Unitarity Constraints on Heavy Higgs Bosons

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We calculate the complete set of $O(G_F M_H^2)$ corrections to the elastic scattering of longitudinally polarized W bosons in the limit $s, M_H^2 \gg M_W^2$, for any value of s/M_H^2. For any Higgs-boson mass such that $M_H \gg M_W$, there exists a critical energy above which partial-wave unitarity is violated and the one-loop correction to the $J=0$ partial wave is large, indicating the breakdown of perturbation theory.

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The standard $SU(2)_L \otimes U(1)$ model predicts the existence of an undiscovered neutral scalar H, the Higgs boson. The Higgs boson (or some similar object) is necessary to break the gauge symmetry and generate the masses of the W and Z gauge bosons. The mass of the Higgs boson is not, however, predicted by the model. Lee, Quigg, and Thacker[1] and Dicus and Mathur[2] showed that if the Higgs-boson mass exceeds a critical value of about 1 TeV, unitarity is violated at tree level for elastic longitudinal-vector-boson scattering at high energy, $s \gg M_H^2$. They interpreted their results to mean that if the Higgs-boson mass is greater than this critical value, then weak interactions will be stronger in the TeV energy regime and perturbation theory will be no longer be valid.

In this Letter we present results for the one-loop contribution to the $J=0$ partial-wave amplitude for the reaction $W_L^+ W_L^- \to W_L^+ W_L^-$ in the limit $s, M_H^2 \gg M_W^2$. (An analytic expression for the amplitude will be presented elsewhere.[3]) We show that when unitarity is violated, the one-loop contribution is large, confirming the breakdown of perturbation theory. Furthermore, the one-loop contribution *increases* the $J=0$ partial-wave amplitude, and thus does not help restore unitarity. However, if we express the amplitude in terms of a running coupling for $s \gg M_H^2$, the one-loop contribution is negative, suggesting that unitarity may be restored by loop corrections.

In the regimes $s \ll M_H^2$ and $s \gg M_H^2$ the logarithmic contributions to the one-loop amplitude for $W_L^+ W_L^-$ scattering have been found previously.[4] Our calculation, however, is valid for all values of s/M_H^2. This allows us to determine the nonlogarithmic contributions to both the low- and high-energy limits of the amplitude.

For $s, M_H^2 \gg M_W^2$, the interactions of the Higgs boson and the longitudinal gauge bosons can be calculated in an effective theory of interacting scalars. For a scattering process involving external longitudinally polarized W's and Z's, the amplitude can be calculated, to $O(M_W^2/s)$, by replacing the external gauge bosons with the corresponding Goldstone bosons of the R_ξ gauge.[1,5,6] In the limit $M_H^2 \gg M_W^2$, interactions of enhanced electroweak strength, $O(G_F M_H^2)$, arise only from diagrams in which the *internal* particles are also Goldstone bosons (or the Higgs boson). We use the effective theory to calculate the one-loop corrections to the $W_L^+ W_L^-$ elastic scattering amplitude for $s, M_H^2 \gg M_W^2$.

In the effective theory, the interactions of the Goldstone bosons and the Higgs scalar are given by

$$\mathcal{L} = -\lambda \left(w^+ w^- + \frac{z^2}{2} + \frac{H^2}{2} + v_0 H + \frac{v_0^2}{2} - \frac{\mu_0^2}{2\lambda} \right)^2, \quad (1)$$

where $w^{\pm}$ and z are the Goldstone bosons, H is the physical Higgs scalar, v_0 is the Higgs-field vacuum expectation value (VEV) which gives rise to spontaneous symmetry breaking, and $\lambda = G_F M_H^2/\sqrt{2}$ is the bare coupling of the $\lambda\phi^4$ theory. (The last two terms, which cancel at tree level, yield a tadpole counterterm which ensures that the physical Higgs field has zero VEV at one-loop level.) This form of the interaction demonstrates that large M_H corresponds to strong interactions between the longitudinal gauge bosons and the Higgs boson. In the Landau gauge, the Goldstone bosons are massless and there is no w-W mixing. The Feynman rules and a description of our renormalization prescription for this effective theory are given in Ref. 7.

