Three-Body Treatment of the Final State in the (^{3}He,pp) Reaction on Medium-Mass Nuclei

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A parameter-free method is proposed for the analysis of transfer reactions leading to an unbound three-body final state. The three-body dynamics are treated in the adiabatic approximation. The technique is applied successfully to new data for the ^{28}Si(^{3}He,pp)^{29}Si reaction.

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The $A(^3\text{He},pp)B$ transfer reaction is of considerable theoretical interest in that the final state consists of three particles in the continuum. New data[1,2] for the reaction have recently become available on medium-mass targets. The observed pp coincidence spectrum shows significant enhancement at low pp relative energy ϵ as a result of the strong final-state interaction between the two protons. This energy dependence is not simply understood within the Watson-Migdal theory of the final-state interaction.[3] In addition, our lack of any phenomenological information on the diproton–residual-nucleus effective interaction in the final state makes the application of distorted-wave Born-approximation prescriptions[1,2,4] problematical. Similar difficulties in the final state of the (d,pn) reaction have been discussed by Austern.[5] There is no published method which provides a consistent theoretical description of the (^{3}He,pp) reaction.

The final state of the reaction can be usefully treated as a three-body system, viz., two emergent protons and the residual nucleus in its final state. In this paper we propose a novel, parameter-free method for an accurate analysis of these three-body dynamics. The nature of the ϵ dependence of the pp coincidence spectrum is investigated and the effective potential required to describe the center-of-mass motion of the pp pair is also derived.

The transition amplitude of the reaction can be written with a three-body $(p+p+B)$ wave function Ψ_ϵ^- in the final state. Explicitly,

$$T(\epsilon)=\langle\Psi_\epsilon^-(\mathbf{x},\mathbf{R})\Phi_B\,|\,V_{1n}+V_{2n}+\Delta U_{\rm res}\,|\,\Phi_3\Phi_A\chi_i^+\rangle\,, \quad (1)$$

where Φ_3, Φ_A, and Φ_B denote the wave functions of the ^{3}He, target, and residual nuclei, respectively, and V_{ij} is the nucleon-nucleon interaction between the emitted protons (1,2) and the transferred neutron (n). In line with common procedure for light-ion reactions, we neglect the small residual interaction term $\Delta U_{\rm res}$ in $T(\epsilon)$ (Ref. 6, p. 78). The entrance-channel distorted-wave function χ_i^+, with energy E_i, is generated by a ^{3}He$+A$ optical potential.

The exit-channel wave function Ψ_ϵ^- is simply related to Ψ_ϵ^+ which obeys the Schrödinger equation

$$(T_R+U_{1B}+U_{2B}+H_{12}-E_f)\Psi_\epsilon^+(\mathbf{x},\mathbf{R})=0\,, \quad (2)$$

where U_{iB} is the proton $(i)+B$ optical potential, evaluated at the mean energy of the emitted protons, T_R is the kinetic-energy operator for their center-of-mass coordinate $\mathbf{R}$ $[=(\mathbf{r}_1+\mathbf{r}_2)/2]$, $\mathbf{x}=\mathbf{r}_1-\mathbf{r}_2$, and E_f is the final-state energy $E_f=E_i+Q$.

Our approach to solving Eq. (2) is to use the adiabatic approximation,[7] i.e., the sub-Hamiltonian $H_{12}=T_{12}+V_{12}$ is replaced by ϵ in Eq. (2). The resulting equation, in which $\mathbf{x}$ appears only as a parameter, is solved numerically[8] by the techniques of Amakawa *et al.*[9] This approximation has been shown to be reliable in the case of deuteron-induced reactions at similar energies.[10] The adiabatic three-body wave function has the form $\Psi_\epsilon^+(\mathbf{x},\mathbf{R})=\phi_{12}(\epsilon,\mathbf{x})\chi_{AD}^+(\mathbf{x},\mathbf{R})$ where, asymptotically, $\chi_{AD}^+(\mathbf{x},\mathbf{R})=\exp(i\mathbf{P}\cdot\mathbf{R})+$outgoing waves. The pp relative wave function at energy ϵ, $\phi_{12}(\epsilon,\mathbf{x})$, satisfies $(H_{12}-\epsilon)\phi_{12}(\epsilon,\mathbf{x})=0$. The adiabatic wave function goes beyond simple factorization in relative and center-of-mass variables[4] as χ_{AD}^+ still depends on $\mathbf{x}$.

