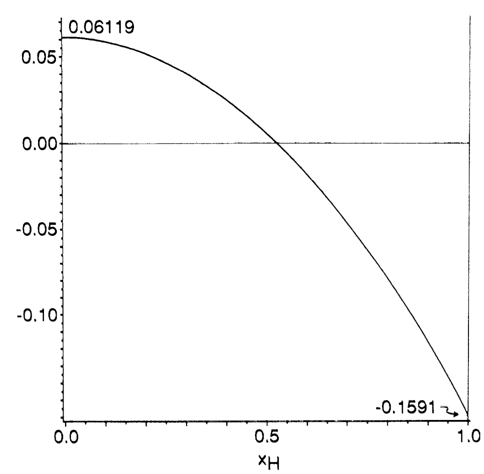

FIG. 4. The $A(x_H)$ term in the harder lepton's energy spectrum.

formed to obtain $I(x_H)$.

Our principal point is that the exceptional properties of $I(x_H)$ warrant its careful analysis for the decay sequence $Z^0 \to \tau^+\tau^- \to \mu^+ e^- X$ at SLC and LEP. A definitive measurement of ξ and of α_H for the τ lepton obviously has fundamental implications within, and beyond, the third family and the standard electroweak model.

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[1]M. L. Perl *et al.*, Phys. Rev. Lett. **35**, 1489 (1975).

[2]B. C. Barish and R. Stroynowski, Phys. Rep. **157**, 1 (1988); K. K. Gan and M. L. Perl, Int. J. Mod. Phys. A **3**, 531 (1988).

[3]T. N. Truong, Phys. Rev. D **30**, 1509 (1984); F. J. Gilman and S. M. Rhie, Phys. Rev. D **31**, 1066 (1985); F. J. Gilman, Phys. Rev. D **35**, 3541 (1987). For a recent statistical study of the decay data, see K. G. Hayes and M. L. Perl, Phys. Rev. D **38**, 3351 (1988).

[4]T. Kamae, in Proceedings of the Twenty-Fourth International Conference on High Energy Physics, Munich, West Germany, 1988 (to be published).

[5]*Polarization at LEP,* edited by G. Alexander *et al.* (CERN Report No. CERN 88-06, 1988); Working Group, CERN Report No. CERN/LEPC/87-6 (unpublished); M. L. Swartz, SLAC Report No. SLAC-PUB-4689, 1988 (unpublished); W. Hollik, DESY Report No. DESY 88-106, 1988 (unpublished), and references therein.

[6]T. L. Trueman [Phys. Rev. D **18**, 3423 (1978)] first proved such factorization in sequential decays for the $\phi\phi$ parity test [N. P. Chang and C. A. Nelson, Phys. Rev. Lett. **40**, 1617 (1978)], which is an analog of Yang's parity test [C. N. Yang, Phys. Rev. **77**, 242 (1950); **77**, 722 (1950)]. Interference of initial- and final-state QED radiation has been studied for the analogous A_{LR} and found to be negligible [S. L. Jadach *et al.*, Z. Phys. C **38**, 609 (1988)].

[7]J. R. Dell'Aquila and C. A. Nelson, State University of New York at Binghamton Reports No. SUNY BING 3/30/88 (unpublished) and No. SUNY BING 3/31/88 (unpublished).

[8]In the limit $M \gg m_\tau$, $T \sim x(1-x)[25+25x-35x^2-10x^3+17x^4-4x^5+\xi^2(1+x-11x^2-x^3+26x^4-16x^5)]+O(\gamma^{-2})$, $TA \sim \xi\alpha_H x(1-x)(-10-10x+62x^2+7x^3-47x^4-16x^5)+O(\gamma^{-2})$, where $\gamma^{-2} \simeq 0.0015$, $\gamma = M/2m_\tau$. Radiative corrections to some distributions for the process $e^+e^- \to \tau^+\tau^-$ have been investigated by S. Jadach and Z. Was, Acta Phys. Pol. B **15**, 1151 (1984); B **16**, 483 (1985); and Z. Was, *ibid.* B **17**, 1099 (1987); and in CERN Report No. CERN 88-06, 1988 (unpublished).

[9]Using leptons from τ decays was discussed by Y. S. Kang, LBL Report No. 15244, 1982 (unpublished). Also see Y.-S. Tsai, Phys. Rev. D **4**, 2821 (1971).

[10]D. C. Kennedy and B. W. Lynn, SLAC Report No. SLAC-PUB-4039, 1988 (unpublished); earlier calculations include M. Veltman, Nucl. Phys. **B123**, 89 (1977); B. W. Lynn, M. E. Peskin, and R. G. Stuart, in *Physics at LEP,* edited by J. Ellis and R. Peccei (CERN Report No. CERN 86-02, 1986); G. Altarelli and G. Martinelli, *ibid.*

[11]D. C. Kennedy *et al.*, SLAC Report No. SLAC-PUB-4039, 1988 (unpublished); G. Burgers, CERN Report No. CERN-TH.5119/88, 1988 (unpublished).

[12]B. Grzadkowski *et al.*, Nucl. Phys. **B281**, 18 (1987); G. Burgers and W. Hollik, CERN Report No. CERN-TH. 5131/88, 1988 (unpublished).

[13]T. Kinoshita and A. Sirlin, Phys. Rev. **113**, 1652 (1959); W. J. Marciano and A. Sirlin, Phys. Rev. D **22**, 2695 (1980); Phys. Rev. Lett. **61**, 1815 (1988).

[14]F. A. Berends *et al.*, Nucl. Phys. **B297**, 429 (1988); **B304**, 921 (1988); Phys. Lett. B **203**, 177 (1988).