Local Growth of Quasicrystals

Most growth processes of physical interest are *local* in the sense that the growth probability at a given site depends only on its local (bounded) neighborhood. A recent Letter[1] claimed the discovery of a local algorithm for growing perfect Penrose tilings[2,3] (PPT). We wish to draw attention to related work by Penrose,[4] demonstrating that such algorithms are impossible, and to shift the focus to the study of defect formation by a more physical growth.

Using the inflation symmetry, Penrose[4] traced the nonlocality of PPT growth to the ambiguity in Fibonacci sequences of Conway "worms,"[3] which bound any forced patch of PPT.[5] Consequently, despite its "local" classification of growth sites, the algorithm introduced in Ref. 1, *cannot* be local precisely because it generates PPT. Its first step requires going around a boundary of a PPT patch, placing all the forced tiles first.[1,3] In the second step, a tile is attached to a 108°-corner site only after an inspection of the *entire surface* assures that no forced sites are left—a clearly nonlocal move. Except when a local growth starts on some special defective seeds,[1,3,5] defects are necessarily created, as illustrated in Fig. 1, which was generated using an Eden growth algorithm with sticking rates $p_f=1$, $p_{108}=0.1$, and $p_o=0.01$ for forced, 108°-corner, and other sites, respectively.[6] Arbitrarily long tears[7] appear after a sufficiently long time even for $p_o=0$ and $p_f/p_{108}\gg 1$. While single-tile defects start appearing at times $\sim(1/p_{108})^2$, the initial growth repeatedly ceases for periods $\sim 1/p_{108}$, leading to unphysically long growth times (there is no growth in the limit $p_{108}=0$): Assuming $p_f/p_{108}\sim e^{60}$, it takes approximately e^{37} times longer to grow a PPT of 10^{23} tiles than a comparable periodic tiling. Most of the single-tile defects can be moved by flips of Conway worms, suggesting the importance of solitonlike dynamics in equilibration of quasicrystals. A study of formation, distribution, and relaxation of defects[6] is essential for a resolution of the current controversy over the nature of disorder in quasicrystals.[1,7,8]

We acknowledge hospitality of the International Centre for Theoretical Physics and the Scuola Internazionale Superiore di Studi Avanzati in Trieste and partial support through the Texas Advanced Technology and Texas A&M Materials Science Programs.

Marko V. Jarić
Center for Theoretical Physics
Texas A&M University
College Station, Texas 77843

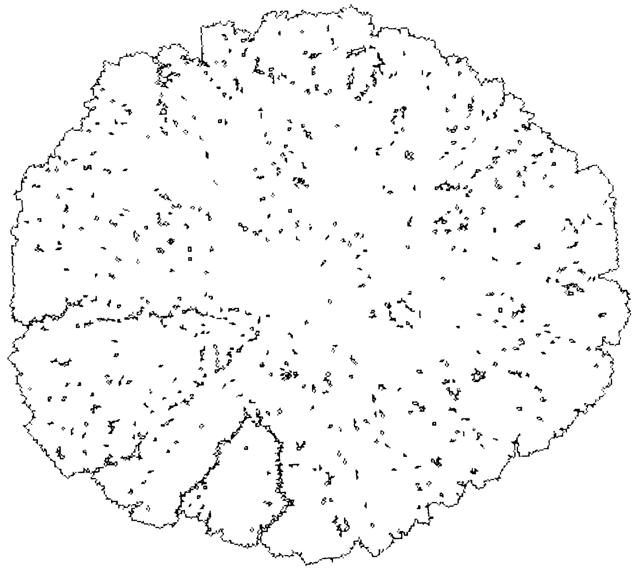

FIG. 1. A 5000-site boundary of a tiling produced by a local growth algorithm.

Marco Ronchetti
Dipartimento di Fisica
Universitá degli Studi di Trento
38050 Povo, Italy

Received 29 August 1988
PACS numbers: 61.50.Em, 61.55.Hg, 64.70.Pf

[1]G. Y. Onoda, P. J. Steinhardt, D. P. DiVincenzo, and J. E. S. Socolar, Phys. Rev. Lett. **60**, 2653 (1988).

[2]R. Penrose, Bull. Inst. Math. Appl. **10**, 266 (1974); Math. Intelligencer **2**, 32 (1979).

[3]M. Gardner, Sci. Am. **236**, 110 (1977).

[4]R. Penrose, in *Aperiodicity and Order, Vol. 1, Introduction to Mathematics of Quasicrystals,* edited by M. V. Jarić (Academic, Boston, 1988).

[5]B. Grünbaum and G. C. Shephard, *Tilings and Patterns* (Freeman, New York, 1987).

[6]For details see M. Ronchetti and M. V. Jarić (unpublished).

[7]V. Elser, in *Aperiodicity and Order, Vol. 3, Icosahedral Structures,* edited by M. V. Jarić (Academic, Boston, 1988), and references therein.

[8]P. W. Stephens and A. I. Goldman, Phys. Rev. Lett. **56**, 1168 (1985); **57**, 2331 (1986); **57**, 2770 (1986); P. J. Steinhardt, *ibid.* **57**, 2769 (1986).