The tree-level matrix element for $W_L^+ W_L^- \to W_L^+ W_L^-$ in the limit $s, M_H^2 \gg M_W^2$ is easily found in the effective theory,[1]

$$\mathcal{M}_0(W_L^+ W_L^- \to W_L^+ W_L^-) = -\sqrt{2} G_F M_H^2 \left(\frac{M_H^2}{s - M_H^2} + \frac{M_H^2}{t - M_H^2} + 2 \right). \quad (2)$$

The tree-level contribution to the $J=0$ partial wave, a_0^0, is then found from Eq. (2) to be[1]

$$a_0^0 \equiv \frac{1}{16\pi s} \int_{-s}^{0} dt\, \mathcal{M}_0(W_L^+ W_L^- \to W_L^+ W_L^-) = -\frac{G_F M_H^2}{8\pi\sqrt{2}} \left[2 + \frac{M_H^2}{s - M_H^2} - \frac{M_H^2}{s} \ln\left(1 + \frac{s}{M_H^2} \right) \right]. \quad (3)$$

Quite general considerations of elastic unitarity require that the $J=0$ partial wave a_0 satisfy

$$|a_0(W_L^+W_L^- \to W_L^+W_L^-)| < 1 . \quad (4)$$

At energies far above the Higgs pole, $s \gg M_H^2$, the lowest-order contribution to the $J=0$ partial wave a_0^0 approaches a constant,

$$|a_0^0(W_L^+W_L^- \to W_L^+W_L^-)| \to \frac{G_F M_H^2}{4\pi\sqrt{2}}, \quad s \gg M_H^2 . \quad (5)$$

Hence, requiring that a_0^0 respect elastic unitarity, Lee, Quigg, and Thacker[1] obtained the bound

$$M_H^2 < 4\pi\sqrt{2}/G_F \simeq (1.2\ \text{TeV})^2 . \quad (6)$$

This bound may be improved[1] to $M_H^2 < 8\pi\sqrt{2}/3G_F \simeq (1\ \text{TeV})^2$ by considering elastic scattering of the state $(2W_L^+W_L^- + Z_L Z_L + HH)/\sqrt{8}$.

In similar spirit, we can find the tree-level contribution to the $J=0$ partial wave in the low-energy limit $s \ll M_H^2$,

$$|a_0^0(W_L W_L \to W_L W_L)| \to G_F s/16\pi\sqrt{2}, \quad s \ll M_H^2 . \quad (7)$$

Note that this amplitude grows linearly with s. This can be understood in terms of a low-energy effective theory which describes the Goldstone-boson interactions in terms of derivative couplings. By requiring $|a_0^0| < 1$, we find a critical energy scale

$$s_c \equiv 16\pi\sqrt{2}/G_F \simeq (2.5\ \text{TeV})^2 . \quad (8)$$

Chanowitz and Gaillard[6] refined this to $s_c \equiv 8\pi\sqrt{2}/G_F \simeq (1.7\ \text{TeV})^2$ by considering elastic scattering of the state $(2W_L^+W_L^- + Z_L Z_L)/\sqrt{6}$. They concluded that for a very massive Higgs boson, perturbation theory breaks down for $s > s_c$.

