Tricritical Point in Compact U(1) Lattice Gauge Theory with Dynamical Fermions

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The compact U(1) lattice gauge theory with dynamical fermions of mass $m=0.1$ and 0.05 with the gauge action $\sum_P(\beta\cos\theta_P+\gamma\cos 2\theta_P)$ is studied numerically along phase-transition lines in the β,γ plane. On an 8^4 lattice, discontinuities appearing in the internal energy and in the chiral order parameter $\langle\bar\psi\psi\rangle$ at $\gamma=0$ are shown to vanish as γ decreases to negative values, indicating an existence of tricritical points. These results are compared with high-statistics data of the internal energy in the pure gauge theory on a 16^4 lattice.

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High-energy behaviors of quantum field theories are governed by their ultraviolet fixed points. Well-known examples are asymptotically free fixed points, which give logarithmic scaling violations. Properties of these fixed points are calculable by a perturbation theory, since they are located in weak-coupling regions. In order to search for new fixed points in strong-coupling regions, however, one should rely on some nonperturbative methods. The lattice formulation of field theory provides a suitable tool for such study. Since the pioneering work of Wilson,[1] it is well known that, if fixed points are located on (multi)critical surfaces, continuum theories constructed at these points have power-law scaling violations with powers given by critical exponents. These power-law scalings are expected to revive technicolor theories[2] and to give a new scenario in grand unified theories.[3]

The first step in this approach is a search for (multi)critical points in strong-coupling regions. Recently, this problem was extensively studied in strongly coupled QED.[4-6] Kogut, Dagotto, and Kocic[4] found that noncompact QED has a second-order chiral phase transition. It was also shown[5] that compact QED with Wilson gauge action has only a first-order phase transition. The purpose of the present paper is to study a phase structure of compact QED with mixed gauge action. Previously, this model was studied by Dagotto and Kogut[6] for a restricted range in the coupling constants' space, where they found only first-order transitions. I show that the transitions become second order as one of the coupling constants decreases to large negative values. There are two main reasons to persist in compact gauge group: (i) It is not clear whether compact and noncompact QED belong to the same universality class. In fact, I show that compact QED has tricritical points whereas the data in Ref. 4 suggest that noncompact QED has critical points. (ii) In compact QED, fermions are confined in the strong-coupling phase, but this problem is not trivial in noncompact QED.

I studied the U(1) gauge theory with N_f dynamical fermions defined by the partition function

$$Z=\int[dU]e^{S_W(U)}\det[M^\dagger(U)M(U)]^{N_f/4}. \qquad (1)$$

Here S_W is the pure gauge action parametrized by two couplings β and γ,

$$S_W=\sum_P(\beta\cos\theta_P+\gamma\cos 2\theta_P)\,, \qquad (2)$$

with θ_P the plaquette angle. I used staggered fermions, so the matrix $M(U)$ is given by

$$M(U)_{i,j}=\tfrac{1}{2}\sum_\mu \eta_{i,\mu}(U_{i,\mu}\delta_{j,i+\mu}-U^*_{j,\mu}\delta_{j,i-\mu})+m\delta_{i,j}\,. \qquad (3)$$

Here i and j refer to lattice points and μ is a unit vector on the lattice. $\eta_{i,\mu}$ are the usual staggered fermion phases and m is the fermion mass. As usual, $M^\dagger M$ is defined only on even lattice points. The simulations have been done by making use of the hybrid molecular-dynamics R algorithm of Ref. 7. Let t be the evolution time of the simulation, and let Δt be the discrete time step used in the discretized molecular-dynamics equation. I took $\Delta t=0.02$ throughout this paper and refreshed the momenta every fifty iterations. Measurements were done every ten iterations.