In the T matrix of Eq. (1), the separation $|\mathbf{y}|$ of the transferred neutron and the outgoing pp pair is forced to be small by V_{1n}, V_{2n}, and Φ_3. Under the usual zero-range assumption (Ref. 6, p. 99) that the transferred neutron wave functions ϕ_n and χ_i^+ are approximately constant over a range $\frac{2}{3}\mathbf{y}$ and $\frac{1}{3}\mathbf{y}$, respectively, we obtain the simpler expression

$$T(\epsilon)=S^{1/2}D_0(\epsilon)\int\chi_f^{-*}(\epsilon,\mathbf{R})\phi_n^*[(B/A)\mathbf{R}]\times\chi_i^+[(B/A)\mathbf{R}]\,d\mathbf{R}\,, \quad (3)$$

with

$$D_0(\epsilon)=\int\phi_{12}^*(\epsilon,\mathbf{x})(\epsilon_3-H_{12})\phi_3(\mathbf{x})\,d\mathbf{x}\,, \quad (4a)$$

and

$$\phi_3(\mathbf{x})=\int\Phi_3(\mathbf{x},\mathbf{y})\,d\mathbf{y}\,. \quad (4b)$$

Here, S is the spectroscopic factor for the $\langle B\,|\,A\rangle$ overlap and $-\epsilon_3$ is the ^{3}He binding energy. The function $\chi_f^-(\epsilon,\mathbf{R})$ is defined by

$$\chi_f^-(\epsilon,\mathbf{R})=\langle\phi_3(\mathbf{x})\,|\,(\epsilon_3-H_{12})\,|\,\Psi_\epsilon^-(\mathbf{x},\mathbf{R})\rangle/D_0(\epsilon)\,, \quad (5)$$

with the bracket denoting integration over $\mathbf{x}$. The func-

tion $\chi_f^-(\epsilon,\mathbf{R})$ is normalized so as to have the standard asymptotic form, $\exp(i\mathbf{P}\cdot\mathbf{R})$+incoming waves. It describes those components of the pp-pair center-of-mass motion with relative energy ϵ and separations $x \leq$ range of ϕ_3, which contribute to the transfer process. We stress that $\chi_f^-(\epsilon,\mathbf{R})$ has no simple relationship with diproton–residual-nucleus elastic scattering and has a meaning only within the transfer amplitude, Eq. (3). It is also convenient to define the effective potential $U_{\rm eff}^{pp}(\epsilon,R)$ which generates $\chi_f^-(\epsilon,\mathbf{R})^*$, that is

$$U_{\rm eff}^{pp}(\epsilon,R)=\frac{(E_f-T_R)\chi_f^-(\epsilon,\mathbf{R})^*}{\chi_f^-(\epsilon,\mathbf{R})^*}=\frac{\langle\phi_3(\mathbf{x})\,|\,(\epsilon_3-H_{12})(U_{1B}^{\dagger}+U_{2B}^{\dagger})\,|\,\Psi_\epsilon^-(\mathbf{x},\mathbf{R})\rangle^*}{\langle\phi_3(\mathbf{x})\,|\,(\epsilon_3-H_{12})\,|\,\Psi_\epsilon^-(\mathbf{x},\mathbf{R})\rangle^*}. \tag{6}$$

The cross section to the final state of the pp pair with energy ϵ can now be written

$$\frac{d^2\sigma}{d\Omega\,d\epsilon}=\frac{\mu_i\mu_f k_f}{(2\pi\hbar^2)^2 k_i}\,|\,T(\epsilon)\,|^2\rho(\epsilon)\,, \tag{7}$$

where $\rho(\epsilon)$ is the phase-space factor[1] and k_f is the final-state wave number. For comparison with the experimental data,[2] the cross section and vector analyzing power A_y are averaged over the energy range 0.966 MeV $\leq\epsilon\leq$ 4.62 MeV.