It is straightforward to extend these considerations to include the one-loop corrections in the limit $s, M_H^2 \gg M_W^2$. We expand a_0 in a perturbation series in $\lambda = G_F M_H^2/\sqrt{2}$,

$$a_0 \equiv a_0^0 + \lambda a_0^1 + \lambda^2 a_0^2 + \cdots . \quad (9)$$

Note that a_0^0, a_0^1, etc., are themselves of $O(\lambda)$. The requirement of elastic unitarity is then, to $O(\lambda^2)$,

$$\mathcal{A}_0^1 \equiv |a_0^0| + \lambda \left[\frac{\text{Re}(a_0^0)\text{Re}(a_0^1) + \text{Im}(a_0^0)\text{Im}(a_0^1)}{|a_0^0|} \right] < 1 . \quad (10)$$

It is this condition which we impose on the $W_L^+W_L^-$ scattering amplitude. For a fixed Higgs-boson mass, we then find a critical energy scale s_c at which Eq. (10) is violated. At energies above s_c, perturbation theory is not expected to be valid. This is a conservative estimate: The perturbation expansion apparently begins to fail for smaller values of s.

At energies far above and far below the Higgs-boson pole, we may neglect the width of the Higgs boson and a_0^0 is real. At the pole, however, the amplitude is infinite unless we include the width. The inclusion of the width via

$$\frac{1}{s - M_H^2} \to \frac{1}{s - M_H^2 + iM_H\Gamma_H} \quad (11)$$

corresponds to summing all orders of perturbation theory for the imaginary part of the s-channel Higgs-boson self-energy diagram. Since we are calculating to only one-loop level for the other diagrams, this prescription is not entirely consistent. Including a finite width in the manner of Eq. (11) spoils the perturbation expansion in λ of Eq. (9). When we include a width, we take a_0^0 to be the tree-level contribution with the replacement of Eq. (11), while a_0^1 is the sum of all the one-loop contributions (except for the imaginary part of the Higgs-boson self-energy diagram, which is already included in the width), also with the replacement of Eq. (11). For a heavy Higgs boson, $M_H \simeq 1$ TeV, the width is large and the effects of a finite width are significant even away from the pole. We will present our results with and without the effects of a finite width. The inclusion of a finite width yields imaginary parts for a_0^0 and a_0^1, which we include in Eq. (10). However, we do not include the imaginary part of a_0^1 that arises from the loop integrations. The effects of this term depend on the inclusion of the width in a_0^0 [see Eq. (10)], so we do not consider them reliable.

In Fig. 1 we show the $J=0$ partial wave as a function of $\sqrt{s}$ for $M_H = 1$ TeV. The solid curves correspond to $|a_0^0|$ with and without the width included in the Higgs-boson s-channel propagator. The dashed curves are for $\mathcal{A}_0^1$, also with and without the width. Note that the inclusion of the width restores unitarity near resonance (although the one-loop–corrected amplitude violates unitarity slightly), but does not affect the high-energy behavior of the amplitude. The inclusion of the width *does* affect the low-energy behavior, however, which is given correctly by the zero-width amplitude.[6] Although the tree-level amplitude approaches a constant for $s \gg M_H^2$, the one-loop–corrected amplitude increases with energy, violating unitarity at $\sqrt{s_c} \simeq 3$ TeV (ignoring the small violation near the pole). The one-loop correction is quite significant above the pole, at least a 50% increase in the amplitude, while below the pole it is more modest.

In Fig. 2 we show the same curves for $M_H = 1234$ GeV, which is the mass for which unitarity is saturated at high energy at tree level [see Eq. (6)]. This is seen in the tree-level curves which approach unity asymptotically at high energies. (The curves approach from below, in contrast with Fig. 6 of Ref. 1.) The one-loop–corrected amplitude violates unitarity for all energies above the pole.

The solid curve in Fig. 3 shows the energy at which unitarity is violated at one-loop level [see Eq. (10)], as a function of the Higgs-boson mass. For small Higgs-boson masses ($M_H \ll 1$ TeV) the critical energy is apparently very large, increasing rapidly with decreasing