This model was analyzed in a coupling-constant region $\beta\geq 0$ and $\gamma\leq 0$ for two values of flavors $N_f=4$ and 1, although a theoretical foundation of the simulation with $N_f\leq 3$ is not yet clear. On an 8^4 lattice, I evaluated the internal energy $E=\beta\langle\cos\theta_P\rangle+\gamma\langle\cos 2\theta_P\rangle$ and the chiral order parameter $\langle\bar\psi\psi\rangle$. The phase diagram is shown in Fig. 1 for fixed mass $m=0.1$. The diagram is divided into three regions. Region I corresponds to the chiral-symmetric phase. In the chiral limit $m=0$, $\langle\bar\psi\psi\rangle$ should become identically zero. Region II corresponds to the phase where chiral symmetry is spontaneously broken. Region III is the antiferromagnetic phase. Topologically, the phase diagram is quite similar to that of pure U(1) gauge theory,[8] where regions I and II are the photon and confining phases, respectively. On the line AB which separates regions II and III, the transitions are always first order. They were located simply by thermal cycles. Two transition lines for $N_f=4$ and 1 are overlapping within large error bars. The line CD separates regions I and II for $N_f=4$, and the line EF for $N_f=1$. The upper-half plane corresponding to $1/g_0^2\equiv\beta+4\gamma\geq 0$

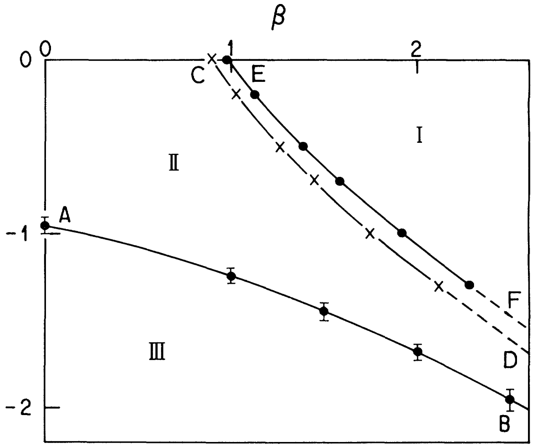

FIG. 1. Phase diagram of the theory in the β, γ plane.

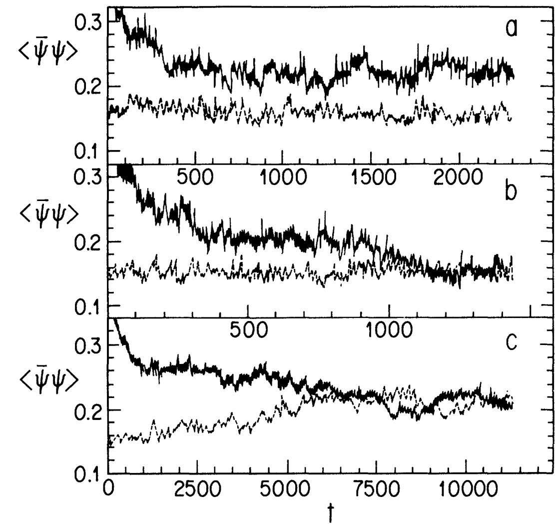

FIG. 2. Time evolutions of the order parameter $\langle\bar{\psi}\psi\rangle$ for $N_f=4$. (a) $\gamma=-0.7$, $\beta=1.439$; (b) $\gamma=-0.7$, $\beta=1.45$; and (c) $\gamma=-1.3$, $\beta=2.11$.

has been studied by Dagotto and Kogut.[6] In this region, they found that the lines CD and EF are always first order and thus useless. Figure 1 shows that the region with $g_0^2<0$ is analytically continued to the region with $g_0^2>0$. This result is quite natural since weak perturbation constraints such as $g_0^2 \geq 0$ have nothing to do with the phase structure in the strong-coupling region.