The present method is applied to the ^{28}Si(^{3}He, pp)^{29}Si($\frac{7}{2}^-$, 3.62 MeV) reaction at 33 MeV. The ^{3}He optical potential and wave function of the transferred neutron are taken from Ref. 2 and U_{iB} from Ref. 11. For ^{3}He we adopt the Phillips momentum-space wave function,[12] constructed via the Faddeev method, but neglect the small singlet D-state components of the overlap. It is assumed that $\phi_{12}(\epsilon,\mathbf{x})$ is a singlet S wave and is calculated with the Reid soft-core potential.[13] Both relative S and D waves are taken into account in the solution of the adiabatic approximation to Eq. (2). This means that asymptotically the pp pair are assumed to be in a singlet S wave but the nuclear potentials U_{iB} are allowed to induce coupling to singlet D waves. For pp relative S waves, the Coulomb force deviates from the point-charge form only for separations $x\geq 2R$, which provide very small contributions in Eq. (3). The Coulomb force can also mix in pp relative D waves; however, these would contribute only through the small nonspherical components of $\phi_3(\mathbf{x})$. Thus, to high accuracy, the pp-nucleus Coulomb interaction can be assumed to act at the center of mass of the pair.

Figure 1 shows the energy-averaged results for $\langle d^2\sigma/d\Omega\,d\epsilon\rangle$ and $\langle A_y\rangle$. The experimental data are reproduced with a spectroscopic factor $S=0.38$, consistent with that from the ^{28}Si(d,p) reaction.[1,2,14] Figure 2 shows the double-differential cross section. The calculations (solid curve) reproduce the bump in the observed spectrum[15] due to the final-state interaction between the two protons. The Watson-Migdal theory (dotted curve) fails to fit.

The nature of the ϵ dependence of the cross section can be understood as follows. In Eq. (7) the ϵ dependence originates in three terms, D_0, ρ, and $\chi_f^-(\epsilon,\mathbf{R})^*$. If the ϵ dependence of χ_f^- is neglected, the ϵ dependence of the cross section is determined solely by the factor $D_0^2(\epsilon)\rho(\epsilon)$, shown by the dashed curve in Fig. 2 (normalized to the solid curve at the peak). This energy dependence is very close to that of the full calculation which is thus almost entirely understood in terms of $D_0^2(\epsilon)\rho(\epsilon)$ and determined only by the relative motion between the two protons. Additionally, this explains why the ϵ dependence of the observed coincidence spectrum is essentially independent of the incident energy[1,2] and of the nuclear final-state population.[2] For this reason, in Fig. 2, the 33-MeV data[2] have been supplemented by data at 52 MeV,[1] which extend to lower ϵ. These data have been normalized to the 33-MeV data in their region of energy overlap. In Ref. 1, $D_0^2(\epsilon)\rho(\epsilon)$ was found to provide a rather poor representation of the 52-MeV data.