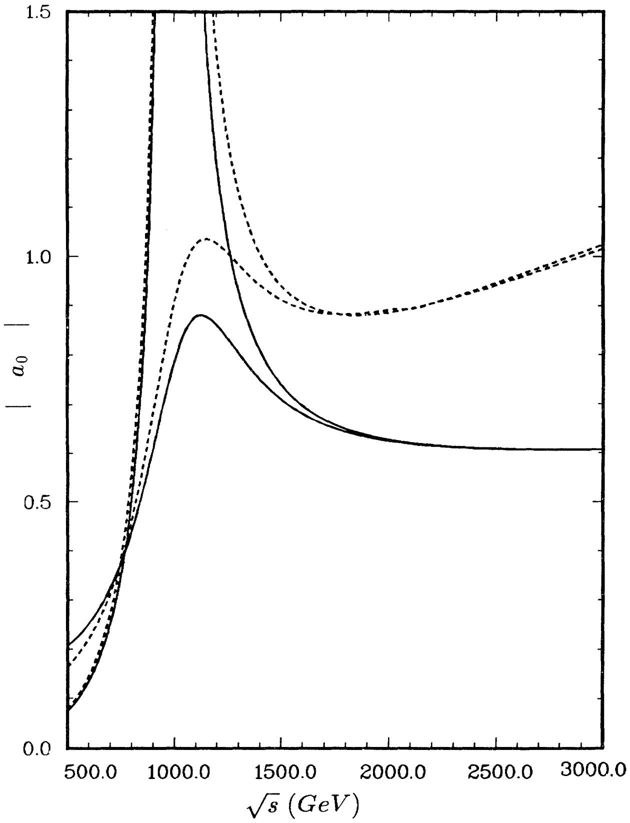

FIG. 1. $J=0$ partial wave for $W_L^+ W_L^-$ elastic scattering as a function of energy for $M_H=1$ TeV. The solid curves are the tree-level contribution, with and without the effects of a finite width. The dashed lines include the one-loop corrections, also with and without finite-width effects.

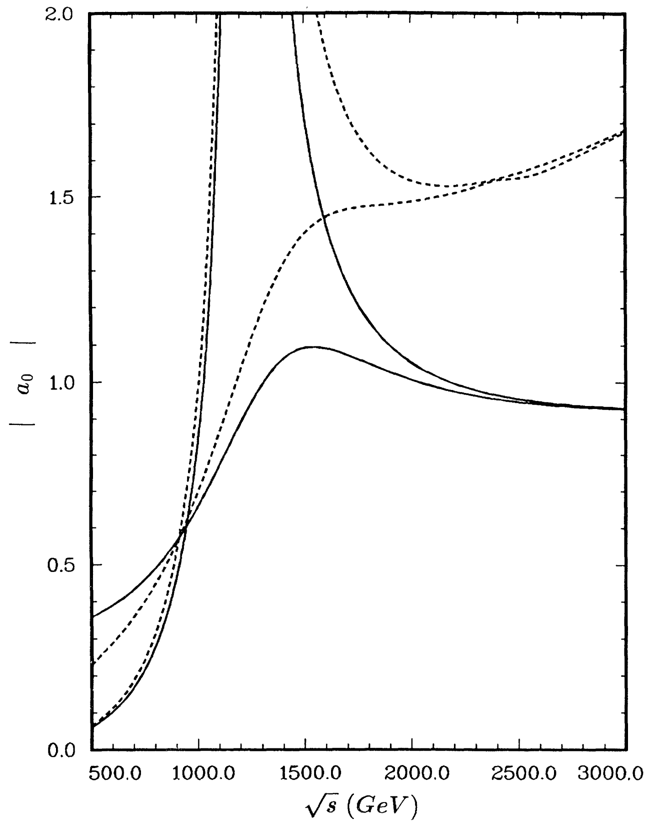

FIG. 2. Same as Fig. 1, except $M_H=1234$ GeV.

Higgs-boson mass. For large Higgs-boson masses ($M_H \gg 1$ TeV), unitarity is violated at roughly 1.4 TeV, with only a weak dependence on the Higgs-boson mass. The region in which critical energy is near the Higgs-boson mass is sensitive to the inclusion of the width, so we do not consider it to be reliable. Accordingly, we have left this region blank in Fig. 3. If unitarity is violated near the pole as well as above it, as in Fig. 1, we give only the critical energy corresponding to the latter point. This occurs for only a narrow range of Higgs-boson masses, 980 GeV $\lesssim M_H \lesssim$ 1050 GeV.