To study the nature of the transitions along the lines CD and EF, I measured discontinuities appearing in energy E and $\langle\bar{\psi}\psi\rangle$ at five values of γ at fixed mass $m=0.1$. I performed long runs starting from both ordered and disordered configurations for several values of β near transition couplings $\beta_c=\beta_c(\gamma)$. The values $\beta_c(\gamma)$ are summarized in Table I. At $\gamma=0.0$, -0.2, -0.7, and -1.0, I observed clear two-state signals both for $N_f=4$ and 1, although measurements became more and more difficult as γ decreased due to long relaxation times. Frequently, I observed two-state signals for more than one value of β within narrow intervals. In those cases, the values of β_c in Table I represent the center values of those intervals. As examples, in Figs. 2(a) and 2(b), I plot two typical time evolutions of $\langle\bar{\psi}\psi\rangle$ at $\gamma=-0.7$ with $N_f=4$ (one time unit corresponds to fifty sweeps per site). Figure 2(a) shows the coexistence of two stable states and Fig. 2(b) shows one stable and one metastable state. In Figs. 3 and 4, I plot the discontinuities ΔE and $\Delta\langle\bar{\psi}\psi\rangle$ as functions of γ. Errors came both from statistical fluctuations within an individual run and from uncertainties in the determinations of β_c. It is clear that the sizes of discontinuities decrease as γ decreases.

At $\gamma=-1.3$, I did not find any evidence of two-state signals both for $N_f=4$ and 1. There are no discontinuities in E and $\langle\bar{\psi}\psi\rangle$. Figure 2(c) shows typical time evolutions of $\langle\bar{\psi}\psi\rangle$ near critical coupling. In Fig. 5, I plot $\langle\bar{\psi}\psi\rangle$ as a function of β for $N_f=4$. For $N_f=1$, I made long runs of order 8000 time steps at $\beta=2.27$, 2.28, and 2.29 and short runs for other values of β, obtaining a smooth curve such as in Fig. 5. Near critical couplings, relaxation times are very large. Typically, two runs starting from ordered and disordered configurations converge to a final value after performing more than 6000

TABLE I. The transition couplings $\beta_c(\gamma)$ as functions of γ for $N_f=4$, 1, and 0.

γ \ N_f	4	1	0
0.0	0.892	0.979	1.0105
−0.2	1.022	1.12	1.16
−0.7	1.439	1.58	1.6235
−1.0	1.74	1.92	1.973

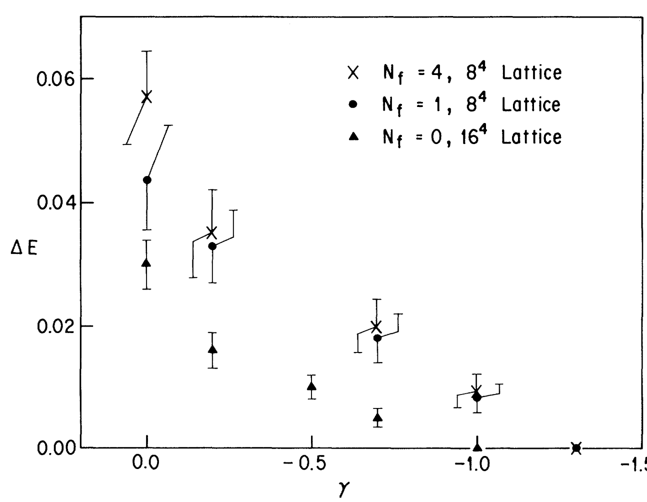

FIG. 3. The discontinuity ΔE as a function of γ.

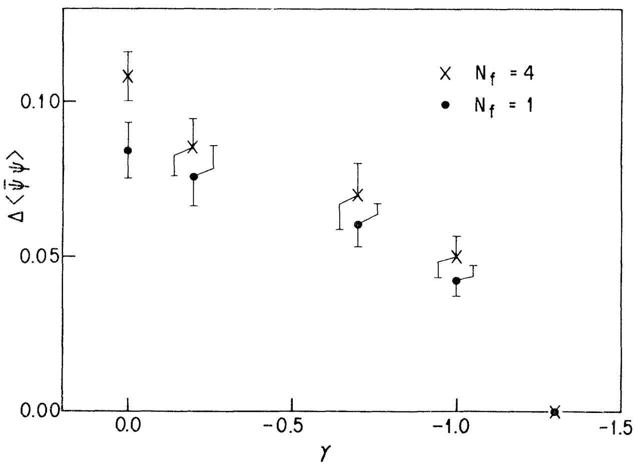

FIG. 4. The discontinuity $\Delta\langle\bar{\psi}\psi\rangle$ as a function of γ.