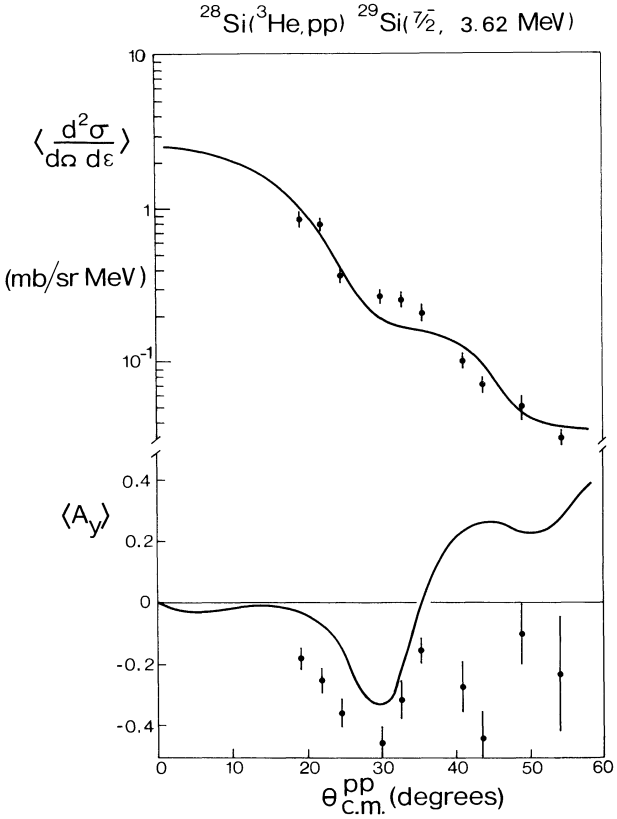

FIG. 1. Energy-averaged cross section and vector-analyzing-power angular distributions for the ^{28}Si(^{3}He, pp)^{29}Si($\frac{7}{2}^-$, 3.62 MeV) reaction at 33 MeV. The curves show the results of the adiabatic final-state-interaction calculations. The data are from Ref. 2.

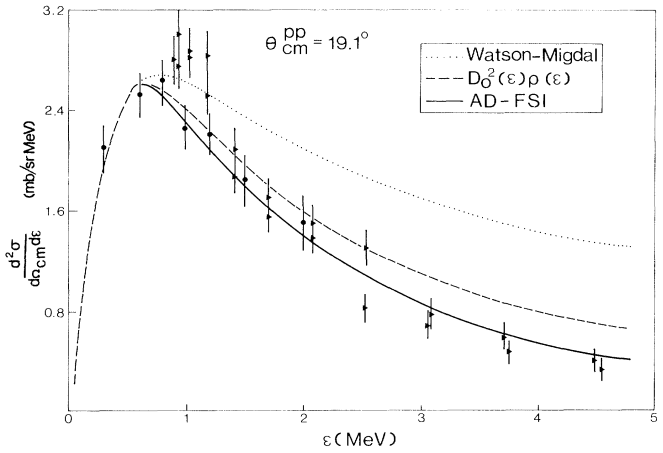

FIG. 2. Double-differential cross-section data for the ^{28}Si(^{3}He,pp)^{29}Si($\frac{7}{2}^-$, 3.62 MeV) reaction at 33 MeV (Ref. 2) (triangles) and 52 MeV (Ref. 1) (circles) as a function of pp relative energy ϵ for $\theta_{c.m.}=19.1°$. The 52-MeV data are normalized to the 33-MeV data. The curves are described in the key and the text.

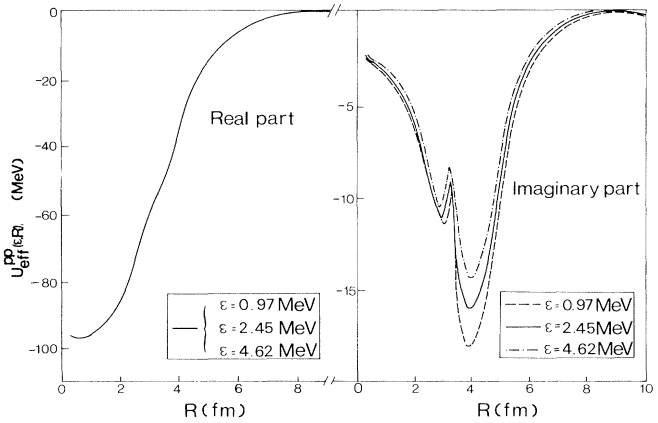

FIG. 3. Real and imaginary parts of the effective interaction $U_{\rm eff}^{pp}(\epsilon,R)$, Eq. (6), in the $L=5$ pp center-of-mass partial wave, as a function of pp relative energy ϵ for the ^{28}Si(^{3}He,pp)^{29}Si reaction at 33 MeV.