We gain some insight into these results by considering the high- and low-energy limits of our calculation. In the limits $s \gg M_H^2$, the one-loop contribution to the amplitude can be expressed in a simple form. For $s \gg M_H^2$, to $O(\lambda^2)$, we have

$$\mathrm{Re}[\mathcal{M}(W_L^+W_L^- \to W_L^+W_L^-)] = -4\lambda\left\{1-\frac{\lambda}{8\pi^2}\left[\frac{1}{2}+\frac{9\pi}{2\sqrt{3}}+\frac{5}{2}\ln\left(\frac{M_H^2}{s}\right)+\frac{5}{2}\ln\left(\frac{M_H^2}{-t}\right)+\ln\left(\frac{M_H^2}{-u}\right)\right]\right\}. \tag{12}$$

This yields, to $O(\lambda^2)$,

$$\mathcal{A}_0^1 = \frac{\lambda}{4\pi}\left\{1+\frac{\lambda}{8\pi^2}\left[6\ln\left(\frac{s}{M_H^2}\right)-4-\frac{9\pi}{2\sqrt{3}}\right]\right\}. \tag{13}$$

The one-loop amplitude grows logarithmically with increasing energy for $s \gg M_H^2$. This explain why, for $M_H \lesssim 1.23$ TeV, unitarity is violated at one-loop level, although it is respected at tree level. The critical energy determined from the high-energy approximation [Eq. (13)] is given by the dashed curve in Fig. 3. For $M_H \sim 1$ TeV, the high-energy approximation yields a poor estimate of the critical energy and one must rely upon the exact one-loop amplitude.

If $s \gg M_H^2$, the logarithm in Eq. (13) is very large and one may question the validity of the perturbative expansion. The coefficient of the logarithm is related to the β function of the $\lambda\phi^4$ theory and we can use the renormalization group to sum the leading logarithms. The result, to $O(\lambda^2(s))$, is

$$\mathcal{A}_0^1 = \frac{\lambda(s)}{4\pi}\left[1-\frac{\lambda(s)}{8\pi^2}\left(4+\frac{9\pi}{2\sqrt{3}}\right)\right], \tag{14}$$

where the running coupling is

$$\lambda(s) = \frac{\lambda}{1-(3\lambda/4\pi^2)\ln(s/M_H^2)} \tag{15}$$

and $\lambda = G_F M_H^2/\sqrt{2}$, as usual. We may regard Eq. (14) as an expansion in powers of $\lambda(s)$. We can determine the energy at which unitarity is violated at tree level by requiring $\lambda(s)/4\pi < 1$. This yields the dot-dashed curve in Fig. 3. Summing the logarithms dramatically reduces the critical energy for small Higgs-boson masses ($M_H \leq 1$ TeV). However, when $\lambda(s)/4\pi = 1$, the second term in the square brackets of Eq. (14) is -1.9, so perturbation theory is apparently not a reliable guide to the onset of unitarity violation in this regime. The one-loop correction is again large when the tree-level amplitude violates unitarity, although now it is of the opposite sign, suggesting that loop corrections could potentially restore unitarity for light Higgs bosons.

At low energies, $s \ll M_H^2$, the amplitude can again be simplified. We find, to $O(\lambda^2)$,

$$\mathrm{Re}[\mathcal{M}(W_L^+W_L^- \to W_L^+W_L^-)] = 2\lambda\left\{\frac{-u}{M_H^2} + \frac{\lambda}{8\pi^2 M_H^4}\left[\left(\frac{st}{6}+\frac{5s^2}{6}\right)\ln\left(\frac{M_H^2}{s}\right) + \left(\frac{st}{6}+\frac{5t^2}{6}\right)\ln\left(\frac{M_H^2}{-t}\right) + \frac{u^2}{2}\ln\left(\frac{M_H^2}{-u}\right) + \left(\frac{9\pi}{2\sqrt{3}}-\frac{76}{9}\right)(s^2+t^2) - \frac{4}{9}u^2\right]\right\}. \quad (16)$$