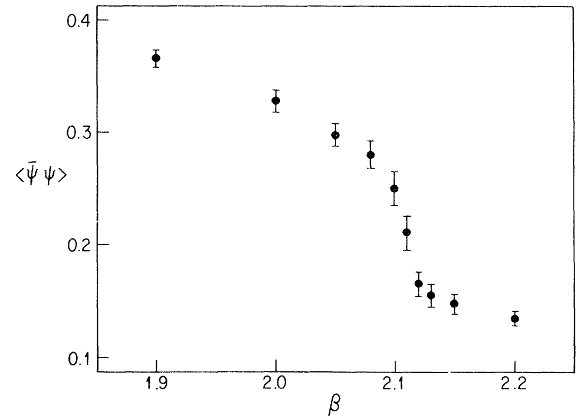

FIG. 5. The order parameter $\langle\bar{\psi}\psi\rangle$ as a function of β at $\gamma=-1.3$ and $N_f=4$.

time steps. (I found that at $\gamma=-0.7$ and -1.0, the system reached equilibrium after 400 and 1400 times steps, respectively.)

The above data strongly indicate that there are tricritical points between $\gamma=-1.0$ and -1.3, where the transitions become second order from first order. As one approaches the second-order critical lines, correlation functions diverge and continuum field theories can be constructed along these lines.[1] Before concluding this paper, I discuss several points which require further investigation.

(i) I cannot rule out the possibility that very small discontinuities have been missed at $\gamma=-1.3$ due to the use of an 8^4 lattice, although it is hard to imagine that discontinuities remain finite as γ goes to $-\infty$. It is instructive to study the finite-size effects for $N_f=0$. In Fig. 3, I also plot the data of ΔE measured in the pure U(1) gauge theory on a 16^4 lattice. The standard Metropolis Monte Carlo method was used and U(1) gauge group was approximated by Z_N with $N=300$.[9] At $\gamma=0.0$, -0.2, -0.5, and -0.7, clear two-state signals were observed. At $\gamma=-1.0$, no clear evidence of first-order transitions was seen. I repeated the analysis on an 8^4 lattice. Clear two-state signals were seen only at $\gamma=0.0$ and -0.2. This means that, at least by the Metropolis method, an 8^4 lattice is too small to detect discontinuities of order $\Delta E \simeq 0.01$. Study of compact QED on a larger lattice is desirable.

(ii) At tricritical points, γ has large negative values. This means that the actions defined at the tricritical points do not satisfy reflection positivity.[10] Although this condition is only a sufficient condition to ensure a bounded spectrum for a transfer matrix, it is necessary to check whether the resultant continuum theory in a Wick-rotated Minkowski space-time is unitary. Concerning this problem, it is worth mentioning my preliminary result on a Monte Carlo renormalization-group (MCRG) study of the pure U(1) gauge theory. The system on a 16^4 lattice with $\beta=1.973$ and $\gamma=-1.0$ was blocked into an 8^4 lattice. The renormalized coupling constants were determined in a space of twelve operators. They are $\beta_{\text{ren}}=0.902$, $\gamma_{\text{ren}}=0.04$, and for other couplings $|\beta_{\text{ren}}^{\text{other}}|<0.07$. Since the renormalized system describes the same physics as the unrenormalized system, the large negative value of unrenormalized γ is only an artifact in a space of two coupling constants. A MCRG study of compact QED is desired. This method is also useful in determining the orders of transitions since renormalization-group flows at second- and first-order transitions are known to be quite different.[11]

(iii) Usually, at tricritical points, there are two relevant operators.[12] If this is the case for compact QED, Fig. 5 is not enough to obtain critical exponents, since I approached the tricritical points only from one direction. Critical exponents govern a scaling law of high-energy behaviors of continuum theories. In particular, nontrivial field theories should have non-Gaussian exponents. More simulations with high statistics and careful analysis on finite-size effects are required to evaluate critical exponents.