This conclusion can, however, be traced to the use of unnecessary and somewhat inaccurate approximations in that calculation for $D_0(\epsilon)$.

While the ϵ dependence of the coincidence spectrum is determined by the relative motion between the two protons, its magnitude and angular dependence are strongly influenced by the center-of-mass motion of the pp pair. To investigate the nature of the center-of-mass motion, the effective potential $U_{\rm eff}^{pp}(\epsilon,R)$ has been calculated and is shown in Fig. 3 for center-of-mass partial wave $L=5$. The weak ϵ dependence seen arises from the fact that, in the last form of $U_{\rm eff}^{pp}$ in Eq. (6), the numerator and denominator have strong but essentially equal ϵ dependence. The effective potential depends only very weakly upon the pp center-of-mass partial wave. It is apparent that the potential has a depth of approximately twice U_{iB}, very unlike that deduced from an analysis based on a folding-model prescription for the final state.[1]

Figure 2 (dotted curve) shows that the ϵ dependence of the coincidence spectrum is not explained by the Watson-Migdal theory. This theory gives the ϵ dependence to be of the form $F(\epsilon)^2\rho(\epsilon)$, where $F(\epsilon)=\sin[\delta_N(\epsilon)]/[\epsilon^{1/2}C(\epsilon)]$ with δ_N the nuclear part of the phase shift and $C(\epsilon)$ the Coulomb penetration factor[16] for pp scattering in the singlet S state. The failure thus arises from a difference between F and $D_0(\epsilon)$. The Watson-Migdal theory successfully describes the spectra of mesons produced in NN collisions[3] where a short-range transition operator is involved. $D_0(\epsilon)$, on the other hand, is determined by an x integration over the range of ϕ_3 ($\lesssim$15 fm). The Watson-Migdal factorization of $\phi_{12}(\epsilon,x)$, into $F(\epsilon)\phi_{12}(\bar{\epsilon},x)/F(\bar{\epsilon})$, where $\bar{\epsilon}$ is the resonance energy, is accurate, however, only for small x (up to 4.0 fm). The factorization is thus not applicable for the calculation of $D_0(\epsilon)$ which deviates markedly from $F(\epsilon)$. The failure of the Watson-Migdal theory is thus due to the poorly localized nature of the pp distribution in ^{3}He. The applicability of the present method is independent of this consideration. A similar failure has been reported for long-ranged charge-exchange processes such as the (d,pp) reaction.[17]

In this paper we propose a novel method for the treatment of the final-state interaction in the (^{3}He,pp) transfer reaction in which the transition amplitude incorporates a three-body ($p+p+B$) wave function in the exit channel. The adiabatic approximation provides an accurate and parameter-free prescription for the calculation of this wave function for medium-mass nuclei from the underlying nucleon-nucleon and nucleon-residual nucleus interactions. A diproton optical potential does not enter. The method gives good agreement with the observed angular and ϵ dependence of the cross section and vector analyzing power A_y. The strong enhancement in the observed pp coincidence spectrum at low ϵ is determined by $D_0(\epsilon)$, Eq. (4), i.e., the relative motion between the two protons, but is little influenced by their center-of-mass motion. The effective potential generating the center-of-mass motion is very weakly dependent on ϵ and has a strong influence upon the magnitude and angular distribution of the cross section and A_y. A reasonable way of simulating the results of the complete three-body model with a standard (zero-range) distorted-wave Born-approximation computer code thus suggests itself: Take $U_{\rm eff}^{pp}(\epsilon,R)$ from an adiabatic calculation at a chosen ϵ as a final-channel distorting potential and replace the conventional zero-range parameter D_0 by $D_0(\epsilon)\rho(\epsilon)^{1/2}$.

Our approach to the final-state interaction is more generally applicable to other nuclear systems, in particular to processes such as (^{4}He,pp) (Ref. 18) and

(^{9}Be,^{4}He,^{4}He) (Ref. 19).

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