The logarithmic terms agree with those of Cheyette and Gaillard[4] who calculated the scattering amplitude in a O(4) nonlinear σ model. The nonlogarithmic terms cannot be obtained from this approach, however. The low-energy contribution to the $J=0$ partial wave is thus

$$\mathcal{A}_0^1 = \frac{\lambda s}{16\pi M_H^2}\left\{1 + \frac{\lambda}{8\pi^2}\frac{s}{M_H^2}\left[\frac{20}{9}\ln\left(\frac{M_H^2}{s}\right) + \frac{12\pi}{\sqrt{3}} - \frac{2441}{108}\right]\right\}. \quad (17)$$

Note that $\lambda/M_H^2 = G_F/\sqrt{2}$, so the expansion parameter is $G_F s$, rather than $\lambda \sim G_F M_H^2$. This is a consequence of the derivative couplings of the Goldstone bosons. Since $s \ll M_H^2$, the one-loop correction to the amplitude is much smaller below the Higgs-boson pole than above it, as shown in Figs. 1 and 2. The low-energy amplitude depends only logarithmically on the Higgs-boson mass, so the energy at which unitarity is violated at one-loop level, for $M_H \gg 1$ TeV, is roughly independent of the Higgs-boson mass. However, the low-energy approximation [Eq. (17)] yields a critical energy which agrees (within 20%) with that of the exact calculation only for $M_H > 4$ TeV. The one-loop correction significantly reduces the energy at which unitarity is violated for a very massive Higgs boson, from 2.5 TeV [see Eq. (8)] to about 1.4 TeV. The one-loop correction is very large at $\sqrt{s} = 1.4$ TeV, more than doubling the three-level amplitude.

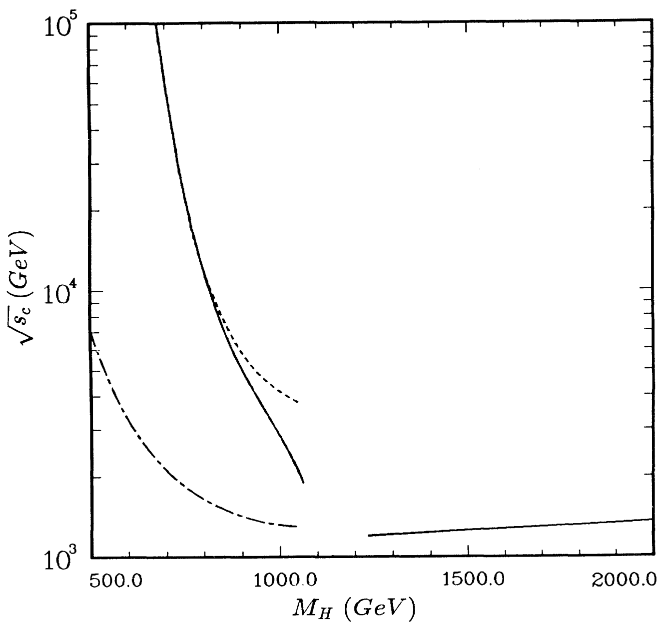

FIG. 3. Critical energy scale at which unitarity [Eq. (10)] is violated. The solid curve is the one-loop result, the dashed curve is the high-energy approximation ($s \gg M_H^2$), and the dot-dashed curve is the result of setting $\lambda(s)/4\pi = 1$.

In conclusion, we have found that for the process $W_L^+W_L^- \to W_L^+W_L^-$ there is some energy at which unitarity is violated for all Higgs-boson masses for $M_H \gg M_W$. When this occurs, the $O(G_F M_H^2)$ one-loop corrections to the amplitude are large indicating that perturbation theory breaks down.

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