(iv) In the above analysis, I fixed the fermion mass at $m=0.1$. To study the finite-mass effect, I made preliminary simulations at $m=0.05$ with $N_f=4$. By measuring discontinuities at $\beta_c(\gamma=-0.7)=1.41$ and $\beta_c(\gamma=-1)=1.71$, I found that $\Delta E_{m=0.05}\simeq\Delta E_{m=0.1}$ and $\Delta\langle\bar{\psi}\psi\rangle_{m=0.05}\simeq 1.6\Delta\langle\bar{\psi}\psi\rangle_{m=0.1}$ for both $\gamma=-0.7$ and -1. Since discontinuities vanish as $A(\gamma-\gamma_t)^B$ near tricritical point γ_t, the above data suggest that the position of the tricritical point does not depend crucially on fermion mass. Of course, more study of the finite-mass effect near tricritical points is necessary to obtain conclusive results.

In conclusion, I have shown that compact QED has tricritical points. As was pointed out in the beginning of the paper, this work is the first step in the construction of new consistent field theories in strong-coupling regions.

There remain important questions such as whether the continuum theories constructed at tricritical points are unitary and nontrivial. In further work, these problems should be pursued based on the present paper.

[1]K. G. Wilson, Phys. Rev. B **4**, 3174 (1971); **4**, 3184 (1971); K. G. Wilson and J. B. Kogut, Phys. Rep. **12C**, 75 (1974).

[2]B. Holdom, Phys. Rev. D **24**, 1441 (1981); K. Yamawaki, M. Bando, and K. Matumoto, Phys. Rev. Lett. **56**, 1335 (1986).

[3]H. Georgi and S. L. Glashow, Phys. Rev. Lett. **47**, 1511 (1981).

[4]J. Kogut, E. Dagotto, and A. Kocic, Phys. Rev. Lett. **60**, 772 (1988).

[5]J. Kogut and E. Dagotto, Phys. Rev. Lett. **59**, 617 (1987).

[6]E. Dagotto and J. Kogut, Nucl. Phys. **B295 [FS21]**, 123 (1988).

[7]S. Gottlieb, W. Liu, D. Toussaint, R. L. Renken, and R. L. Sugar, Phys. Rev. D **35**, 2531 (1987).

[8]G. Bhanot, Nucl. Phys. **B205 [FS5]**, 168 (1982).

[9]These simulations were done for a Monte Carlo renormalization-group study of the pure U(1) gauge theory with A. Gonzalez-Arroyo. More details on the simulations will be discussed in a separate paper. For previous works, see H. G. Evertz, J. Jersák, T. Neuhaus, and P. Zerwas, Nucl. Phys. **B251 [FS13]**, 279 (1985); R. Gupta, M. A. Novotny, and R. Cordery, Phys. Lett. B **172**, 86 (1986); A. N. Burkitt, Nucl. Phys. **B270 [FS16]**, 575 (1986); C. B. Lang, Nucl. Phys. **B280 [FS18]**, 255 (1987); A. Hasenfratz, Phys. Lett. B **201**, 492 (1988).

[10]K. Osterwalder and E. Seiler, Ann. Phys. (N.Y.) **110**, 440 (1978).

[11]A. Gonzalez-Arroyo, M. Okawa, and Y. Shimizu, Phys. Rev. Lett. **60**, 487 (1988); A. Hasenfratz and P. Hasenfratz, Nucl. Phys. **B295 [FS21]**, 1 (1988).

[12]See, for example, I. D. Lawrie and S. Sarbach, in *Phase Transitions and Critical Phenomena,* edited by C. Domb and J. L. Lebowitz (Academic, London, 1984), Vol. 9. Strictly speaking, it may also happen that there is one relevant and one marginal operator. In fact, it is possible that the pure U(1) gauge theory belongs to this category (see Ref. 9).